Strand spaces:
A framework to prove protocols
and find counterexamples

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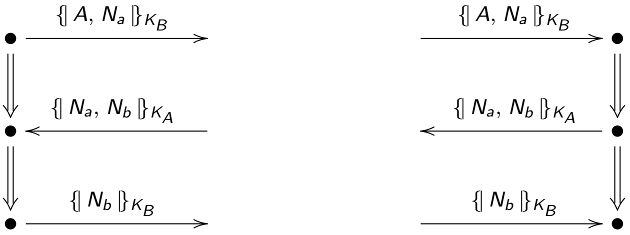
Strands

- Strand: A finite linear sequence $\bullet \Rightarrow \bullet \Rightarrow \bullet \cdots$ of events

  transmission reception neutral

- Strand may represent
  - single local session of a protocol, or
  - an adversary action
- Each event called a node
- Transmission, reception sometimes written $+, -$ resp
- Node n is labeled with a message $\text{msg}(n)$

Example: Needham-Schroeder



Protocol: finite set Π of roles

Strands of Π: all substitution instances

Adversary Strands

a: basic value $\quad$ t_i: any msg

$$\bullet \xrightarrow{a}$$

$$\begin{tikzcd}
t_1 \arrow[d] & t_2 \arrow[d] & \{\!|\, t_1 \,|\!\}_{t_2} \\
\bullet \arrow[r, Rightarrow] & \bullet \arrow[r, Rightarrow] & \bullet \arrow[u]
\end{tikzcd}
\qquad
\begin{tikzcd}
t_1 \arrow[d] & t_2 \arrow[d] & \mathrm{tag}\ t_1,\ t_2 \\
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\end{tikzcd}$$

$$\begin{tikzcd}
\bullet \arrow[r, Rightarrow] & \bullet \arrow[r, Rightarrow] & \bullet \arrow[d] \\
\{\!|\, t_1 \,|\!\}_{t_2} \arrow[u] & {t_2}^{-1} \arrow[u] & t_1
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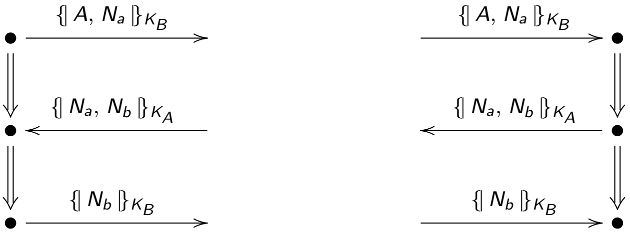
Executions are bundles, 1



Executions are bundles, 1

$$
\begin{tikzcd}[column sep=huge, row sep=large]
\bullet \arrow[r, "\{\!|\, A,\, N_a \,|\!\}_{K_B}"] \arrow[d, Rightarrow] & \bullet \arrow[d, Rightarrow] \\
\bullet \arrow[d, Rightarrow] & \bullet \arrow[l, "\{\!|\, N_a,\, N_b \,|\!\}_{K_A}"'] \arrow[d, Rightarrow] \\
\bullet \arrow[r, "\{\!|\, N_b \,|\!\}_{K_B}"] & \bullet
\end{tikzcd}
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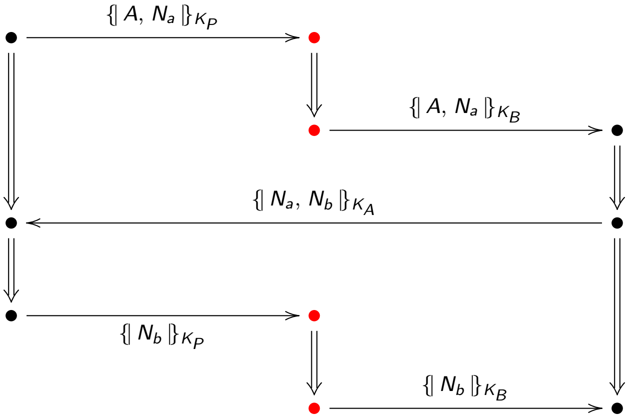
Executions are bundles, 2



Some adversary strands for bundle 2

$$
\begin{tikzcd}
{} \arrow[r, "\{\!|\, A,\ N_a \,|\!\}_{\mathrm{pubk}(C)}"] & \bullet \arrow[d, Rightarrow] & & \\
\bullet \arrow[r, "\mathrm{pubk}(C)^{-1}"'] & \bullet \arrow[d, Rightarrow] & & \\
& \bullet \arrow[r, "A,\ N_a"] & \bullet \arrow[d, Rightarrow] & \\
& \bullet \arrow[r, "\mathrm{pubk}(B)"'] & \bullet \arrow[d, Rightarrow] & \\
& & \bullet \arrow[r, "\{\!|\, A,\ N_a \,|\!\}_{\mathrm{pubk}(B)}"] & {}
\end{tikzcd}
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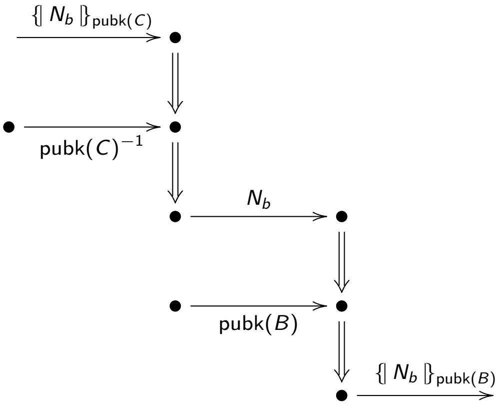

