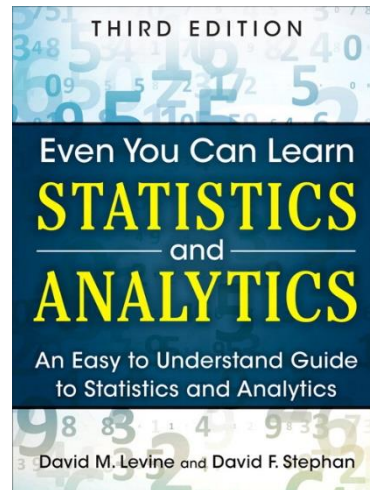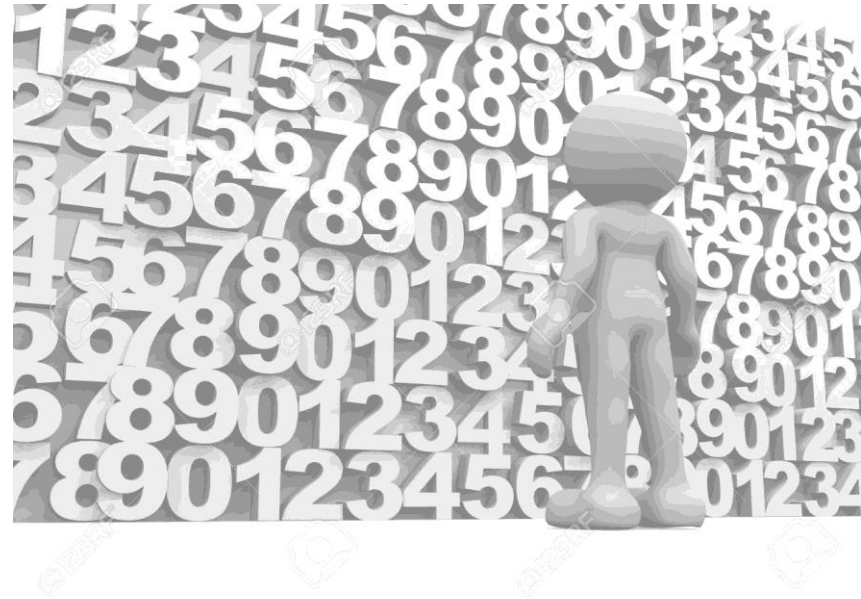# IMGD 2905

Descriptive Statistics

Chapter 3

Summarizing Data

- With lots of playtesting, there is a lot of data
  - This is a good thing!
- But raw data is often just a pile of numbers
  - Rarely of interest
  - Or even sensible
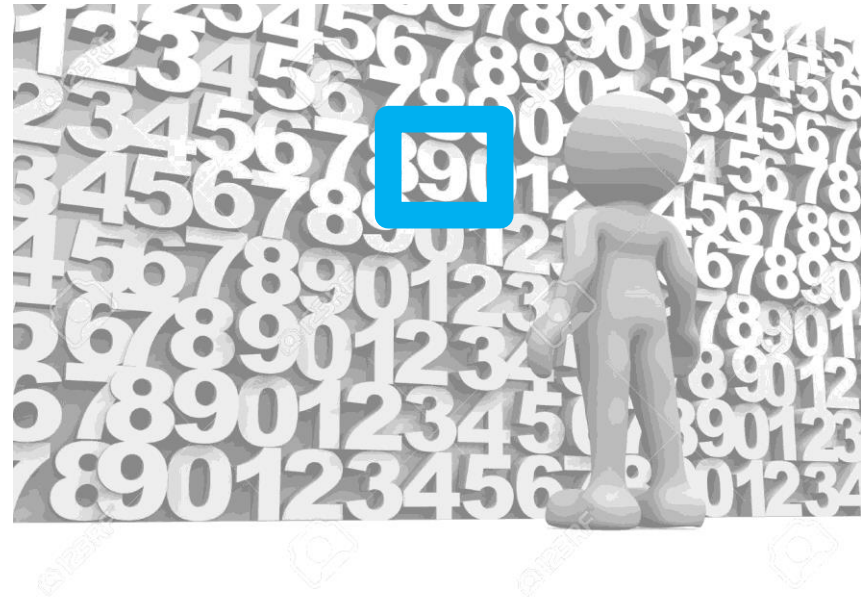- Q: How to summarize all this information?

Summarizing Data

- With lots of playtesting, there is a lot of data
  - This is a good thing!
- But raw data is often just a pile of numbers
  - Rarely of interest
  - Or even sensible
- Q: How to summarize all this information?

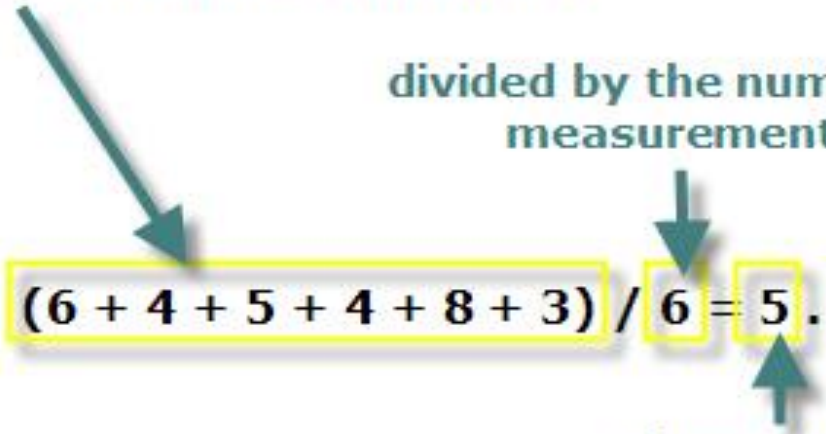Measures of central tendency

Examples? Pros and Cons?

Measure of Central Tendency: Mean

The sum of the measurements

divided by the number of measurements

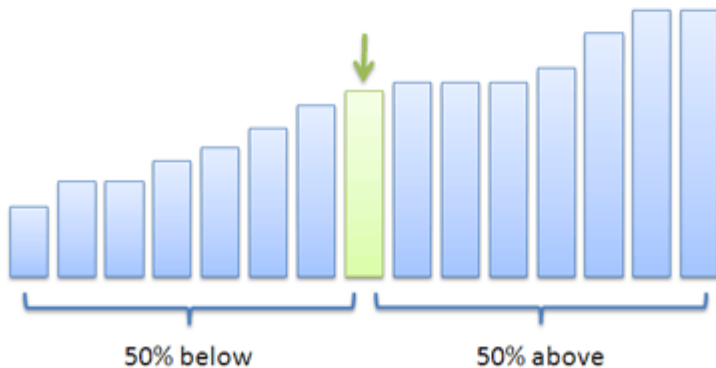$(6 + 4 + 5 + 4 + 8 + 3) / 6 = 5.$

gives you the mean.

http://www.cdn.sciencebuddies.org/Files/463/9/MeanEquation.jpg

- Also called the "arithmetic mean" or "average"

- In Excel, `=AVERAGE(range)`
  `=AVERAGEIF()` – averages if numbers meet certain condition

Measure of Central Tendency: Median

- Sort values low to high and take middle value

Median

10 11 13 **15** 16 23 26

middle number

50% below 50% above

13 22 26 38 36 42 49 50 77 81 98 110

Median = 45.5

- In Excel, =MEDIAN(range)

Measure of Central Tendency: Mode

- Number which occurs most frequently

- Not so useful in many cases

- → Best use for categorical data
  - e.g., most popular Champion group in League of Legends

- In Excel, =MODE()

