The Goals of Protocols

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Small semantic theory

For crypto protocols

- Skeletons $\mathbb{A}$:
  - Fragmentary parts of executions
  - Some skeletons are non-fragmentary: realized

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$\mathbb{A}$ is realized iff every test in $\mathbb{A}$ is solved

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  1. $\mathbb{A}_1 \in S$ implies $\mathbb{A}_1$ is realized
  2. $\mathbb{A}_1 \in S$ implies $\mathbb{A}_0 \rightsquigarrow^{*} \mathbb{A}_1$
  3. If $\mathbb{A}_0 \rightarrowtail \mathbb{B}$ and $\mathbb{B}$ is realized, then for some $\mathbb{A}_1 \in S$ $\mathbb{A}_1 \rightarrowtail \mathbb{B}$

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S: set of realized, accessible skeletons
that separates $\mathbb{A}_0$ from other realized skeletons

Shapes

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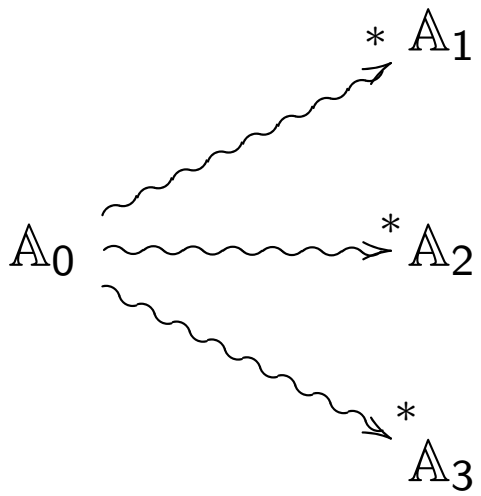
"Node-injective" maps

What security goals can CPSA validate?



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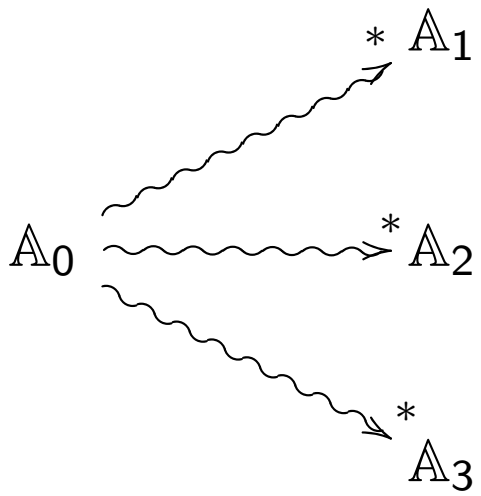
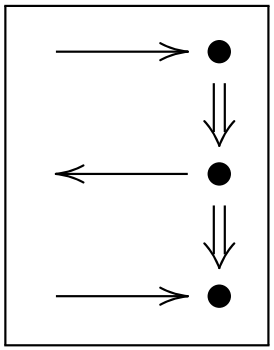


$$\mathbb{A}_0 \rightsquigarrow / \rightsquigarrow^* \cdot$$

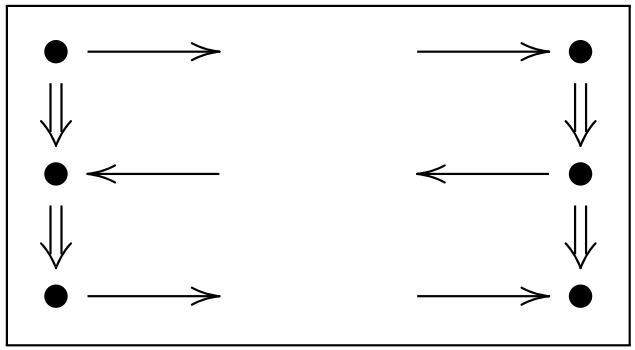
These are authentication and confidentiality goals resp. although allowing great variety