Bundle: Definition

Let $\mathcal{B}$ be a finite directed acyclic graph V, E where

V consists of nodes

E is $(\Rightarrow_E \cup \rightarrow_E)$ where:

$n_1 \Rightarrow_E n_2$ implies $n_1 \Rightarrow n_2$

$n_1 \rightarrow_E n_2$ implies n_1 transmission,
n_2 reception, and
$\mathrm{msg}(n_1) = \mathrm{msg}(n_2)$

$\mathcal{B}$ is a bundle if

1. $n_2 \in V$ and $n_1 \Rightarrow n_2$ implies $n_1 \in V$ and $n_1 \Rightarrow_E n_2$ — start at beginning
2. $n_2 \in V$ is a reception node implies there is a unique $n_1 \in V$ such that $n_1 \rightarrow_E n_2$ — what's heard was said

Bundle ordering $\preceq_{\mathcal{B}}$

Let $\mathcal{B}$ be a bundle

- Define $\preceq_{\mathcal{B}}$ to be $(\Rightarrow_E \cup \rightarrow_E)^*$
- So $n_1 \preceq_{\mathcal{B}} n_2$ means there is a path in $\mathcal{B}$ from n_1 to n_2

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Well-founded means:

Every non-empty $S \subseteq \text{nodes}(\mathcal{B})$ has $\preceq_{\mathcal{B}}$-minimal members

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Serves as an induction principle — Bundle induction

If S has no $\preceq_{\mathcal{B}}$-minimal members, $S = \emptyset$

Messages

Basic messages:

Names for principals

Keys basic keys are either symmetric or asymmetric

Data maybe used as nonces etc

Built up using

Encryption of t using K is $\{\!|\, t \,|\!\}_K$

Tagged pair of t_1, t_2, tagged with *tag* is $\mathit{tag}\ \ t_1,\ t_2$

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Special tag *nil* : Write $\mathit{nil} \;\; t_1, \; t_2$ as $t_1, \; t_2$

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Messages are an inductively defined structure

Two notions of subterm: $\sqsubseteq$ and $\ll$

Both are reflexive, transitive relations

$\sqsubseteq$ generated by: "ingredient"

$$t_1 \sqsubseteq tag\ \ t_1,\ t_2 \qquad\qquad t_2 \sqsubseteq tag\ \ t_1,\ t_2$$

$$t_1 \sqsubseteq \{\!| \ t_1 \ |\!\}_K$$

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$\ll$ generated by: "occurs in"

$$t_1 \ll tag \;\; t_1,\; t_2 \qquad\qquad t_2 \ll tag \;\; t_1,\; t_2$$

$$t_1 \ll \{\!|\, t_1 \,|\!\}_K \qquad\qquad K \ll \{\!|\, t_1 \,|\!\}_K$$

Origination

a originates at $n \in \text{nodes}(\mathcal{B})$ iff

- $a \sqsubseteq \text{msg}(n)$
- n is a transmission node
- $a \not\sqsubseteq \text{msg}(m)$ whenever $m \Rightarrow^{+} n$

I.e. a is transmitted as an ingredient of $\text{msg}(n)$, and n is its first use as an ingredient

Ingredient $\sqsubseteq$ just uses plaintext, not key

Fresh choice

a is freshly chosen in $\mathcal{B}$ means a originates uniquely in nodes($\mathcal{B}$):

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i.e. a originates at a unique node n

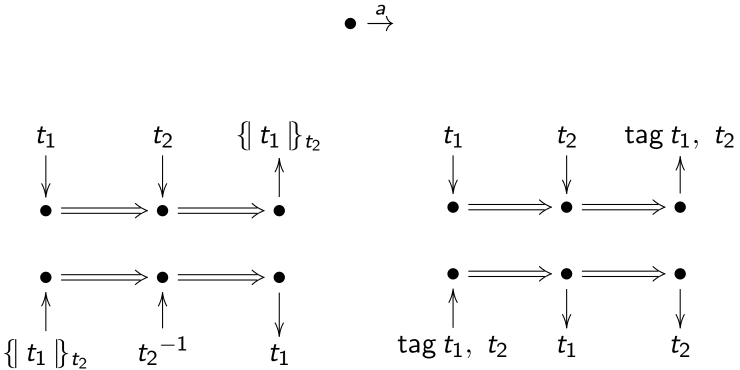
Uncompromised keys

A key K is uncompromised if it originates nowhere:

$$\text{for every } n \in \text{nodes}(\mathcal{B}),\ K \not\sqsubseteq \text{msg}(n)$$

Adversary never uses non-originating keys

If adversary uses K, it must have originated



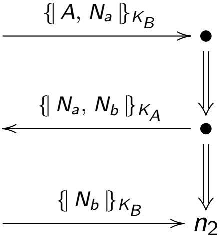
Prove all things. Hold fast
to what is good.