6, 3, 9, 6, 6, 5, 9, 3

Cedar, Alder, Cedar, Pine, Cedar,
Cedar, Alder, Alder, Pine, Cedar

mode → Alder - III
      (Cedar) - IX
      Pine - II
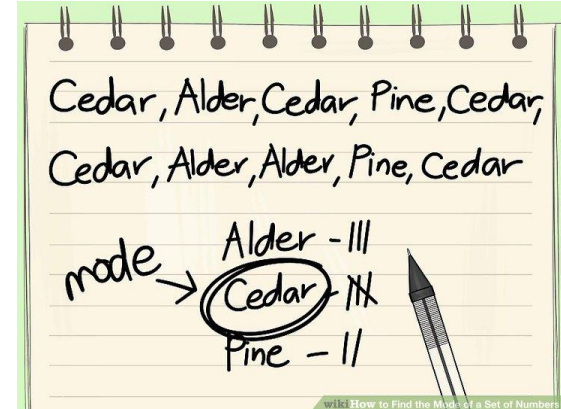
Depiction: Mean, Median, Mode?
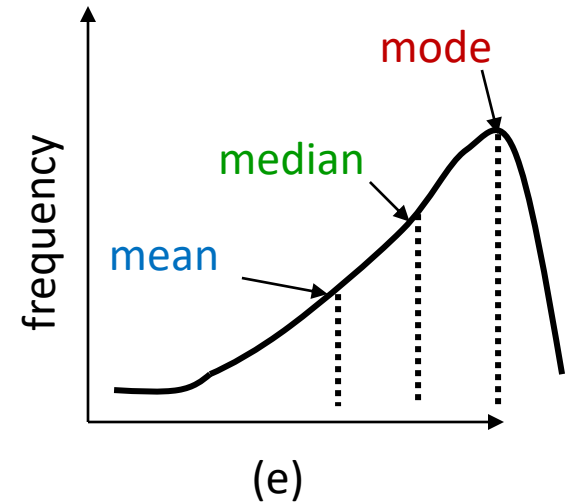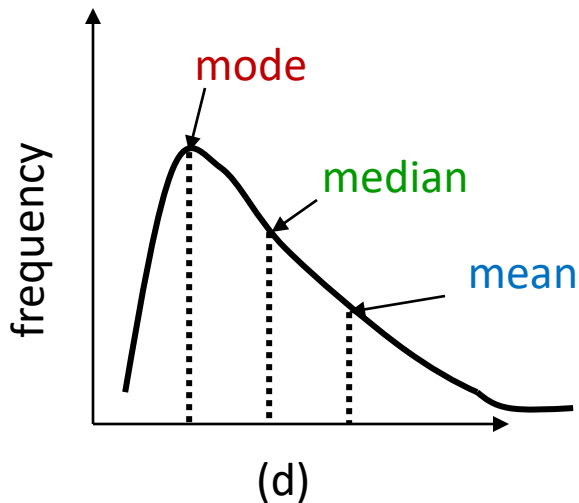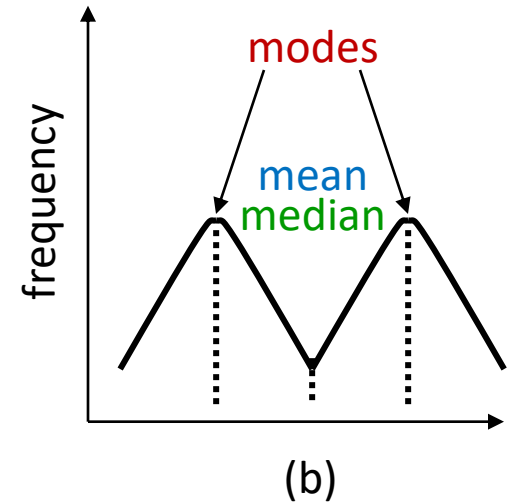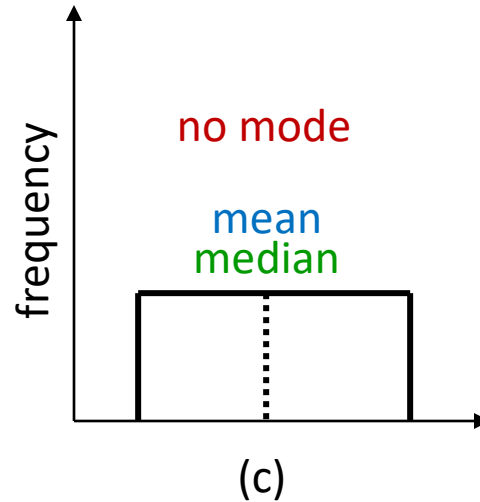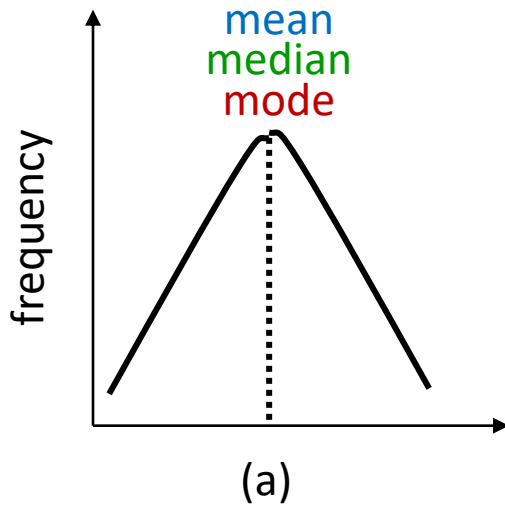
Which to Use, Mean, Median, Mode?

Which to Use, Mean, Median, Mode?

- Mean many statistical tests with sample
  - Estimator of population mean
  - Uses all data
- Median is useful for skewed data
  - e.g., income data (US Census) or housing prices (Zillo)
  - e.g., *Overwatch* team (6 players): 5 people level 5, 1 person level 275
    - Mean is 50 - not so useful since no one at this level
    - Median is 5 - more representative
  - Does not use all data. "Resistant" to extremes (e.g., 275)
  - But what if were exam scores? Hard to "bring up" grade
- Mode is useful primarily for categorical data only
  - Most played League champion, most popular maze, …

Other Measures of Position

- May not always want center
  - e.g., want to know weapon that gets most kills in PUBG
- What other positions may be desired?
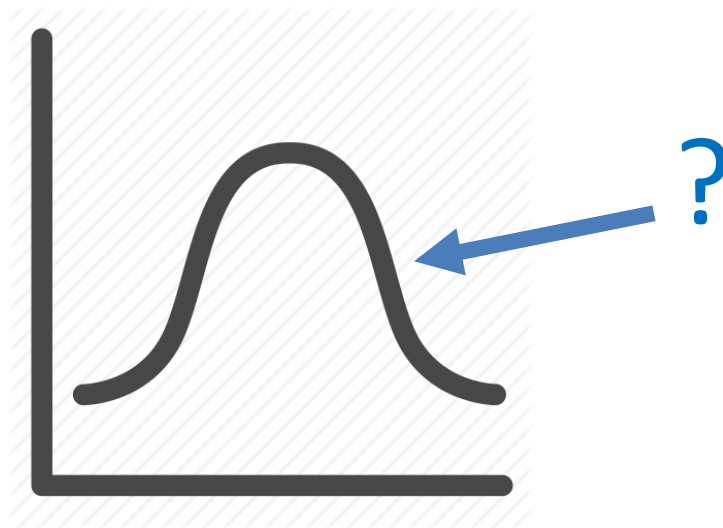
Other Measures of Position

- May not always want center
  - e.g., want to know best LoL Champions

?

- Maximum / Minimum
  - Not discussed more
- Trimmed Mean
- Quartiles
- Percentiles
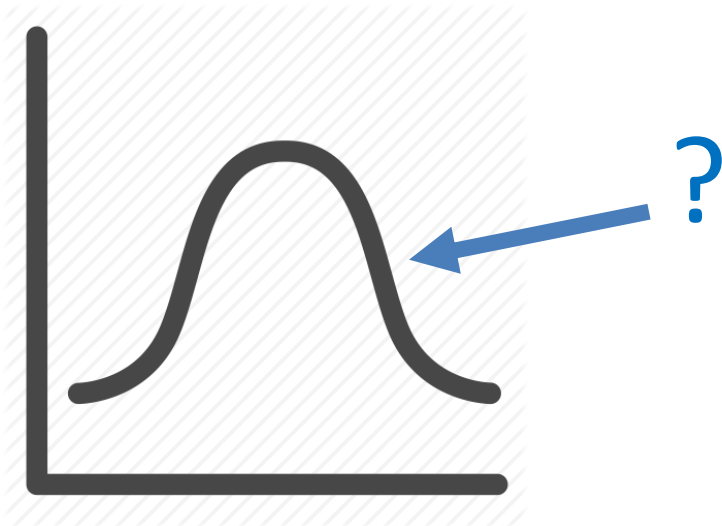
Trimmed Mean

