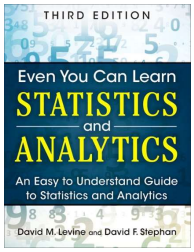
IMGD 2905

Descriptive Statistics

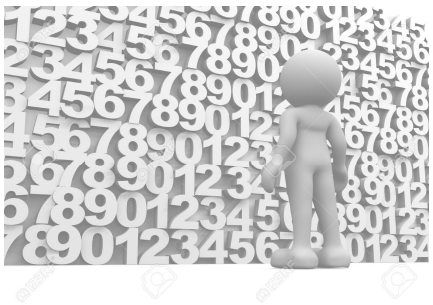
Chapter 3



1

Summarizing Data

- With lots of playtesting, there is a lot of data
  - This is a good thing!
- But raw data is just a pile of numbers
  - Rarely of interest
  - Or even sensible
- Q: How to summarize all this information?



2

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Measures of central tendency

Examples?

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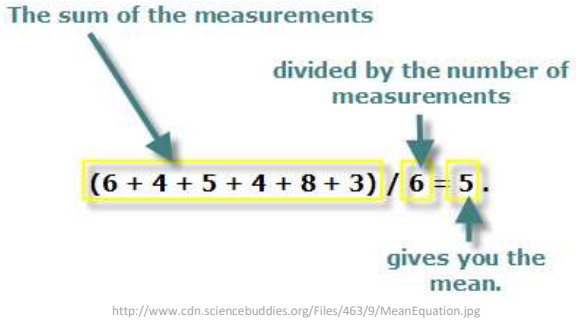
Measure of Central Tendency: Mean



http://www.cdn.sciencebuddies.org/Files/463/9/MeanEquation.jpg

- Also called the "arithmetic mean" or "average"
- In Excel, =AVERAGE(range)

  =AVERAGEIF() – averages if numbers meet certain condition

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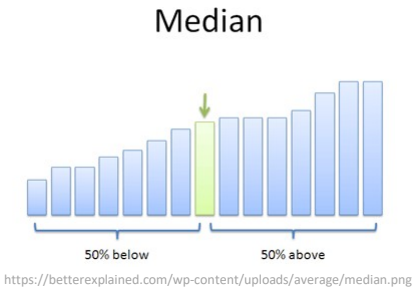
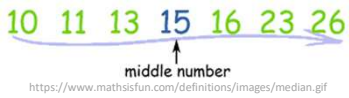
Measure of Central Tendency: Median

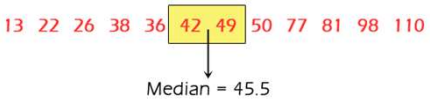
- Sort values low to high and take middle value



https://betterexplained.com/wp-content/uploads/average/median.png



https://www.mathsisfun.com/definitions/images/median.gif



http://www.nedarc.org/statisticalHelp/basicStatistics/measuresOfCenter/images/median.gif

- In Excel, `=MEDIAN(range)`

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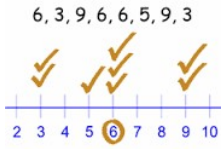
Measure of Central Tendency: Mode

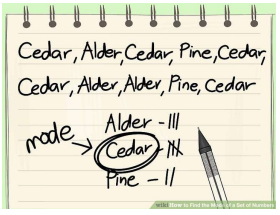
- Number which occurs most frequently
- Not too useful in many cases

