

# Cryptoprotocol transformations that preserve goals

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Dedicated to [Bob Dylan](#)  
on his 70<sup>th</sup> birthday

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(Grant CNS-0952287).

8 Mar 2013  
BiSS

# A Protocol Design Problem

- Protocol designers

*reuse, adapt, combine*

existing protocols

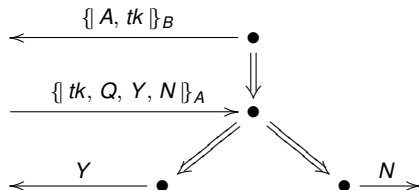
- Want to achieve additional

*authentication and confidentiality goals*

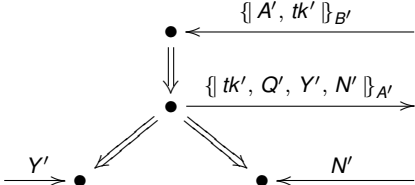
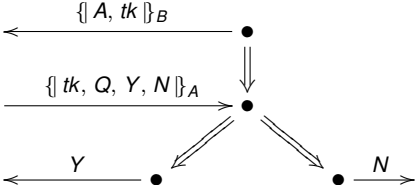
- Under what conditions are

*source protocol goals preserved?*

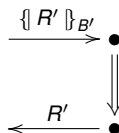
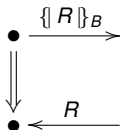
# Example: Q & A



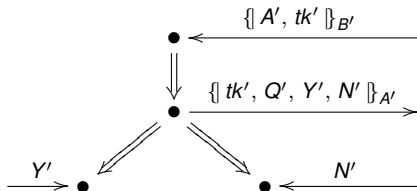
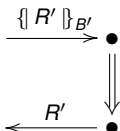
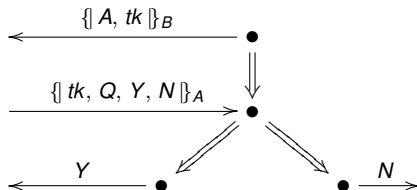
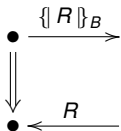
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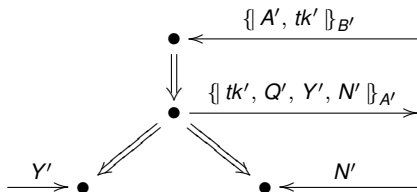
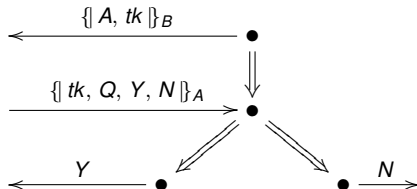
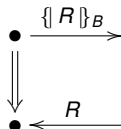
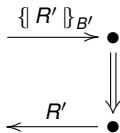
# HD: A simple protocol



# Q & A uses the HD idea



# Q & A uses the HD idea repeatedly



# Protocol Transformations

- Illustrated several protocol transformations

$$F: \text{HD} \rightarrow \text{Q \& A}$$

- Each  $F$  maps
  - ▶ transmissions to transmissions
  - ▶ receptions to receptionspreserving order etc.



# Protocol Design Problem

- Protocol designers transform existing protocols
- Want to achieve additional

*authentication and confidentiality goals*

- When do transformations preserve security goals?
- Key idea:

*Don't focus on syntactic similarity;  
Focus on similarity in the analysis process*

# Protocol analysis as scenario-finding

- Protocol analysis activity
  - ▶ Start with a partial execution
  - ▶ Enrich it, seeking a full execution
  - ▶ Branch when alternate enrichments possible

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$$A \xrightarrow{\lambda} B$$

# Protocol analysis as scenario-finding

Preserving goals means preserving analysis

- Protocol analysis activity
  - ▶ Start with a partial execution
  - ▶ Enrich it, seeking a full execution
  - ▶ Branch when alternate enrichments possible
- Scenario-finding in protocol  $\Pi$  is a labeled transition system

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- $\Pi_2$  preserves goals of  $\Pi_1$  if (roughly)