- Take "trimming" off top and bottom (typically 5% or 10%)
  - Reduces effects of extreme values, like median
- In Excel, =TRIMMEAN(array,percent)
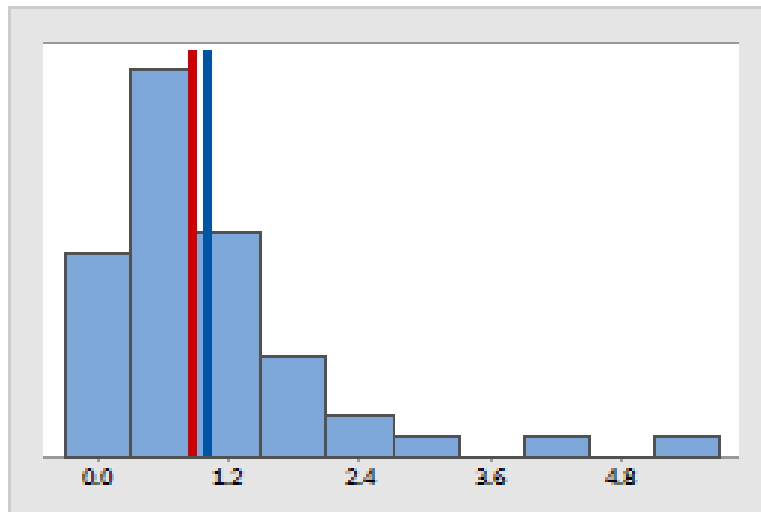
Blue – original mean
Red – trimmed mean

http://support.minitab.com/en-us/minitab/17/histogram_mean_vs_trimmed_mean.png

Quartiles

- Sort values
- First quartile (Q1) is 25% from bottom
- Third quartile (Q3) is 75% from bottom
- (What is second quartile?)
- In Excel, =QUARTILE(array,n)

https://www.hackmath.net/images/quartiles.png

	First Quartile		Median Second Quartile		Third Quartile	
First Quarter		Second Quarter		Third Quarter		Fourth Quarter
24, 25, 26,		27, 30, 32,		40, 44, 50,		52, 55, 57
	Q_1 26½			Q_2 36		Q_3 51

MathBits.com

https://mathbitsnotebook.com/Algebra1/StatisticsData/quartileboxview2.png
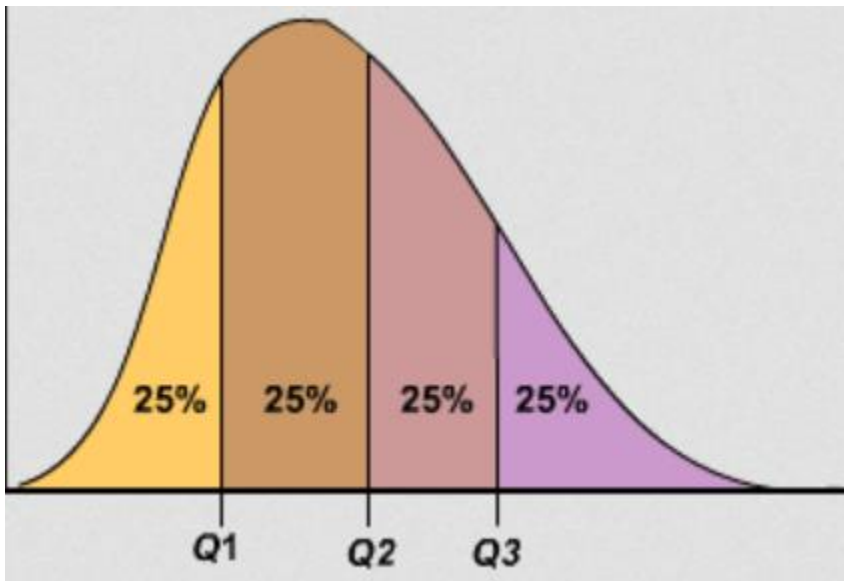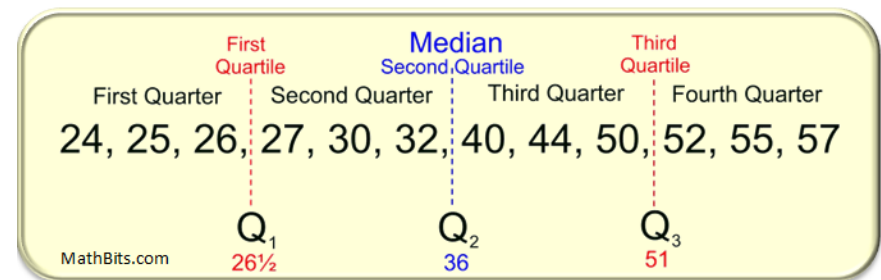
Percentiles

- Generalization of quartiles
- N^{th} percentile is data point n% from bottom of data
- Interpolate as for first quartile
- In Excel, =PERCENTILE(array,k) (k: 0 to 1)
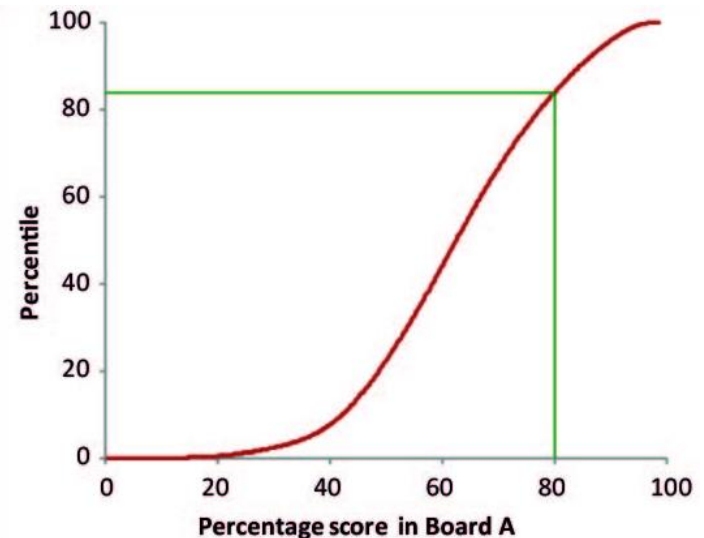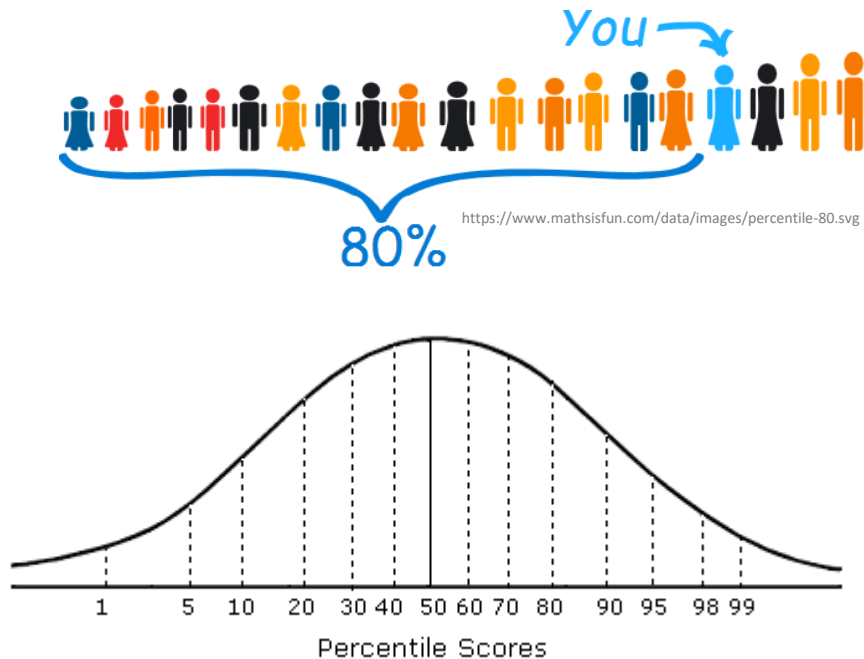
You →

80%

https://www.mathsisfun.com/data/images/percentile-80.svg

Percentile Scores

1   5   10   20   30 40   50 60 70   80   90   95   98 99

http://www.psychometric-success.com/images/AA1301.gif

Percentile

Percentage score in Board A

http://www.isical.ac.in/~jeexiiscore_normal/PercentilesAdvantages.htm

Summarizing Data, Part 2

- Ok, pile of numbers can now be summarized as *one* number
  - Mean, median, mode
- But is that enough?
- Q: What other major aspect of numbers haven't we summarized?

