CS2223, HW1: Orders of Growth*

Course website: `http://web.cs.wpi.edu/~cs2223/b12/`. Hand in your answer by midnight, 31 Oct., so that we can discuss some problems in class Thursday. Use Turnin at `https://turnin.cs.wpi.edu/`.

The textbook, *CLRS*, has *exercises* as well as *problems*. The exercises come at the end of each section, while the problems come as a separate group at the end of the chapters. This can be confusing. Both problems and exercises are included in this week's list.

Working in groups and talking about the problems is strongly encouraged. More enjoyable and more educational. You can also discuss them with Fei, Linglong, Xianjing, and with me. Our office hours are listed at `http://web.cs.wpi.edu/~cs2223/b12/#personnel`.

This version has answers in blue for the non *CLRS* questions.

A. Show these O, Θ facts. For each problem, show that the claimed fact is true by showing a threshold N_0 and a constant c (or constants c_1, c_2). Write down the inequality (or inequalities) the claim requires, and use algebra to simplify them to show they're true.

Choose "tight" values for N_0 and c_i: You can round off, but each value should be within 1 of where the equations become true (keeping your other choices fixed).

1. $f_1(n) = 4n + 6$, $g_1(n) = 2n^2$: Show that $f_1 \in O(g_1)$.

   $N_0 =$ 3 (other choices possible...)

   $c =$ 1

   The inequality: $4n + 6 \leq 2n^2$ for $n > N_0$

   $$0 \leq n^2 - 2n - 3, \text{ equivalently (factoring) } 3 \leq n$$

2. $f_2(n) = 2^n + 10$, $g_2(n) = 2^{n+2}$: Show that $f_2 \in \Theta(g_2)$.

   $N_0 =$ 2 (other choices possible...)

*Due: Wednesday night, 31 Oct.

$c_1 = 1$

$c_2 = 4$

The inequalities: $2^n + 10 \leq c_1 2^{n+2} \leq c_2(2^n + 10)$, for $n > 2$

$2^n + 10 \leq 4 \cdot 2^n \leq 4(2^n + 10)$
$14 \leq 16 \leq 56$, and this will remain true for $n > 2$.

B. Show this is false. Professor Egregious tells his class that

$$9^n \in O(3^n)$$

Show that he's wrong, by showing that any N_0, c he chooses leads to an inequality that cannot be true for all $n > N_0$. Use algebra to show an explicit condition on c that cannot work.

$$\begin{aligned} 9^n &\leq c \cdot 3^n \\ (3 \cdot 3)^n &\leq c \cdot 3^n \\ 3^n \cdot 3^n &\leq c \cdot 3^n \\ 3^n &\leq c \end{aligned}$$

which is always false for $n > \log_3 c$.

C. Put these functions in order. Put these functions in a list by increasing order of growth. That means, in your final list, if f comes before g, then $f \in O(g)$. If $f \in \Theta(g)$, either one can come first.

Below, say which pairs of functions have $f \in \Theta(g)$.

$n \log_2 n$	$12\sqrt{n}$	$1/n$	$n^{\log_2 n}$
$100n^2 + 6n$	$n^{0.51}$	$n^2 - 324$	$50n^{0.5}$
$2n^3$	3^n	$2^{32}n$	$\log_2 n$

The list of functions: **The pairs** that are Θ:

$1/n$, $\log_2 n$, $12\sqrt{n}$, $50n^{0.5}$, $n^{0.51}$, $2^{32}n$, $n \log_2 n$,
$100n^2 + 6n$, $n^2 - 324$, $2n^3$, $n^{\log_2 n}$, 3^n

$(12\sqrt{n}, 50n^{0.5})$ and $(100n^2 + 6n, n^2 - 324)$

Some Exercises and Problems from *CLRS*.

Section 2.2. Exercises 2.2-1, 2.2-2, 2.2-3, 2.2-4; page 29.

Chapter 2. Problems 2-3, parts a-b, 2-4 parts a-c; pp. 39–42.

Section 3.1. Exercises 3.1-1, 3.1-4; p.53.

Chapter 3. Problems 3-1, 3-2, 3-3 part a; pp. 61–62.