→ Best use for categorical data

  - e.g., most popular Hero group in Heroes of the Storm

- In Excel, `=MODE()`





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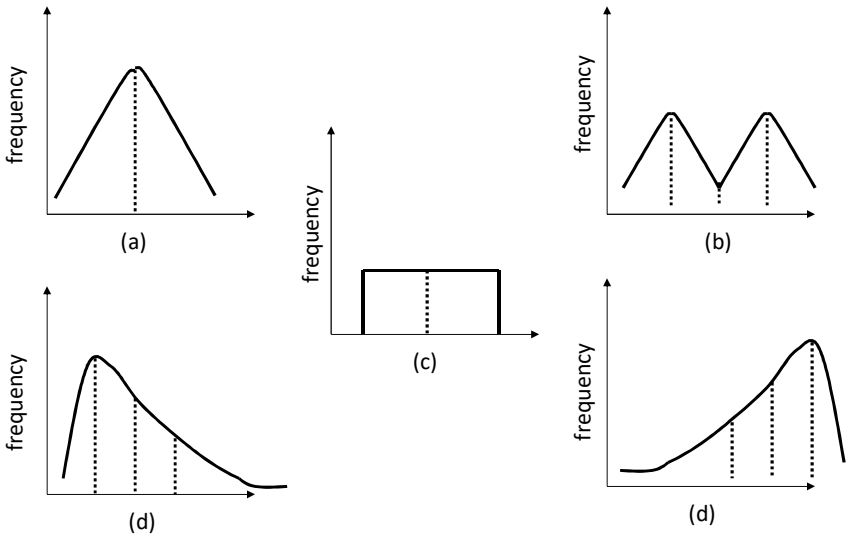
Depiction: Mean, Median, Mode?



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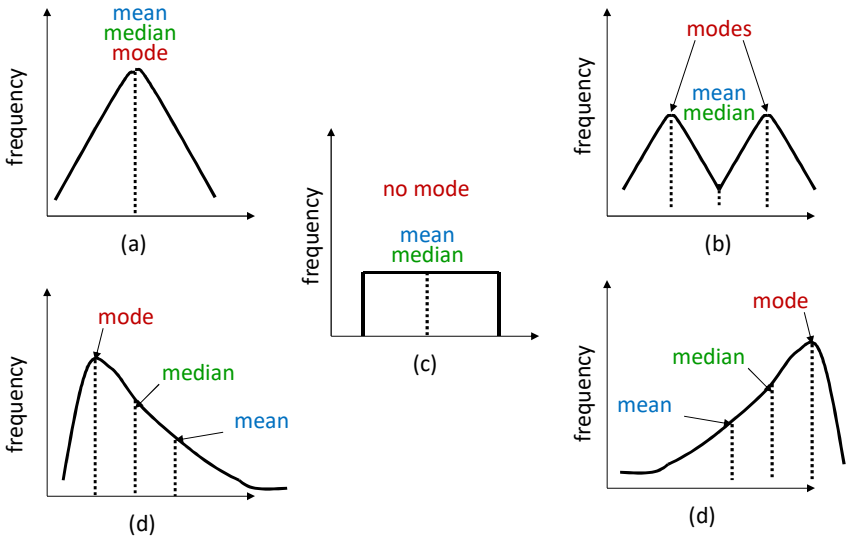
Depiction: Mean, Median, Mode?



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Which to Use, Mean, Median, Mode?

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Which to Use, Mean, Median, Mode?

- Mean many statistical tests with sample
  - Estimator of population mean
  - Uses all data
- Median is useful for skewed data
  - e.g., income data (US Census) or housing prices (Zillo)
  - e.g., *Overwatch* team (6 players): 5 people level 5, 1 person level 275
    - Mean is 50 - not so useful since no one at this level
    - Median is 5 - more representative
  - Does not use all data. "Resistant" to extremes (e.g., 275)
  - But what if were exam scores? Hard to "bring up" grade
- Mode is useful primarily for categorical data only
  - Most played League champion, most popular maze, …

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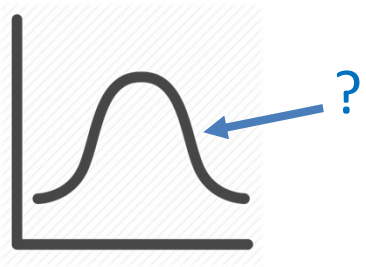
Other Measures of Position

- May not always want center
  - e.g., want to know best League Champions
- What other positions may be desired?