Summarizing Data, Part 2

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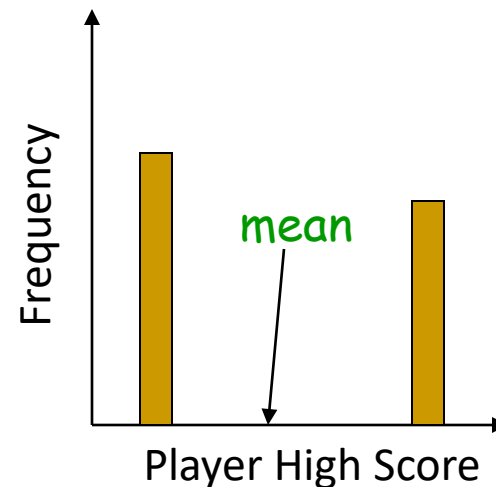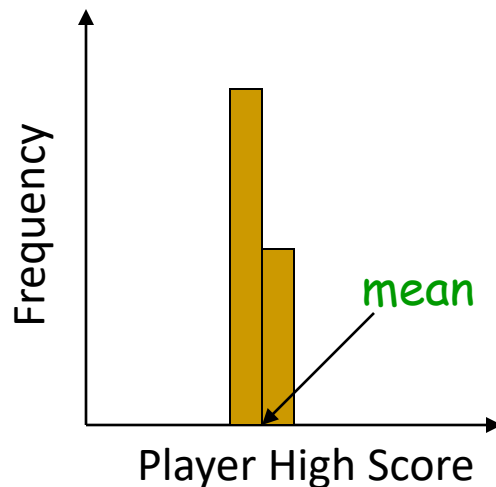
Measures of variation
(*aka* measures of *dispersion,* or measures of *spread*)

Summarizing Data, Part 2

"Then there is the man who drowned crossing a stream with an average depth of six inches." – W.I.E. Gates

- Summarizing by single number rarely enough → need statement about dispersion (aka variation)

Above: does single number (mean) tell you enough about data?

- Is data clumped or spread out?

Less Spread

https://mathbitsnotebook.com/Algebra1/StatisticsData/STSpread.html

More Spread

MathBits.com

MathBits.com

Mean: 10
Stdev: 2

Mean: 12
Stdev: 5

Cumulative Probability

http://rovdownloads.com/blog/tips-on-interpreting-pdf-cdf-and-icdf-3/
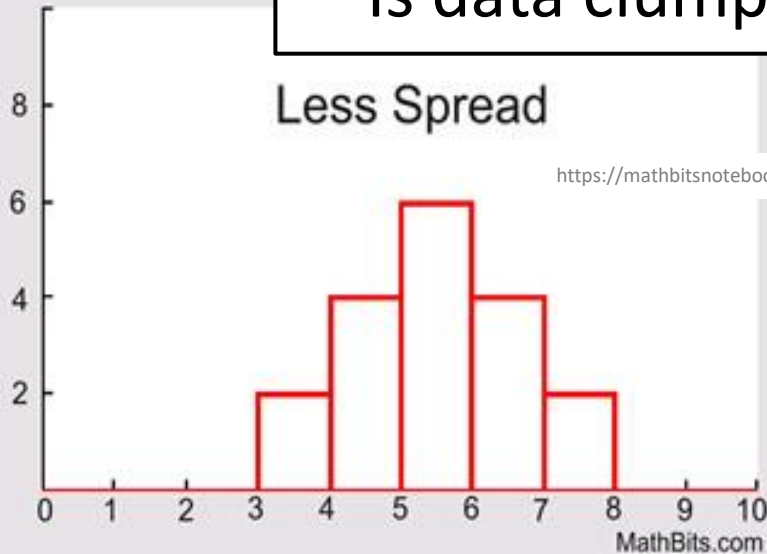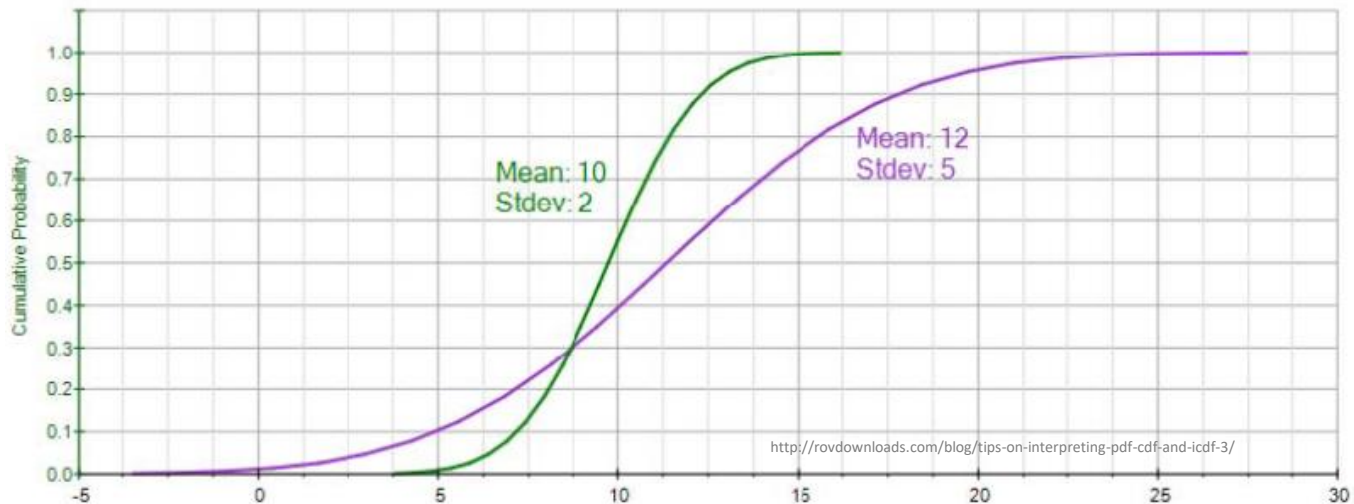
Dispersion Overview (2 of 3)

Is data clumped or spread out?
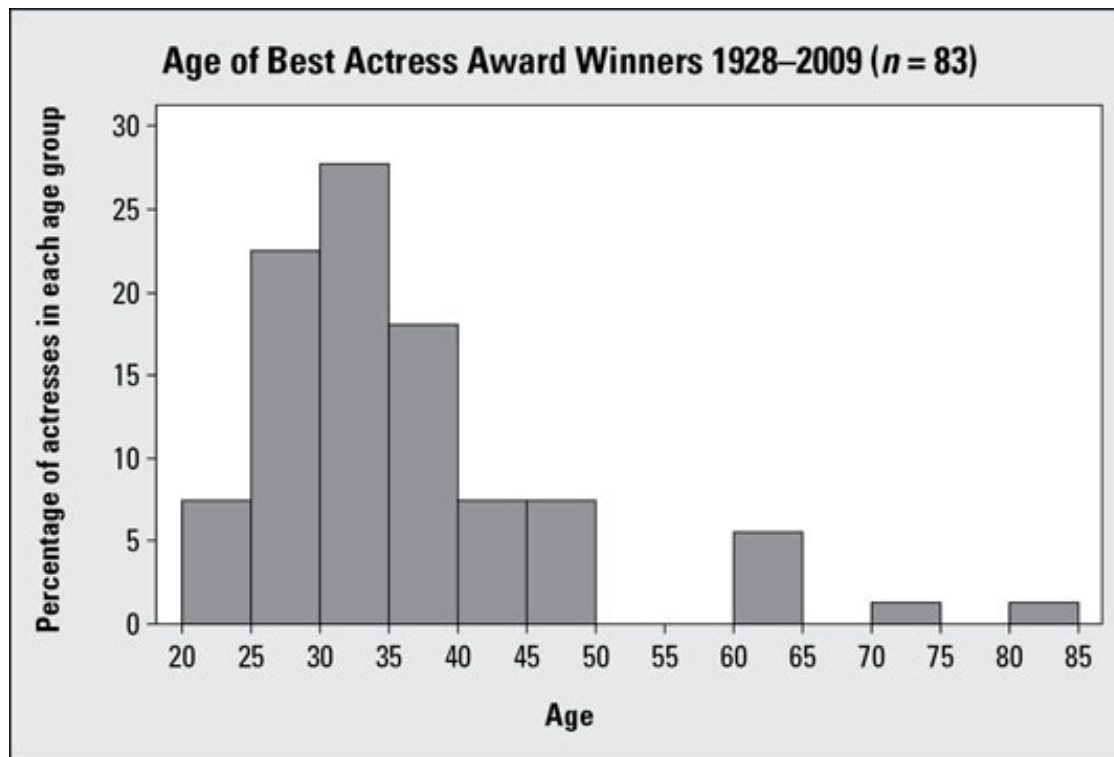
Dispersion Overview (3 of 3)