St. Paul, 1 Thessalonians 5:21
with thanks to Imre Lakatos

Proving Authentication: Needham-Schroeder

for the responder

Suppose that N_b is uniquely originating, and K_A^{-1} is non-originating



N_b is found outside $\{\!| N_a, N_b |\!\}_{K_A}$ in $\text{msg}(n_2)$.
Thus, there is a $\preceq_{\mathcal{B}}$-minimal node m such that N_b is found outside $\{\!| N_a, N_b |\!\}_{K_A}$ in $\text{msg}(m)$

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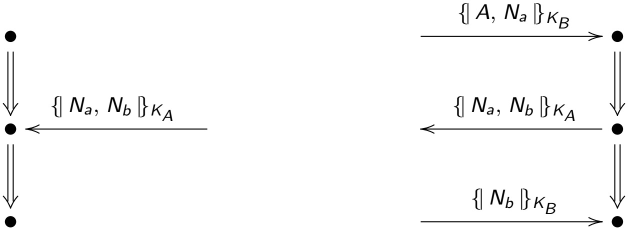

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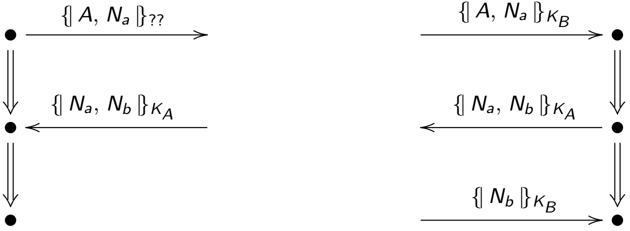

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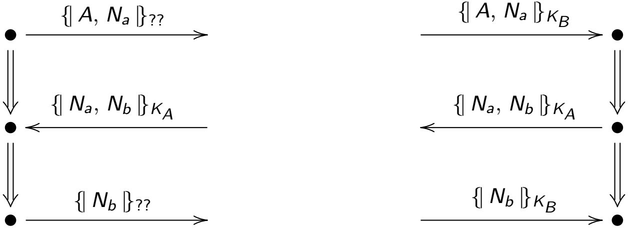

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What did we prove about NS?

If $\mathcal{B}$ is a bundle where

1. $K_A^{-1} \in \text{non}_{\mathcal{B}}$ and $N_b \in \text{unique}_{\mathcal{B}}$
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then $\mathcal{B}$ has a full initiator strand with parameters A, C, N_a, N_b

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for some C

Proving Authentication: Needham-Schroeder-Lowe

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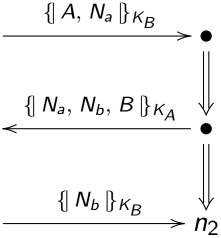

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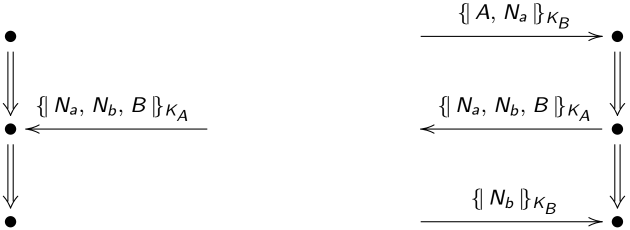

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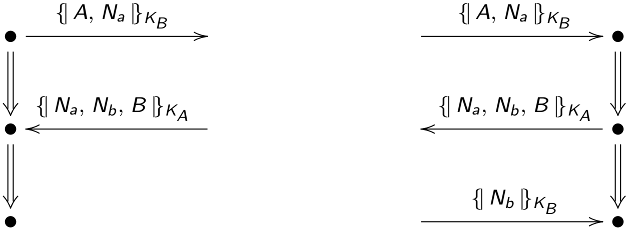

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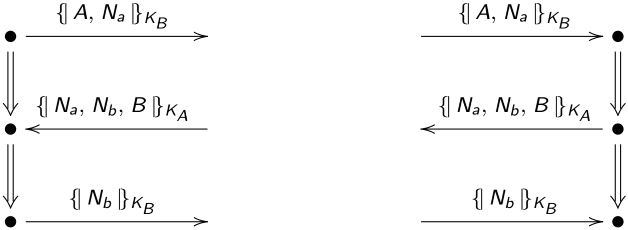

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Quick Summary

- Breaking and proving protocols: A tight duality
- Strand theory focuses on causal relations
- Questions: What about
  - mechanized support?
  - big, real protocols? For instance, TLS

Subjects for tomorrow