NS Weak Authentication

From responder's point of view



$\xrightarrow{H}$



$\mathrm{Resp}[A, B, N_a, N_b]$

$N_b \in \mathsf{unique}, \quad \mathsf{pubk}(A)^{-1} \in \mathsf{non}$

$\mathrm{Init}[A, B', N_a, N_b] \quad \mathrm{Resp}[A, B, N_a, N_b]$

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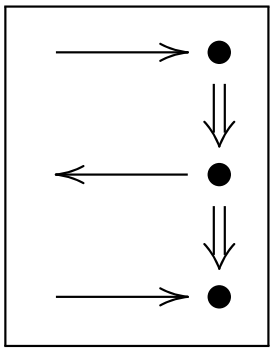
Stronger Authentication

From responder's point of view



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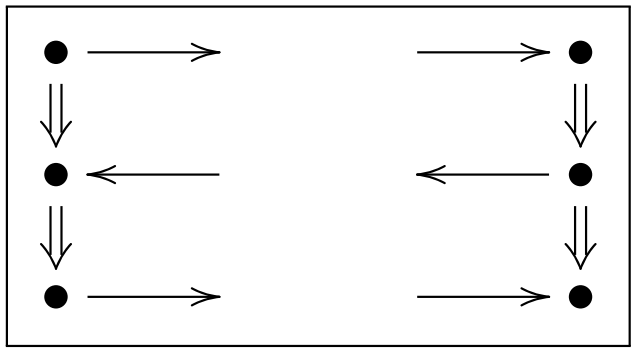

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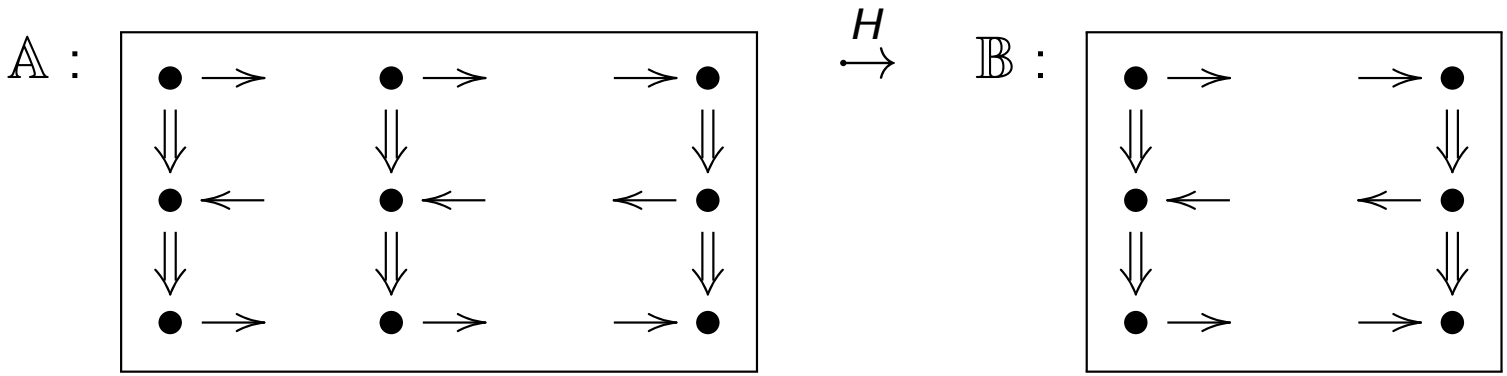
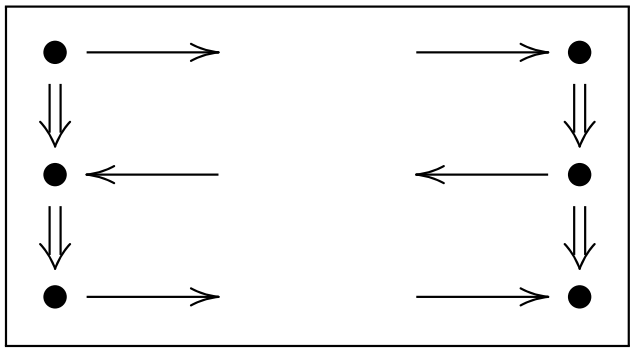
An Injectiveness Property