Is data clumped or spread out?
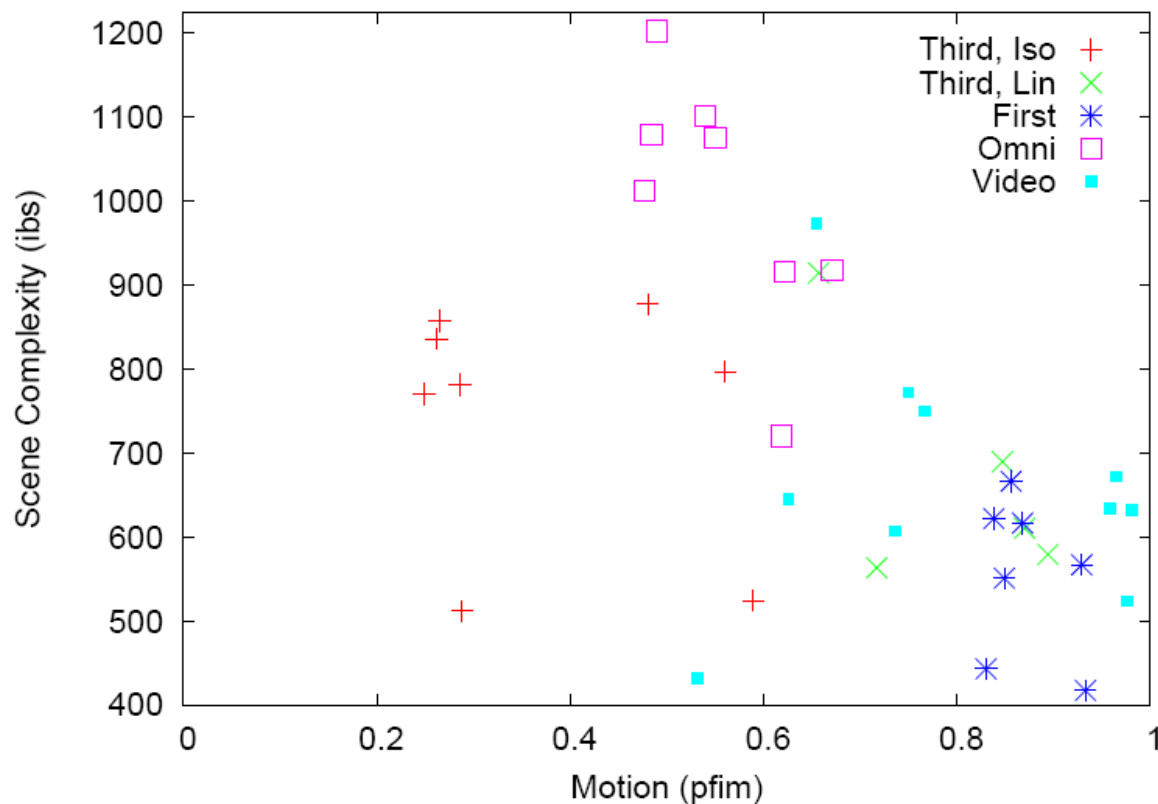
"Motion and Scene Complexity for Streaming Video Games"

What are Some Measures of Dispersion? → Groupwork
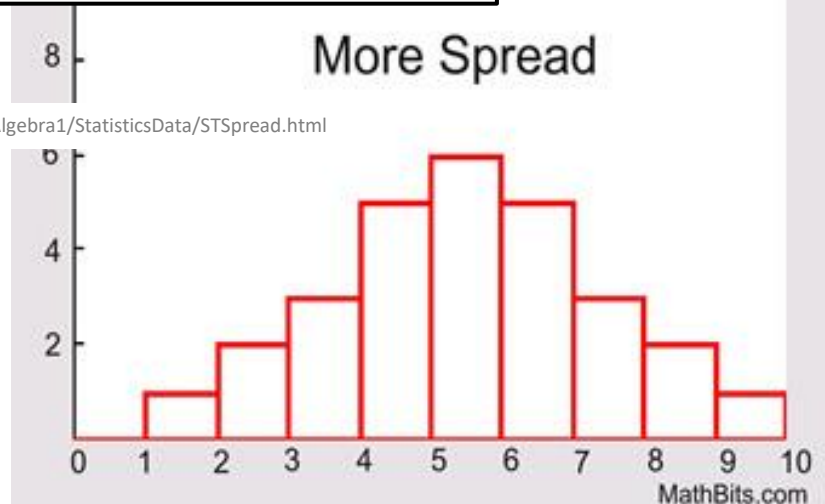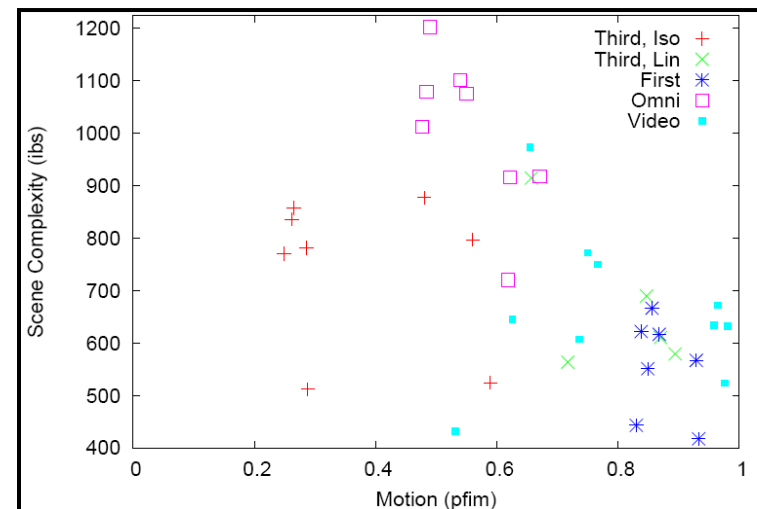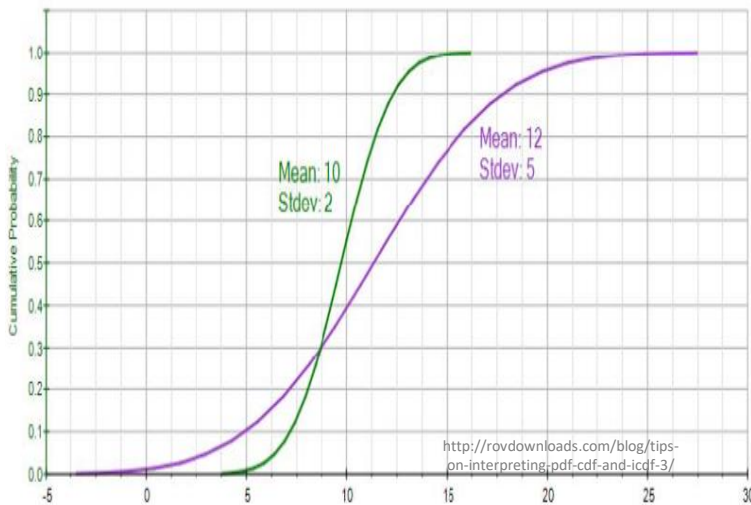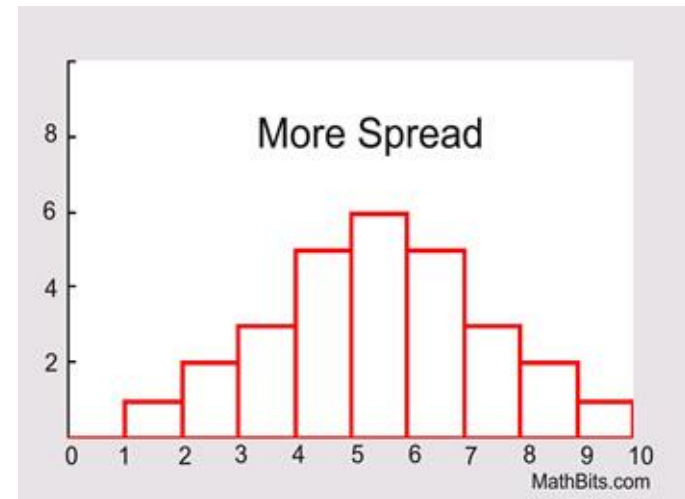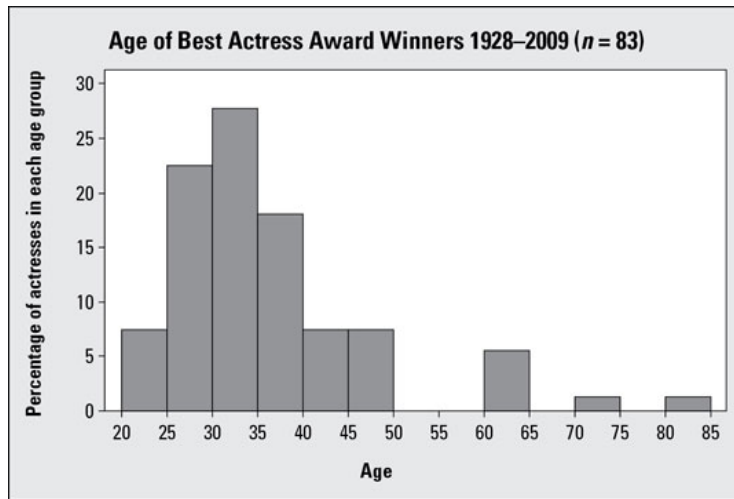
Groupwork