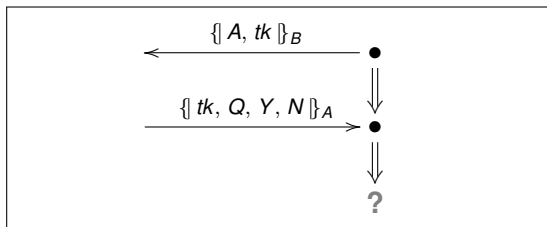
$$\begin{array}{l} \text{scenario-finding in } \Pi_1 \xrightarrow{\sim}_1 \text{ simulates} \\ \text{scenario-finding in } \Pi_2 \xrightarrow{\sim}_2 \end{array}$$

and

*When  $\xrightarrow{\sim}_1$  can progress, so can  $\xrightarrow{\sim}_2$*

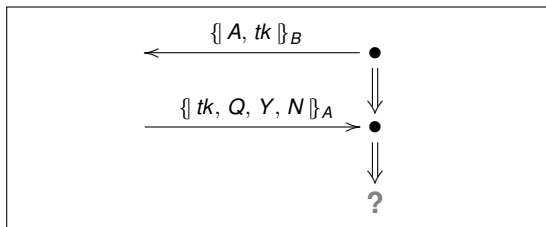
# Protocol Analysis for Q & A

One scenario



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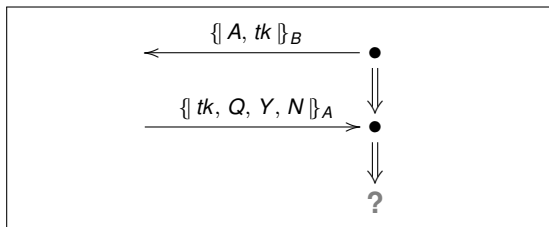
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What must be true if the Answerer offers a token and gets a query?

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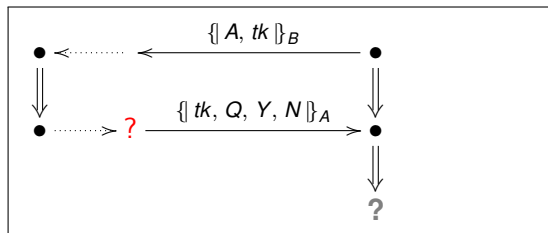
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It depends on the assumptions. Assume:

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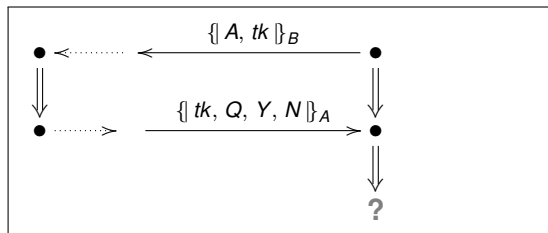
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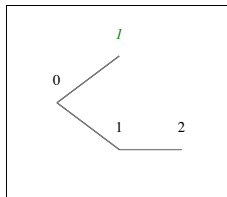
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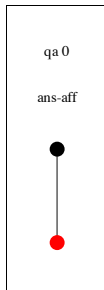
# Q & A: Scenario in CPSA

$tk$  fresh,  $\text{privk}(A)$ ,  $\text{privk}(B)$  uncompromised

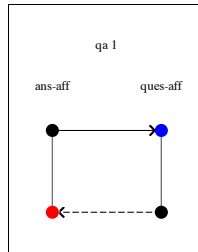
Tree 0.



Item 0, Child: 1, Seen Child: 1.