$$N_a, N_b \in \text{unique}, \quad \text{pubk}(A)^{-1} \in \text{non}$$

An Implicit Authentication Property



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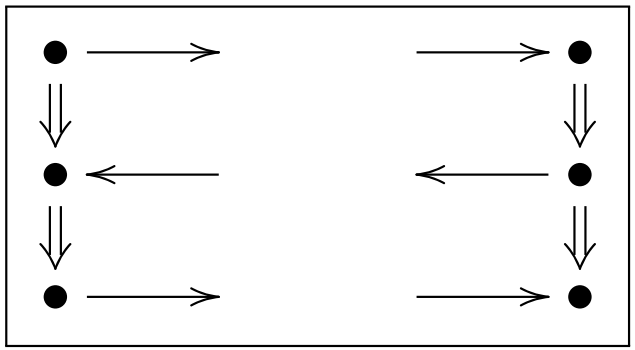
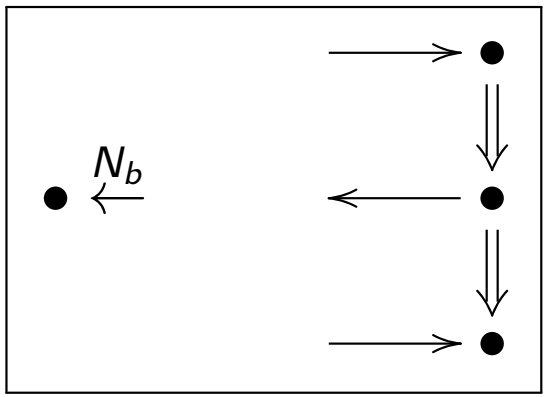


$\text{Init}[X, Y, N_a, N_b]$ $\text{Resp}[A, B, N_a, N_b]$ $\text{Init}[A, B, N_a, N_b]$ $\text{Resp}[A, B, N_a, N_b]$

A Confidentiality Property

$\mathbb{A}$:



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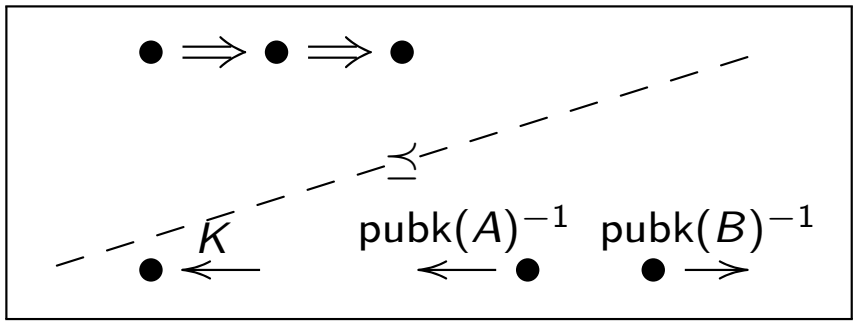
Forward Secrecy



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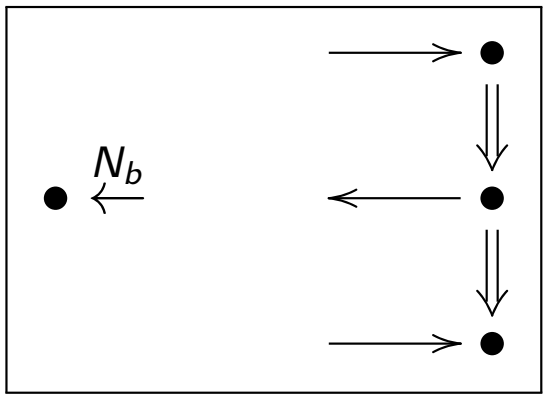