Group A: 2 4 6 8 10

Group B: 2 9 9 10 10

- Different ways to report *dispersion* with **one** number?

- What are pros and cons of each?

- Icebreaker, Groupwork, Questions

https://web.cs.wpi.edu/~imgd2905/d22/groupwork/3-dispersion/handout.html

Range

- Difference between smallest and largest value
- Somewhat obvious, but doesn't tell you much about "clumping"
  - Minimum may be zero
  - Maximum can be from outlier
    - Event not related to phenomena studied (e.g., 0 on project)
  - Maximum gets larger with # samples, so no "stable" point

In Excel, `=MAX(array)-MIN(array)`

12, 25, 27, 29, 36, 38, 40, 43, 50, 54, 62

Range = 62 - 12 = 50

http://idolosol.com/images/range-3.jpg

Project 2

Range = 96 – 69 = 27

Min

Max

mean

median

Cumulative Distribution

Score (percent)
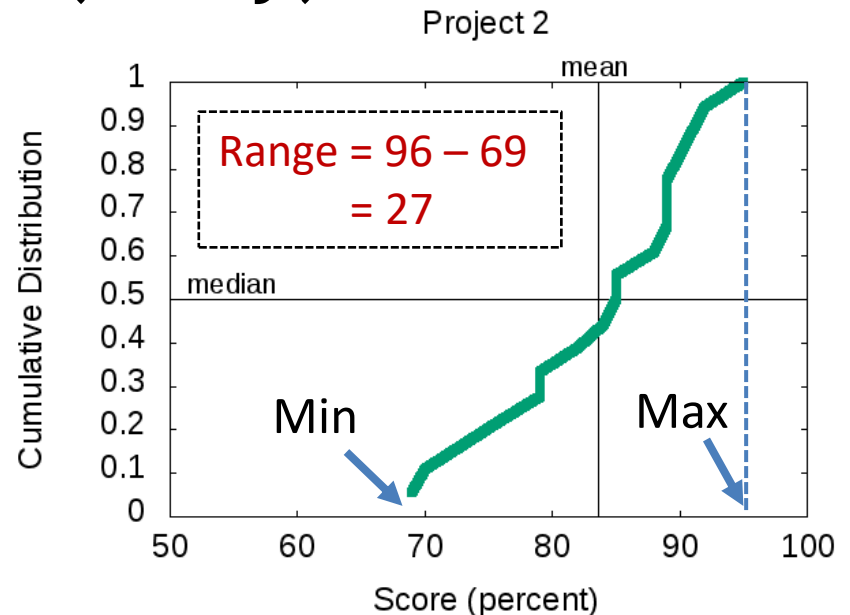
Variance

- Compute mean of sample
- Compute how far each value in sample is from mean
  - Some can be less than mean, some greater
  - → So <u>square</u> this difference (what does squaring do?)
- Divide by number of sample values – 1
  - The "-1" corrects "bias" when trying to estimate *population variance* using *sample variance*

"sum up all"   "mean"

$$\text{Sample Variance} = s^2 = \frac{\Sigma(X - \overline{X})^2}{n - 1}$$

Variance Example

- Sample kills in *PUBG* matches
  - 12, 20, 16, 18, 19
  - What is sample variance?

- First, mean = 85 / 5 = 17

Kills	X − mean	(X − mean)^2
12	-5	25
20	3	9
16	-1	1
18	1	1
19	2	4

s^2 = (25 + 9 + 1 + 1 + 4) / (5 − 1) = 40 / 4 = 10 kills squared

- In Excel, =VAR(array)

"Larger" means
"more spread"
… but units odd

Standard Deviation

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}}$$

- Square-root of variance
- Usually, use standard deviation instead of variance
  - Why? → Same *units* as data (e.g., "kills" in previous example)
- Can compare standard deviation to mean (*coefficient of variation*, next)
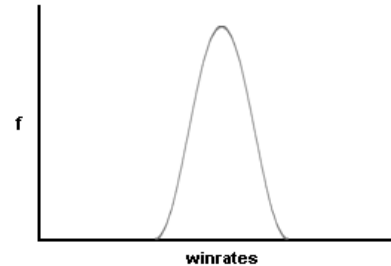- But first:
  - Mendenhall's Empirical Rule
  - Z-score

Average "distance" of points from mean

$$c = \sqrt{(a^2 + b^2)}$$
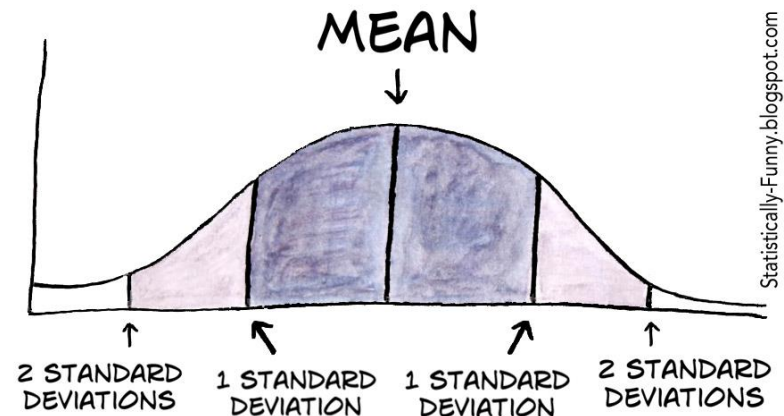
Low Standard Deviation

f

winrates

A "thin" curve means that your winrates remain close to the mean average.

High Standard Deviation

f
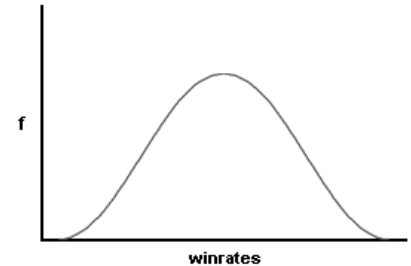
winrates

A "fat" curve means that there is a wider spread of your winrates.

MEAN
↓

2 STANDARD DEVIATIONS   1 STANDARD DEVIATION   1 STANDARD DEVIATION   2 STANDARD DEVIATIONS

Statistically-Funny.blogspot.com

Mendenhall's Empirical Rule

1. About 68% data within one standard deviation of mean
   - interval between mean-s and mean+s contains about 68% of data