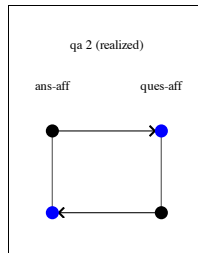


Item 1, Parent: 0, Child: 2.



```
(defstrand ans-aff 2 q tk y n a b)
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```

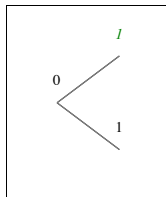
Item 2, Parent: 1.



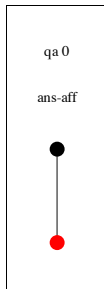
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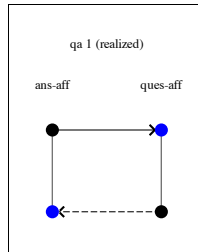
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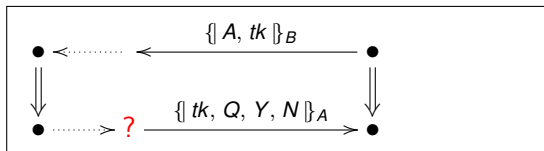
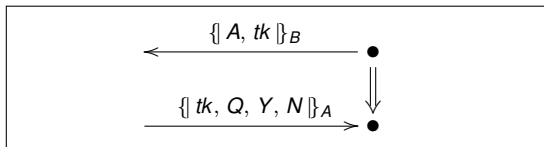


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# “Skeletons”

Assume:

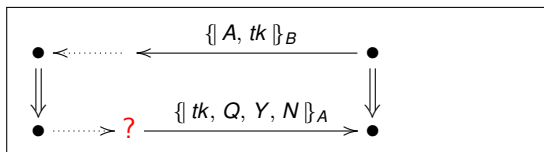
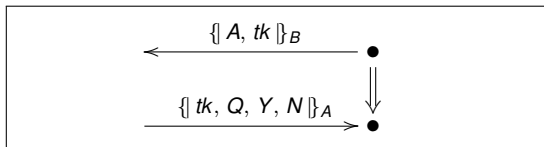
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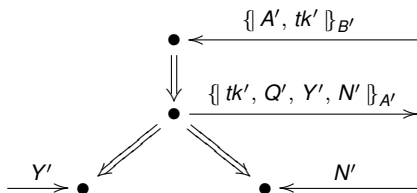
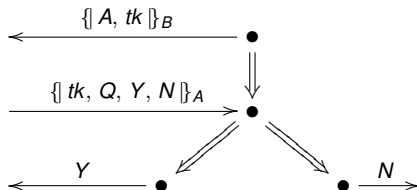
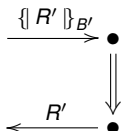
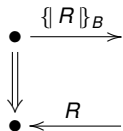
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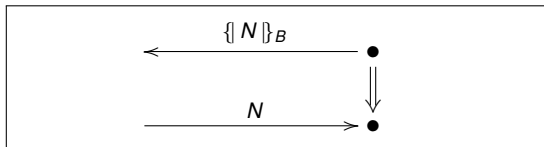


Skeleton  $\mathbb{A}$  is **realized** if possible execution

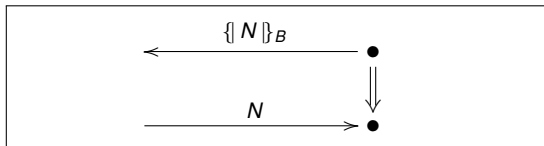
# Q & A uses the HD idea



# Protocol Analysis for HD



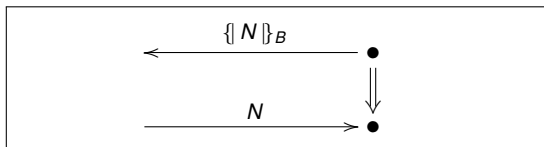
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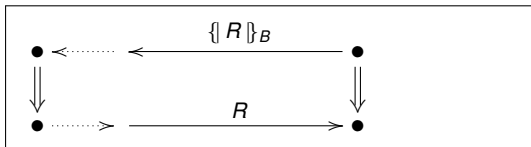


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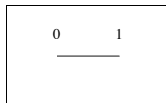
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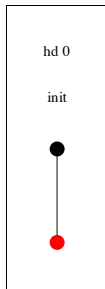
# HD example in CPSA

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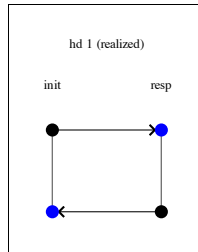
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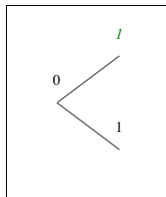


