The Shapes of Protocols

Finding out what can happen

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Let's start simple

Blanchet's Simple Example Protocol

$$
\begin{tikzcd}
\{\!|\, \{\!|\, k \,|\!\}_{\mathsf{sk}(A)} \,|\!\}_{\mathsf{pubk}(B)} & \{\!|\, s \,|\!\}_k \arrow[d] \\
\bullet \arrow[u] \arrow[r, Rightarrow] & \bullet
\end{tikzcd}
$$

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What can happen, from initiator's point of view

Can K be disclosed?

$$\begin{tikzcd} \{\!| \{\!| k |\!\}_{\mathsf{sk}(A)} |\!\}_{\mathsf{pubk}(B)} & \bullet \arrow[l] \arrow[d, Rightarrow] & & \bullet & k \arrow[l] \\ & {} & & & \end{tikzcd}$$

$$\mathsf{pubk}(B)^{-1} \in \mathsf{non} \qquad k \in \mathsf{unique}$$

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Adversary can't do it
since $\mathsf{pubk}(B)^{-1} \in \mathsf{non}$ and $k \in \mathsf{unique}$

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Are there any unintended services?

Unintended Services for k?

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$$\begin{tikzcd}
\bullet \arrow[r, Rightarrow] & {\color{red}\bullet} \arrow[d] \\
\{\!|\, \{\!|\, k \,|\!\}_{\mathsf{sk}(A)} \,|\!\}_{\mathsf{pubk}(B)} \arrow[u] & \{\!|\, s \,|\!\}_k
\end{tikzcd}$$

Not a service for k because $k \not\sqsubseteq \{\!|\, s \,|\!\}_k$

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So this is impossible
a secrecy goal

Diagram above is dead

The Nonce Test

Generalizing previous reasoning

Suppose $c \in$ unique originates at regular n_0
and in msg(n_1), $c \sqsubseteq$ msg(n_1) is found outside all the encryptions $S =$

$$\{\!|\, t_1 \,|\!\}_{K_1},\ \ldots,\ \{\!|\, t_j \,|\!\}_{K_j}$$

Then either:

1. One of the decryption keys $K_1^{-1}, \ldots, K_j^{-1}$ is disclosed before n_1, or
2. Some regular m_1 sends c outside S and
   - $m_1 \preceq n_1$
   - if $m_0 \Rightarrow^+ m_1$, c was found in msg(m_0) only within S if at all

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We say that c escapes from S at m_1

Found outside

c is found outside S in t means:

Regarding t as an abstract syntax tree

there is a path p through the tree where

- $\mathrm{last}(p) = c$
- p never traverses key subterm of encryption $\{\!|\, t_1 \,|\!\}_K$
- p never traverses any $e \in S$

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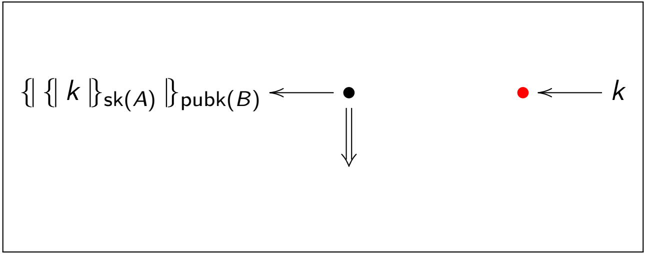
You can get to an ingredient occurrence of c
without crossing anything in S

An Example



k is found outside $\{\!|\ k\ |\!\}_{\mathsf{sk}(A)}$ in k
but only within it in $\{\!|\ \{\!|\ k\ |\!\}_{\mathsf{sk}(A)}\ |\!\}_{\mathsf{pubk}(B)}$

Found only within

c is found only within S in t means:

Regarding t as an abstract syntax tree

for every path p through the tree where

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You can't get to any ingredient occurrence of c
without crossing something in S

What can happen, from initiator's point of view

Another query $\mathbb{B}_1$ $\qquad$ t_0 is $\{\!|\, \{\!|\, k \,|\!\}_{\mathsf{sk}(A)} \,|\!\}_{\mathsf{pubk}(B)}$

$$\begin{tikzcd} \bullet \arrow[r] \arrow[d, Rightarrow] & t_0 \\ \bullet & \{\!|\, s \,|\!\}_k \arrow[l] \end{tikzcd}$$

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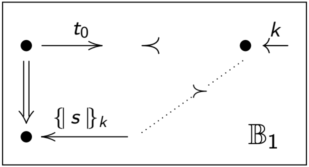
Either k is disclosed,
or $\{\!|\, s \,|\!\}_k$ comes from a regular source

One of the two possible explanations $\mathbb{B}_1$

t_0 is $\{\!|\, \{\!|\, k \,|\!\}_{\mathsf{sk}(A)} \,|\!\}_{\mathsf{pubk}(B)}$



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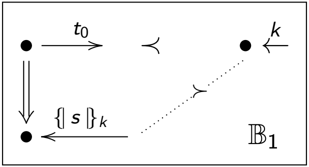

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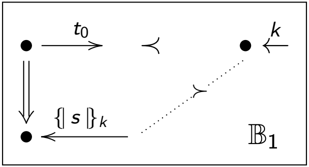