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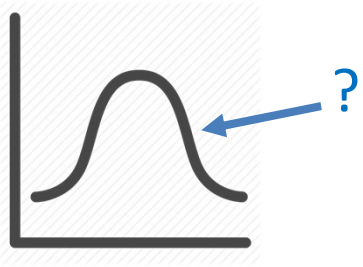
Other Measures of Position

- May not always want center
  - e.g., want to know best League Champions



- Maximum / Minimum
  - Not discussed more
- Trimmed Mean
- Quartiles
- Percentiles

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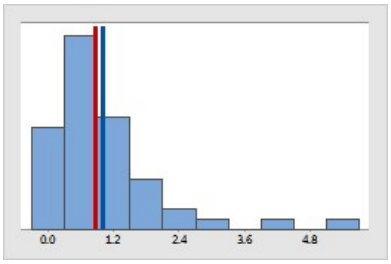
Trimmed Mean

- Take "trimming" off top and bottom (typically 5% or 10%)
  - Reduces effects of extreme values, like median
- In Excel, `=TRIMMEAN(array,percent)`



http://support.minitab.com/en-us/minitab/17/histogram_mean_vs_trimmed_mean.png

Blue – original mean
Red – trimmed mean

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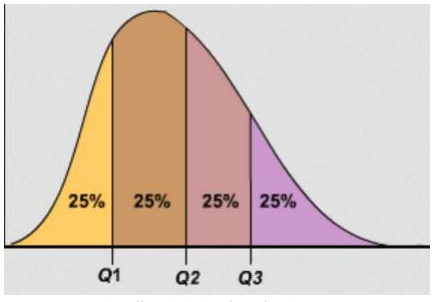
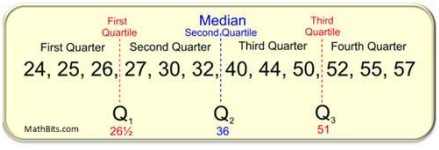
Quartiles

- Sort values
- First quartile (Q1) is 25% from bottom
- Third quartile (Q3) is 75% from bottom
- (What is second quartile?)
- In Excel, `=QUARTILE(array,n)`



https://www.hackmath.net/images/quartiles.png



https://mathbitsnotebook.com/Algebra1/StatisticsData/quartileboxview2.png

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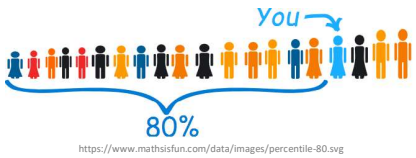
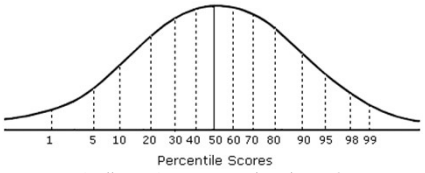
Percentiles

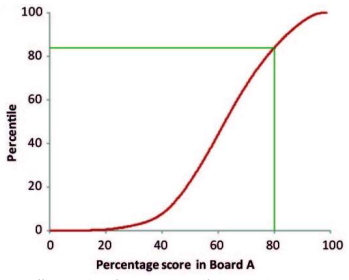
- Generalization of quartiles
- N^{th} percentile is data point n% from bottom of data
- Interpolate as for first quartile
- In Excel, `=PERCENTILE(array,k)` (k: 0 to 1)



https://www.mathsisfun.com/data/images/percentile-80.svg



http://www.psychometric-success.com/images/AA1301.gif



http://www.isical.ac.in/~jeexiiscore_normal/PercentilesAdvantages.htm

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Summarizing Data, Part 2

- Ok, pile of numbers can now be summarized as *one* number
  - Mean, median, mode
- But is that enough?
- Q: What other major aspect of numbers haven't we summarized?

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Summarizing Data, Part 2

- Ok, pile of numbers can now be summarized as *one* number
  - Mean, median, mode
- But is that enough?
- Q: What other major aspect of numbers haven't we summarized?