```
(defstrand init 2 r b)  
(defstrand resp 2 r b)
```

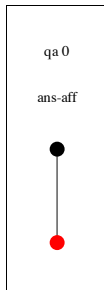
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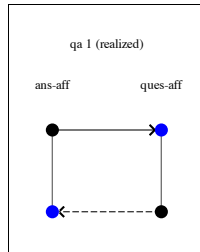
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# Comparing HD and Q & A Analyses

## Weakened Q & A Scenario

- Start with corresponding scenarios (skeletons)
- HD analysis can always take step to a skeleton  $\mathbb{A}$  when Q & A analysis can take step to corresponding  $\mathbb{B}$
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These two properties imply:  
 $\Pi_2$  *preserves all security goals achieved by  $\Pi_1$  for this starting point*

# Security goals

An authentication goal

$$\begin{array}{lll} \text{Init}(\mathbf{s}) \wedge & \text{AtPos2}(\mathbf{s}, \mathbf{m}) \wedge & \text{Peer}(\mathbf{s}, \mathbf{b}) \wedge \\ \text{Nonce}(\mathbf{s}, \mathbf{v}) \wedge & \text{Uncompr}(\text{inv}(\text{pk}(\mathbf{b}))) \wedge & \text{FreshAt}(\mathbf{n}, \mathbf{v}) \end{array}$$

implies

$$\begin{array}{lll} \exists \mathbf{s}' . & \text{Resp}(\mathbf{s}') \wedge & \text{AtPos2}(\mathbf{s}', \mathbf{m}') \quad \text{Self}(\mathbf{s}', \mathbf{b}) \\ & \text{Nonce}(\mathbf{s}', \mathbf{v}) & \text{Prec}(\mathbf{n}, \mathbf{m}') \quad \text{Prec}(\mathbf{m}', \mathbf{m}) \end{array}$$



# Security goals

A (bogus) confidentiality goal

$$\begin{array}{l} \text{Init}(\mathbf{s}) \wedge \quad \text{AtPos2}(\mathbf{s}, m) \wedge \quad \text{Peer}(\mathbf{s}, \mathbf{b}) \wedge \\ \text{Nonce}(\mathbf{s}, \mathbf{v}) \wedge \quad \text{Uncompr}(\text{inv}(\text{pk}(\mathbf{b}))) \wedge \quad \text{FreshAt}(n, \mathbf{v}) \wedge \\ \quad \quad \quad \text{Heard}(\mathbf{s}', \mathbf{v}) \end{array}$$

implies

falsehood

# Form of a security goal

$$\forall \bar{x} . (\phi \supset \exists \bar{y} . \psi_1 \vee \dots \vee \psi_j)$$

where  $\phi$  and each  $\psi_j$  is a conjunction of atomic formulas

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Key point:  
 $\phi$  and the  $\psi_i$  are  
*preserved under homomorphisms*

# Skeletons form models for a language $\mathcal{L}(\Pi)$

Some vocabulary shared among all  $\Pi$

$\text{Prec}(m, n)$      $\text{Fresh}(v)$      $\text{FreshAt}(n, v)$      $\text{Uncompr}(v)$

Some vocabulary is  $\Pi$ -specific

$\text{Init}(s)$      $\text{AtPos1}(s, n)$      $\text{AtPos2}(s, n)$   
 $\text{Peer}(s, b)$      $\text{Resp}(s)$      $\text{Lsn}(s)$   
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“Homomorphism” defined in normal way for  $\mathcal{L}(\Pi)$ -structures  $\mathbb{A}$

# Characteristic formula, characteristic skeleton

$\phi$  is a *characteristic formula* for  $\mathbb{A}$  under  $\eta_*$ :

$$\mathbb{B} \models_{\eta} \phi \quad \text{iff} \quad \exists H . H: \mathbb{A} \rightarrow \mathbb{B} \text{ and } \eta = H \circ \eta_*$$