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Principle: Dead if any substructure is dead

The other possible explanation $\mathbb{B}_2$

t_0 is $\{\!|\, \{\!|\, k \,|\!\}_{\mathsf{sk}(A)} \,|\!\}_{\mathsf{pubk}(B)}$

$$\mathbb{B}_2 \qquad \begin{tikzcd}
\bullet \arrow[r, "t_0"] \arrow[d, Rightarrow] & \prec \arrow[r, "\{\!|\, \{\!|\, k \,|\!\}_{\mathsf{sk}(C)} \,|\!\}_{\mathsf{pubk}(D)}"] & \bullet \arrow[d, Rightarrow] \\
\bullet & \succ \arrow[l, "\{\!|\, s \,|\!\}_k"'] & \bullet \arrow[l, "\{\!|\, s \,|\!\}_k"']
\end{tikzcd}$$

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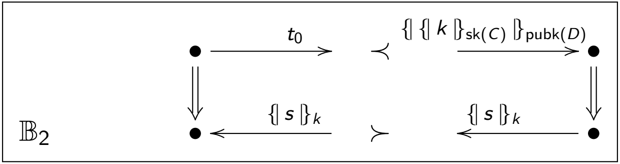
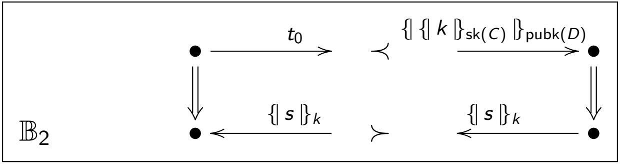


$$\mathsf{pubk}(B)^{-1} \in \mathsf{non} \qquad k \in \mathsf{unique}$$

Do we know $C = A$ and $D = B$ in $\mathbb{B}_2$?

Nonce test applies to k in $\mathbb{B}_2$

t_0 is $\{\!|\ \{\!|\ k\ |\!\}_{\mathsf{sk}(A)}\ |\!\}_{\mathsf{pubk}(B)}$



Any service to build a new message t_1
with $k \sqsubseteq t_1$?

Unintended Services transforming k?

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\begin{tikzcd}
\bullet \arrow[r, "t_0"] \arrow[d, Rightarrow] & {} \arrow[r, phantom, "\prec"] & {} \arrow[r, "\{\!| \{\!| k |\!\}_{\mathsf{sk}(C)} |\!\}_{\mathsf{pubk}(D)}"] & \bullet \arrow[d, Rightarrow] \\
\bullet & {} \arrow[l, "\{\!| s |\!\}_k"'] \arrow[r, phantom, "\succ"] & {} & \bullet \arrow[l, "\{\!| s |\!\}_k"']
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$$\mathsf{pubk}(B)^{-1} \in \mathsf{non} \qquad k \in \mathsf{unique}$$

Do we know $C = A$ and $D = B$ in $\mathbb{B}_2$?
Yes, since $C \neq A$ or $D \neq B$
would imply it's dead

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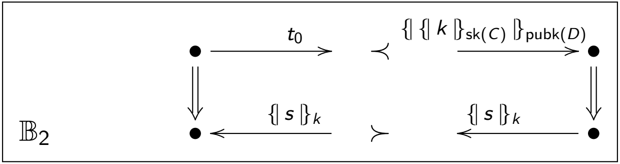

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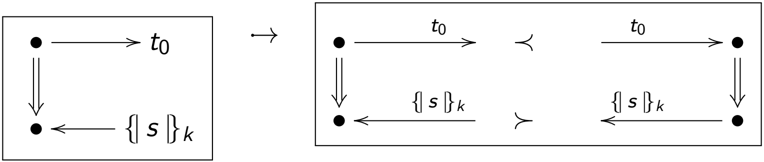
What did we prove?

t_0 is $\{\!|\, \{\!|\, k \,|\!\}_{\mathsf{sk}(A)} \,|\!\}_{\mathsf{pubk}(B)}$



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Any bundle containing at least $\mathbb{B}$
contains at least $\mathbb{B}_3$

A Tool to do this Reasoning: CPSA

Crypto Protocol Shape Analyzer

- Works with bundle fragments called skeletons starting from some $\mathbb{A}_0$
- While some skeleton $\mathbb{A}_i$ has unexplained parts, CPSA picks one
- Considers all enrichments $\mathbb{A}_j$ to explain it
- If none available, skeleton $\mathbb{A}_i$ is dead
- Branching stops when all parts explained
- Conclusion:

all bundles containing $\mathbb{A}_0$
contain one of its fully explained enrichments $\mathbb{A}_j$

The Encryption Test

Suppose that $\{\!|\, t \,|\!\}_K \sqsubseteq \text{msg}(n_1)$ where $n_1 \in \text{nodes}\mathbb{B}$. Then either:

1. Key K is disclosed before n_1 occurs,
   so that the adversary could construct $\{\!|\, t \,|\!\}_K$ from t; or
2. A regular $+$ node $m_1 \preceq n_1$ with

$$\{\!|\, t \,|\!\}_K \sqsubseteq \text{msg}(m_1)$$

   May choose m_1 least such