2. About 95% within 2 standard deviations of mean

3. Almost all data within 3 standard deviations of mean

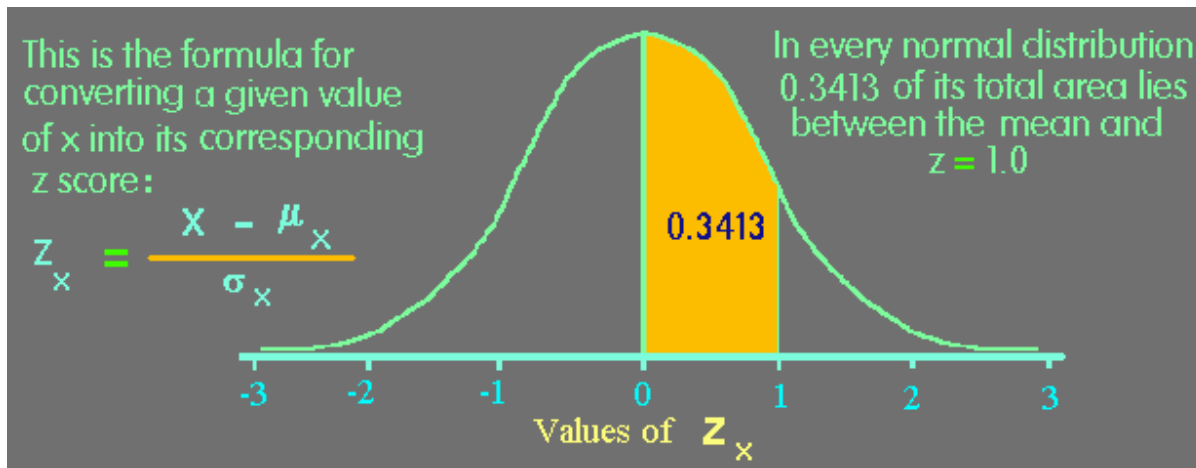https://mathbitsnotebook.com/Algebra1/StatisticsData/normalgrapha.jpg
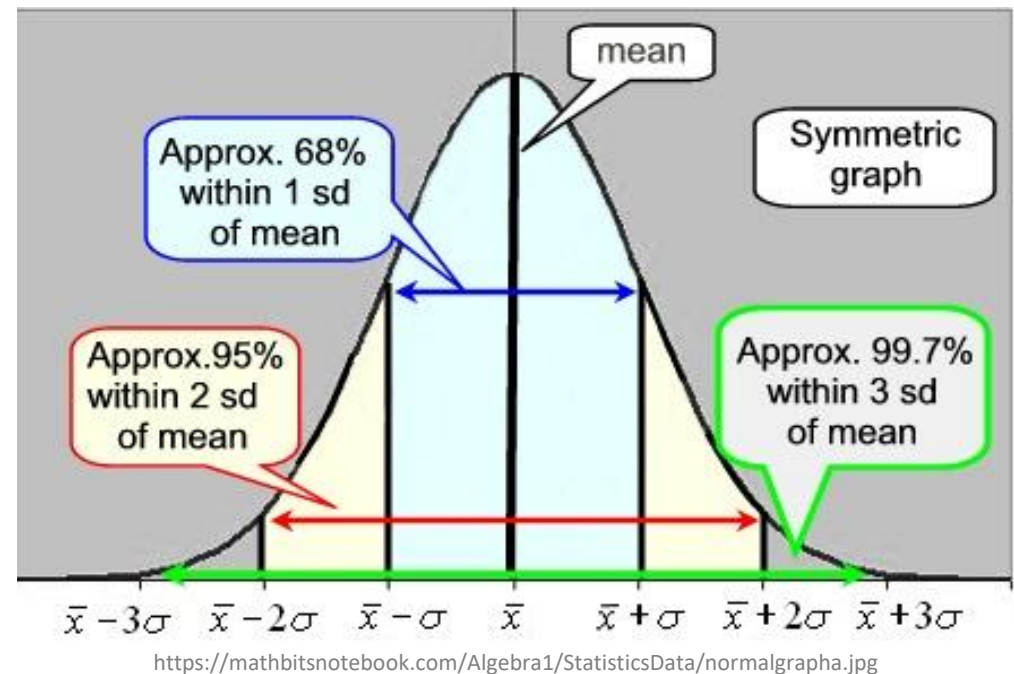
Rule holds for normal ("Bell curve") distribution

Z-Score

- Measure of how "far" from center (mean) **single** data point is
  - *Not* measure of dispersion for whole data set

$$z = \frac{X - \bar{X}}{s}$$

This is the formula for converting a given value of x into its corresponding z score:

$$z_x = \frac{X - \mu_x}{\sigma_x}$$

In every normal distribution 0.3413 of its total area lies between the mean and z = 1.0

0.3413

Values of z_x

https://www.animatedsoftware.com/pics/stats/sgzscor2.gif

Example

Mean	469
Std dev	119
X	650

Z-score for X?

(650 – 469)/119   1.52

Coefficient of Variation (CV)

- Size of standard deviation **relative** to mean
  - e.g., large sd & large mean, not so spread
  - but large sd & small mean, more spread
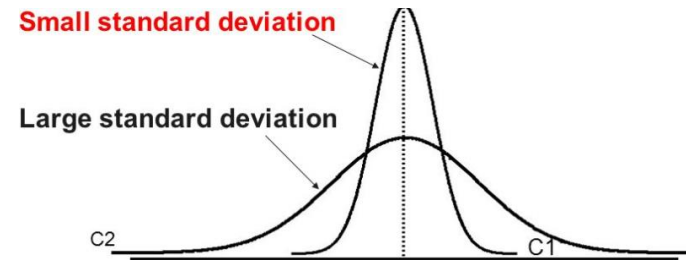- Standard deviation divided by mean
  - Can do this since same units!
- CV is "unit-less", so measure of spread independent of quantity
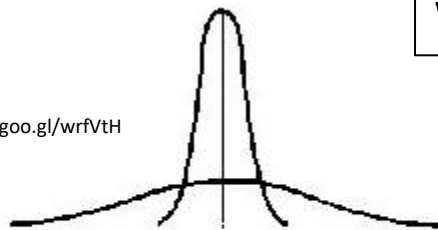 - E.g. seconds, clicks, spaces

Shown as percent (multiply by 100)

$$CV = \frac{s}{\bar{x}} \times 100$$

Small standard deviation
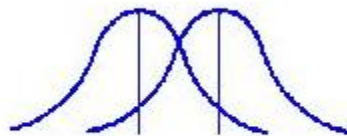
Large standard deviation

C2   C1

http://images.slideplayer.com/35/10391754/slides/slide_59.jpg

What is the relative CV for each curve?
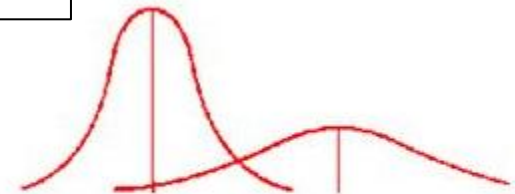
http://goo.gl/wrfVtH

Same Means
Different Standard Deviations

Different Means
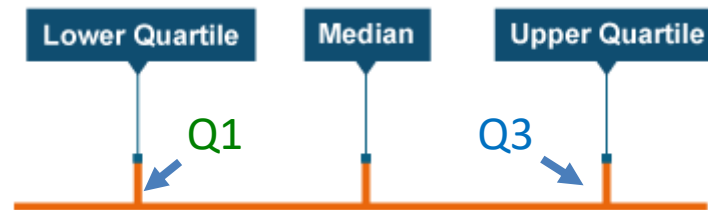Same Standard Deviations

Different Means
Different Standard Deviations

Semi-Interquartile Range

- ½ distance between Q3 (75th percentile) and Q1 (25th percentile)

http://www.bbc.co.uk/staticarchive/9629000486ef4b1a40efa565c162cb779e0bd82c.png

$$\frac{Q3 - Q1}{2}$$

- <u>Guideline</u>: use semi-interquartile (SIQR) for index of dispersion whenever using median as index of central tendency

Index of Dispersion Example

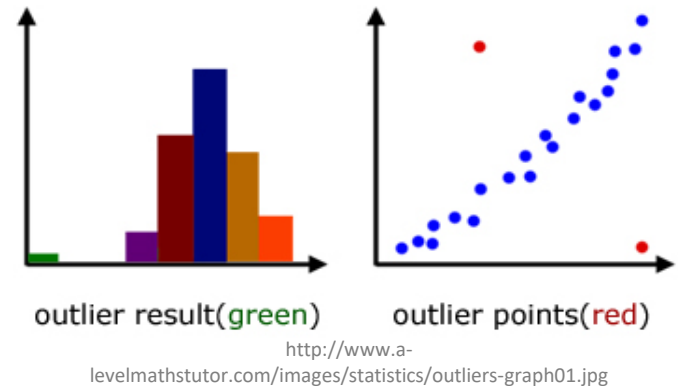| (sorted) Lap Times |
|---|
| 1.9 |
| 2.7 |
| 3.9 |
| **4.1** |
| 4.2 |
| 4.2 |
| 4.4 |
| **4.5** |
| 4.5 |
| 4.8 |
| 4.9 |
| **5.1** |
| 5.1 |
| 5.3 |
| 5.6 |
| 5.9 |