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Characterizing $\mathbb{A}$ by a formula

$$\begin{aligned}
&\mathtt{RespThd}(n) \wedge \mathtt{RespScd}(m) \wedge \mathtt{Coll}(m, n) \wedge \mathtt{Lsn}(\ell) \wedge \\
&\mathtt{Hear}(\ell, k) \wedge \mathtt{Peer}(m, a) \wedge \mathtt{Self}(m, b) \\
&\wedge \mathtt{MyNonce}(m, c) \wedge \mathtt{YourNonce}(m, d) \\
&\wedge \mathtt{Non}(\mathtt{inv}(\mathtt{pk}(a))) \wedge \mathtt{UnqAt}(m, c).
\end{aligned} \tag{1}$$

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Goal formula: (1) implies falsehood

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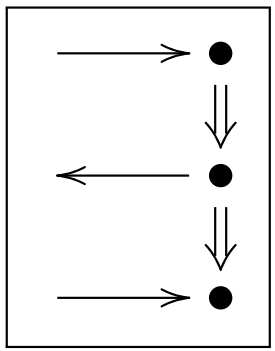

$\xrightarrow{H}$

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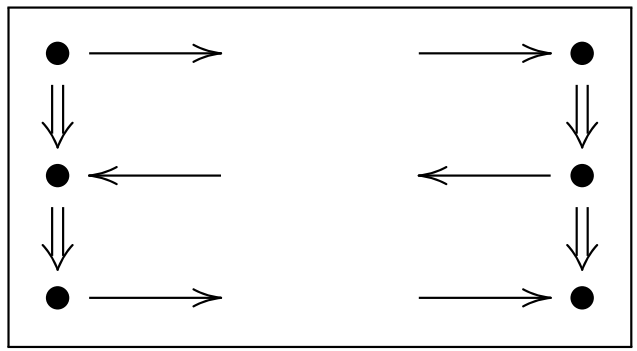

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Characterizing $\mathbb{B}$ by a formula

$$
\begin{aligned}
&(2) \wedge \texttt{InitThd}(n') \wedge \\
&\texttt{Self}(n', a) \wedge \texttt{Peer}(n', b) \wedge \\
&\texttt{YourNonce}(n', c) \wedge \texttt{MyNonce}(n', d)
\end{aligned}
\tag{3}
$$

Characterizing $\mathbb{B}$ by a formula

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Goal formula: (2) implies $\exists n'$. (3)

The Goal Language $\mathcal{GL}(\Pi)$ of a Protocol

- Protocol-independent vocabulary

Functions:	$\texttt{pk}(a)$	$\texttt{sk}(a)$	$\texttt{inv}(k)$
	$\texttt{lts}(a, b)$		
Relations:	$\texttt{Preceq}(m, n)$	$\texttt{Coll}(m, n)$	$=$
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- Protocol specific vocabulary for Π
  - Role predicates, e.g.

    `RespFst(n)` `RespScd(n)` `RespThd(n)` `Lsn(n)`

  - Parameter predicates, e.g.

    `Self(n,a)` `MyNonce(n,c)` `YourNonce(n,c)`

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- Every security goal equivalent to a set of formulas

$$\phi \supset \bigvee_{i<k} \exists \vec{y_k} \,.\, \phi_i$$

where ϕ and ϕ_i are conjunctions of atoms

Skeletons are Models, Goals are Implications

- Each skeleton $\mathbb{A}$ is a finite structure
  - Has a "characteristic formula" $\mathsf{cf}(\mathbb{A})$ expressing its content
  - $\mathsf{cf}(\mathbb{A})$ is a conjunction of atomic formulas
  - $\mathbb{A} \rightarrow \mathbb{B}$ if and only if $\mathbb{B} \models_\eta \mathsf{cf}(\mathbb{A})$

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- What if $\mathbb{A}$ has shape set S in protocol Π?
- Π bundles then satisfy:

$$\mathsf{cf}(\mathbb{A}) \supset \bigvee_{\mathbb{B} \in S} \exists \vec{y} \,.\, \mathsf{cf}(\mathbb{B})$$

where $\vec{y}$ includes all variables free in $\mathsf{cf}(\mathbb{B})$ but not in $\mathsf{cf}(\mathbb{A})$