Measures of variation
(*aka* measures of *dispersion,* or measures of *spread*)

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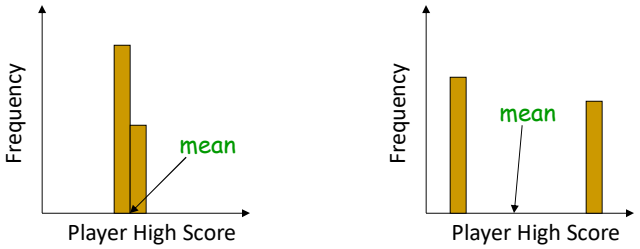
Summarizing Data, Part 2

"Then there is the man who drowned crossing a stream with an average depth of six inches." – W.I.E. Gates

- Summarizing by single number rarely enough → need statement about variation



Above: does single number (mean) tell you enough about data?

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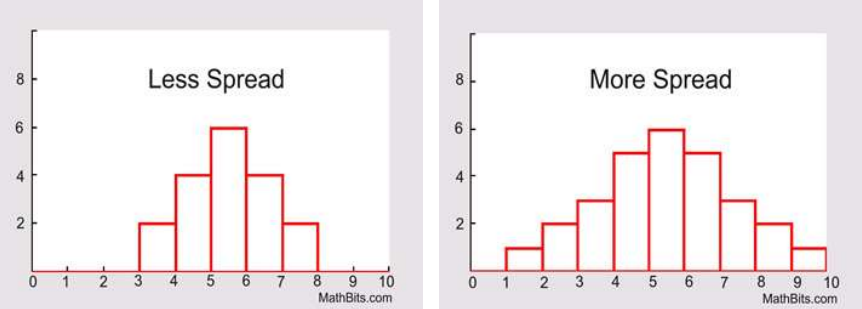
Variation Overview (1 of 3)

- Is data clumped or spread out?



https://mathbitsnotebook.com/Algebra1/StatisticsData/STSpread.html

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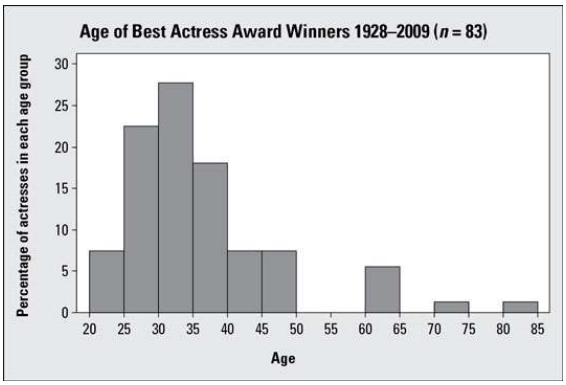
Variation Overview (2 of 3)

- Is data clumped or spread out?



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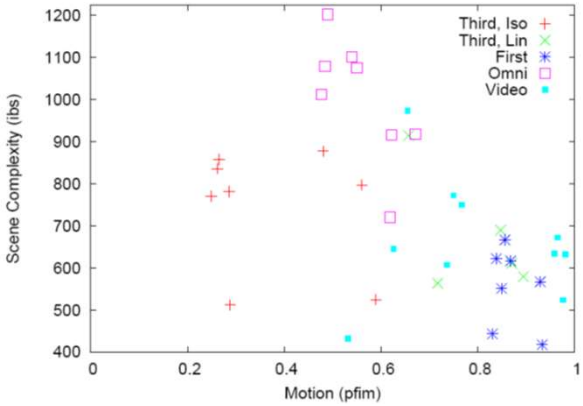
Variation Overview (3 of 3)

- Is data clumped or spread out?



