Lecture 20: Equivalent Circuits

- Logic Gates
- Equivalent Circuits
- Karnaugh Maps

Logic Gates

- Electronic devices that produce an output that is a simple Boolean function of its input symbols.
- Hardware basis of digital computers.

Basic Gates

- Tannenbaum, Figure 3-2.

Exclusive OR

- Tanenbaum Figure 3-8

Example

- Distributive Law:
  A + BC = (A+B)(A+C)
  (we proved this in class)

Complete Boolean Functions

- Design and fabrication of circuits are simpler if only 1 or 2 kinds of gates are used.
- We still need to be able to implement any Boolean Function.
- A functionally complete set of gates is a set where the three basic functions (AND, OR, NOT) can be synthesized.

- Functionally complete sets:
  - AND, OR, NOT
  - AND, NOT
  - OR, NOT
  - NAND
  - NOR
- Example: for AND, NOT to be complete, we must be able to build OR from them (use DeMorgan's theorem):

- NAND and NOR are sufficient by themselves!
- Tannenbaum Figure 3-4.
- NAND and NOR are often the preferred building blocks for systems.

Example – Majority Function

- Majority function – are there more 0's or 1's?

A	B	C	M
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

- Use the combinations giving 1 to draw the equation in Sum of Products (SOP) Form

Drawing a Circuit from SOP Form

1. define the truth table and write the equation in sum-of-products form.
2. provide an inverter to generate the complement of each input
3. to make diagrams easier to draw, draw a vertical wire for each input and for its complement
4. draw an AND gate for each term in the equation
5. wire the AND gates to the appropriate inputs
6. feed the outputs of all AND gates to an OR gate

Simplification of Circuits

- Algebraic simplification
- Karnaugh Maps

Karnaugh Map Formats

- Figure 2-1, AoA

Warning!

- Take a closer look at the table formats.
- The progression of values is 00 01 11 10!
- If you use the normal progression of values (00 01 10 11), the map will not work!
- This is a common error with using Karnaugh Maps.

Using Karnaugh Maps

- Put a 1 in the appropriate box in the Karnaugh Map for each term in the equation.
- Find terms that can be combined by circling groups of adjacent 1's; enclose as many 1's as possible in groups of powers of 2 (groups of 8, groups of 4, groups of 2, groups of 1)
  - Use the minimal number of circles needed
  - Circles may overlap
  - Better to have larger overlapping circles than smaller ones that do not overlap!
- Your resulting function will have as many terms as there are circles.

- majority function example

- example 2-4 to 2-6 (AoA)

- example 2.7 – 2.9 (AoA)

- partial pattern list for 4x4 (AoA)

- 4-element examples

Simplification is More than Gate Count!

- example from Hill & Peterson