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$\mathbb{A}$  is a *characteristic skeleton* for  $\phi$  under  $\eta_*$ :

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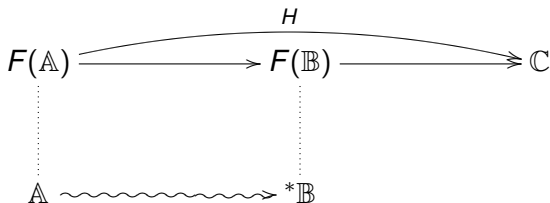
$\phi$  and the  $\psi_i$  are  
*preserved under homomorphisms*

The *starting point* of a security goal  
is the characteristic skeleton of  $\phi$



# What we want

For  $F$  to preserve goals



## Definition: Analysis LTS (ALTS)

$S \subseteq \text{Skel}(\Pi)$

$\wedge$  contains **dead**

(no realized images)

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$\cdot \rightsquigarrow \cdot \subseteq S \times \Lambda \times S$  is a ALTS iff

(A) If  $A \in S$ , then  $\exists B . A \rightsquigarrow B$  iff  $A$  is not realized;

(B) If  $A \xrightarrow{\ell} B$ , then:

① If  $\ell = \text{dead}$ , then  $\rightsquigarrow$  stutters, and  $A$  has no realized images;

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- 3 If  $J: A \rightarrow C$ , with realized  $C$ , then

$\exists B'$  s.t.  $A \xrightarrow{\ell} B'$  and

$J = K \circ H$ , where  $A \xrightarrow{H} B' \xrightarrow{K} C$

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- 3 If  $J: A \rightarrow C$ , with realized  $C$ , then

$\exists B'$  s.t.  $A \rightsquigarrow^{\ell} B'$  and

$J = K \circ H$ , where  $A \xrightarrow{H} B' \xrightarrow{K} C$

$\{B: A \rightsquigarrow^{\ell} B\}$  called the  $\ell$ -cohort for  $A$

# Transformations and ALTS

Assume  $F: \Pi_1 \rightarrow \Pi_2$  a transformation  
 $G: \Lambda_1 \rightarrow \Lambda_2$  a label map with  $G^{-1}(\{\text{dead}\}) = \{\text{dead}\}$   
 $\rightsquigarrow_1, \rightsquigarrow_2$  ALTS for  $\Pi_1, \Pi_2$  resp.

$F, G$  preserve progress iff  $\mathbb{A} \rightsquigarrow_1^{\ell}$  implies  $F(\mathbb{A}) \rightsquigarrow_2^{G(\ell)}$

$\rightsquigarrow_1$  simulates  $\rightsquigarrow_2$  for  $F, G$  iff  $F(\mathbb{A}) \rightsquigarrow_2^{G(\ell)} \mathbb{B}'$  implies

for some  $\mathbb{B}$ ,  $F(\mathbb{B}) = \mathbb{B}'$  and  $\mathbb{A} \rightsquigarrow_1^{\ell} \mathbb{B}$

# Theorem: $\rightsquigarrow_1^*$ witnesses for $\Pi_2$ executions

Assume  $F: \Pi_1 \rightarrow \Pi_2$  a transformation

$G: \Lambda_1 \rightarrow \Lambda_2$  a label map with  $G^{-1}(\{\text{dead}\}) = \{\text{dead}\}$

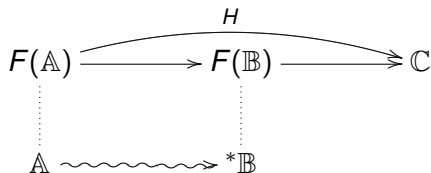
$\rightsquigarrow_1, \rightsquigarrow_2$  ALTS for  $\Pi_1, \Pi_2$  resp.

with

progress and simulation

For every  $\Pi_2$ -realized  $\mathbb{C}$ , if  $H: F(\mathbb{A}) \rightarrow \mathbb{C}$

There is a  $\Pi_1$ -realized  $\mathbb{B}$  such that  $\mathbb{A} \rightsquigarrow_1^* \mathbb{B}$  and:



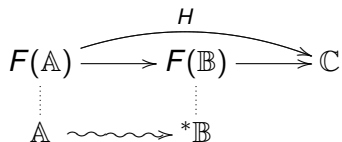


# Preserving security goals

If  $\Pi_1$  achieves goal  $\Psi$

$$\forall \vec{x} . (\phi_0 \supset \exists \vec{y} . \phi_1 \vee \dots \vee \phi_k)$$

and  $F: \Pi_1 \rightarrow \Pi_2$  satisfies



then  $\Pi_2$  achieves  $Tr_F(\Psi)$