"Motion and Scene Complexity for Streaming Video Games"

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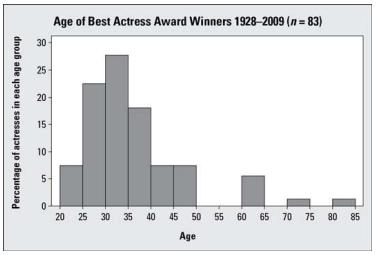
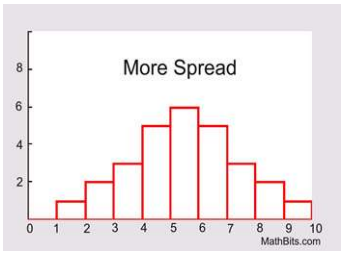
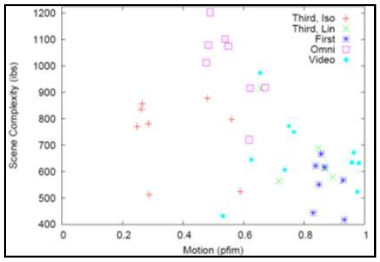
What are Some Measures of Variation?







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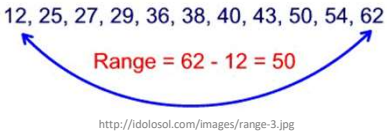
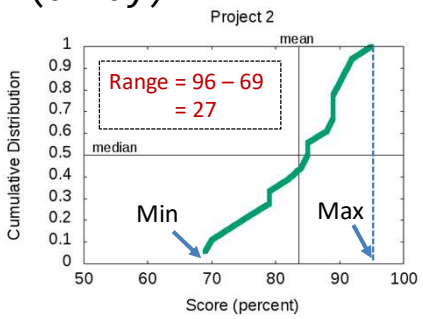
Range

- Difference between smallest and largest value
- Somewhat obvious, but doesn't tell you much about "clumping"
  - Minimum may be zero
  - Maximum can be from outlier
    - Event not related to phenomena studied (e.g., 0 on project)
  - Maximum gets larger with # samples, so no "stable" point

In Excel, `=MAX(array)-MIN(array)`





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Variance

- Compute mean of sample
- Compute how far each value in sample is from mean
  - Some can be less than mean, some greater
  - → So square this difference (why square?)
- Divide by number of sample values – 1
  - The "-1" corrects "bias" when trying to estimate *population variance* using *sample variance*

"sum up all" "mean"

$$\text{Sample Variance} = s^2 = \frac{\Sigma(X - \overline{X})^2}{n-1}$$

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Variance Example

- Sample kills in *League of Legends* match
  - 12, 20, 16, 18, 19
  - What is sample variance?
- First, mean = 85 / 5 = 17

Kills	X – mean	$(X - \text{mean})^2$
12	-5	25
20	3	9
16	-1	1
18	1	1
19	2	4

$s^2 = (25 + 9 + 1 + 1 + 4) / (5 - 1) = 40 / 4 =$ 10 kills squared

- In Excel, =VAR(array)

"Larger" means "more spread" … but units odd

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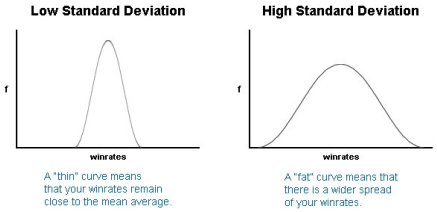
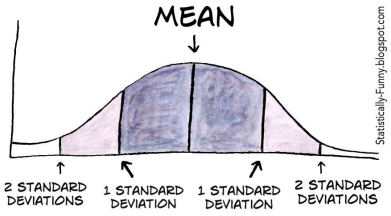
Standard Deviation

- Square-root of variance
- Usually, use standard deviation instead of variance
  - Why? → Same *units* as data (e.g., "kills" in previous example)
- Can compare standard deviation to mean (*coefficient of variation*, next)
- But first:
  - Mendenhall's Empirical Rule
  - Z-score

$$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n - 1}}$$





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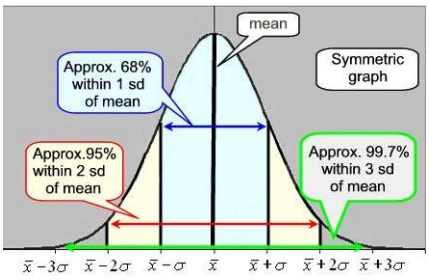
Mendenhall's Empirical Rule