- First, sort. Then, compute:
  - Mean = 4.4
  - Min = 1.9, Max = 5.9
  - Median = [16 / 2] = 8th = 4.5
  - Q1 = 16 / 4 = 8th = 4.1
  - Q3 = 3 * 16 / 4 = 12th = 5.1

- *SIQR* = (Q3 - Q1) / 2   = 0.5
- *Variance*   = 0.96
- *Stddev*   = 0.98
- *CV* = stddev/mean   = 0.22
- *Range* = max – min   = 4

Groupwork

- Rank *measures of dispersion* by sensitivity) to outliers
  - Variance
  - Range
  - Coefficient of Variation
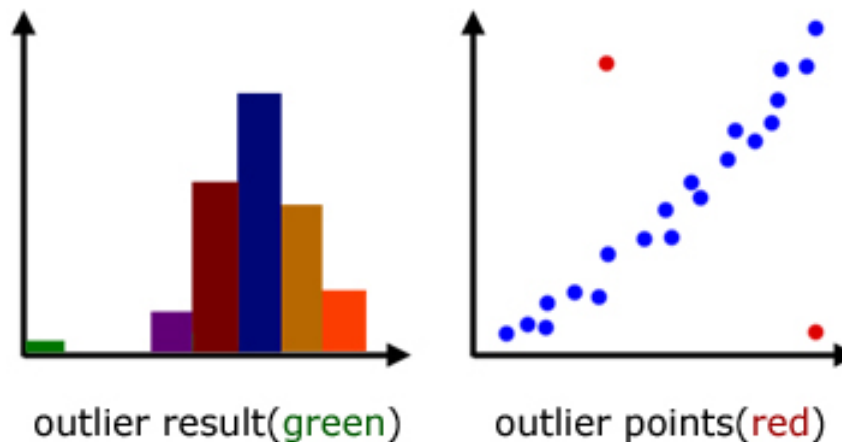 - Semi-interquartile Range

outlier result(green) outlier points(red)

http://www.a-levelmathstutor.com/images/statistics/outliers-graph01.jpg

https://web.cs.wpi.edu/~imgd2905/d20/

Ranking of Affect by Outliers?

Measure of Dispersion

- Range
- Standard Deviation
- Coefficient of Variation
- Semi-interquartile Range

Most to Least

?

outlier result(green)   outlier points(red)

Ranking of Affect by Outliers?

Measure of Dispersion

- Range
- Standard Deviation
- Coefficient of Variation
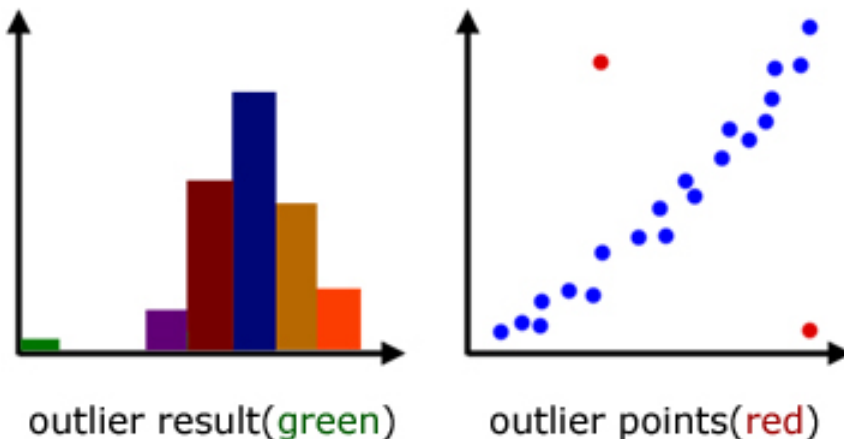- Semi-interquartile Range

Most to Least

- Range   susceptible
- Variance
  - Standard Deviation
  - Coefficient of Variation
- SIQR   resistant

outlier result(green)

outlier points(red)

Only for quantitative data!
 categorical can't quantify spread since no 'distance'
 Instead, give categories for given percentile of samples
  e.g., "90% of samples are in 3 categories" (Pareto chart)

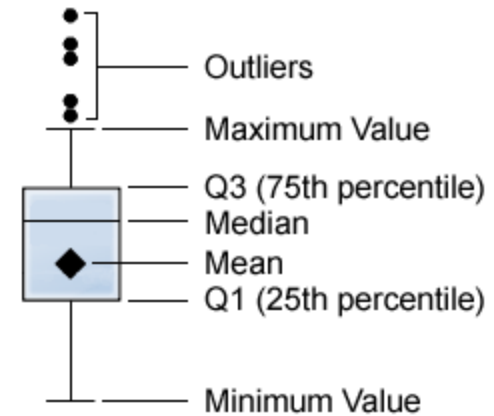# Depicting Dispersion in Charts

- Histogram

- Cumulative distribution

- Box-and-Whiskers

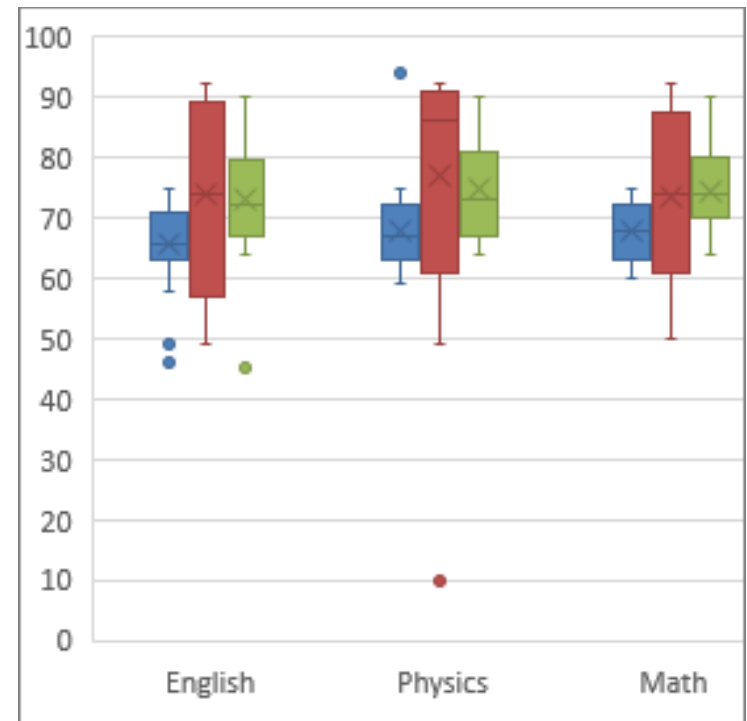- Error Bars

Box-and-Whiskers Chart

- Way of showing variation
- Highlight middle 50% (interquartile range, IQR)
  - "Box"
- Lines go to smallest non-outlier
  - "Whiskers"
- Points indicate outliers
- Middle line shows median
- Sometimes with mean
- Outlier? → Data value "way out there", "far" from the rest
  - Formally, 1.5+ IQRs away from quartile
- Available in Excel

Also called "boxplot"

Outliers
Maximum Value
Q3 (75th percentile)
Median
Mean
Q1 (25th percentile)
Minimum Value

http://support.sas.com/documentation/cdl/en/vaug/65747/HTML/default/images/boxplot.png

https://support.office.com/en-us/article/Create-a-box-and-whisker-chart-62f4219f-db4b-4754-aca8-4743f6190f0d
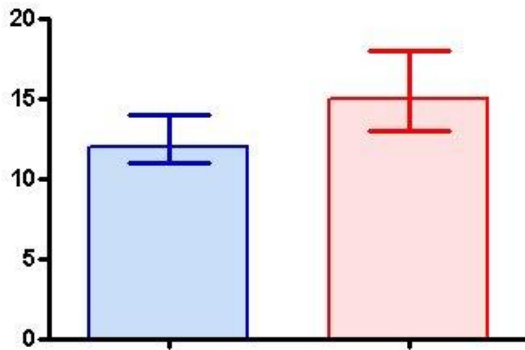
Error Bars for Columns and Points

- Line through graph point parallel to axis with "caps"
- Denotes uncertainty (variation) in value
- Excel: click "+" → "Error Bars" → "type"

- Often:
  - 1 standard deviation
- Can be (discuss later):
  - 1 standard error
  - 1 confidence interval

State clearly!

https://s3.amazonaws.com/cdn.graphpad.com/faq/804/images/804b.jpg

http://www.excel-easy.com/examples/images/error-bars/error-bars.png