1. About 68% data within one standard deviation of mean
   - interval between mean-s and mean+s contains about 68% of data
2. About 95% within 2 standard deviations of mean
3. Almost all data within 3 standard deviations of mean



https://mathbitsnotebook.com/Algebra1/StatisticsData/normalgrapha.jpg

Rule assumes normal ("Bell curve") distribution

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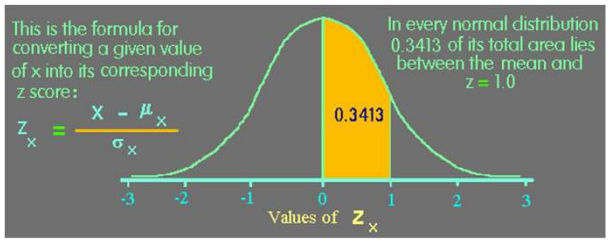
Z-Score

- Measure of how "far" from center (mean) single data point is
  - *Not* measure of dispersion for whole data set

$$z = \frac{X - \bar{X}}{s}$$



https://www.animatedsoftware.com/pics/stats/sgzscor2.gif

Example	
Mean	469
Std dev	119
X	650
Z-score for X?	
(650 – 469)/119	1.52

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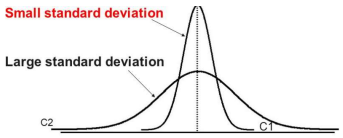
Coefficient of Variation (CV)

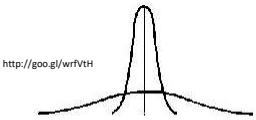
- Size of standard deviation relative to mean
  - e.g., large sd & large mean, not so spread
  - but large sd & small mean, more spread
- Standard deviation divided by mean
  - Can do this since same units!
- CV is "unit-less", so measure of spread independent of quantity
  - E.g. seconds, clicks, spaces

Shown as percent (multiply by 100)

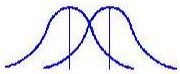
$$CV = \frac{s}{\bar{x}} \times 100$$



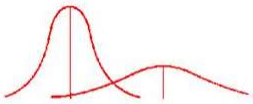
http://images.slideplayer.com/35/10391754/slides/slide_59.jpg



Same Means
Different Standard Deviations



Different Means
Same Standard Deviations



Different Means
Different Standard Deviations

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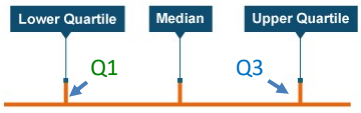
Semi-Interquartile Range

- ½ distance between Q3 (75th percentile) and Q1 (25th percentile)



http://www.bbc.co.uk/staticarchive/9629000486ef4b1a40efa565c162cb779e0bd82c.png

$$\frac{Q3 - Q1}{2}$$

- Guideline: use semi-interquartile (SIQR) for index of dispersion whenever using median as index of central tendency

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Index of Variation Example

(sorted) Lap Times
1.9
2.7
3.9
4.1
4.2
4.2
4.4
4.5
4.5
4.8
4.9
5.1
5.1
5.3
5.6
5.9

- First, sort. Then, compute:
  - Mean = 4.4
  - Min = 1.9, Max = 5.9
  - Median = [16 / 2] = 8th = 4.5
  - Q1 = 16 / 4 = 8th = 4.1
  - Q3 = 3 * 16 / 4 = 12th = 5.1

- *SIQR* = (Q3 - Q1) / 2 = 0.5
- *Variance* = 0.96
- *Stddev* = 0.98
- *CV* = stddev/mean = 0.22
- *Range* = max – min = 4

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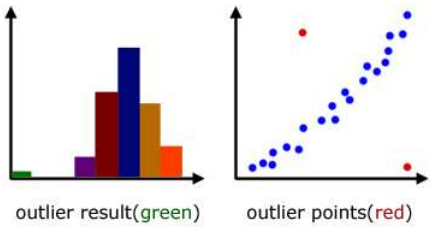
Ranking of Affect by Outliers?

Measure of Variation

- Variance
- Range
- Standard Deviation
- Coefficient of Variation
- Semi-interquartile Range

Most to Least

?



http://www.a-levelmathstutor.com/images/statistics/outliers-graph01.jpg

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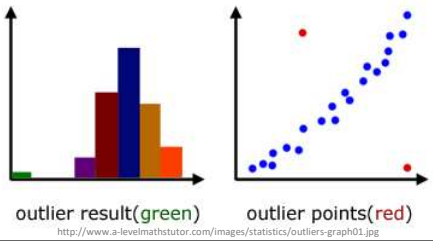
Ranking of Affect by Outliers?

Measure of Variation

- Variance
- Range
- Standard Deviation
- Coefficient of Variation
- Semi-interquartile Range

Most to Least

- Range — susceptible
- Variance
  - Standard Deviation
  - Coefficient of Variation
- SIQR — resistant



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Index of Variation Summary

- Ranking of affect by outliers
  - Range — susceptible
  - Variance
    - Standard deviation
    - Coefficient of variation
  - Semi-interquartile range — resistant
- Note, all only applied to quantitative data!
  - For categorical data, can't quantify spread since no 'distance' between
  - Instead, give number of categories for given percentile of samples
    - e.g., "90% of samples are in 3 categories"
    - (Pareto chart provides this)

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Depicting Variation in Charts

- Histogram (done)
- Cumulative distribution (done)
- Box-and-Whiskers (new)
- Error Bars (new)

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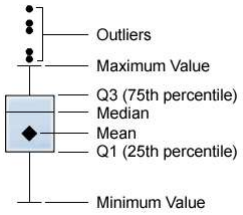
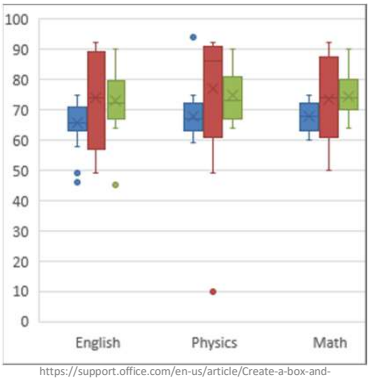
Box-and-Whiskers Chart

- Way of showing variation
- Highlight middle 50% (interquartile range, IQR)
  - "Box"
- Lines go to smallest non-outlier
  - "Whiskers"
- Points indicate outliers
- Middle line shows median
- Sometimes with mean
- Outlier? → Data value "way out there", "far" from the rest
  - Formally, 1.5+ IQRs away from quartile
- Available in Excel 2016

Sometimes called "boxplot"



http://support.sas.com/documentation/cdl/en/vaug/65747/HTML/default/images/boxplot.png



https://support.office.com/en-us/article/Create-a-box-and-whisker-chart-62f4219f-db4b-4754-aca8-4743f6190f0d

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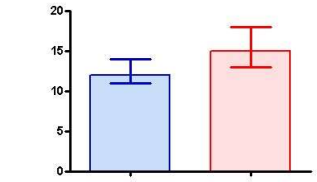
Error Bars

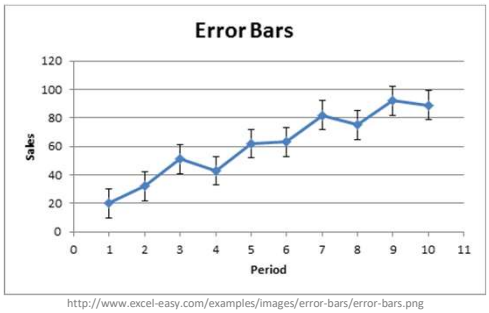
- Line through graph point parallel to axis with "caps"
- Denotes uncertainty (variation) in value
- Excel: click "+" → "Error Bars" → "type"

- Often:
  - 1 standard deviation
- Can be (discuss later):
  - 1 standard error
  - 1 confidence interval

State clearly!



https://s3.amazonaws.com/cdn.graphpad.com/faq/804/images/804b.jpg



http://www.excel-easy.com/examples/images/error-bars/error-bars.png

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