1. Run the Ford-Fulkerson algorithm to find the maximum flow in the following flow network: (10 points)

2. Prove or give a counterexample: In any flow network with directed edges and positive capacities associated with each edge and for any minimum cut \((S, T)\) of the network, increasing the capacity of any edge from \(S\) to \(T\) will increase the maximum flow, i.e. the amount of flow the network can carry. (15 points)

3. In a directed graph a set of paths is edge-disjoint if their edge sets are disjoint, i.e. no two paths share an edge (although they may share vertices). Given a directed graph \(G = (V, E)\) with two distinguished vertices \(s\) and \(t\), give an efficient algorithm to find the maximum number of edge-disjoint \(s-t\) (directed) paths in \(G\). (Hint: Use a flow network.). (15 points)

4. In a directed graph a set \(F \subseteq E\) of edges separates \(s\) from \(t\) if, after removing the edges \(F\) from the graph, no \(s-t\) paths remain in the graph. Prove the following: In every directed graph with nodes \(s\) and \(t\), the maximum number of edge-disjoint \(s-t\) paths is equal to the minimum number of edges whose removal separates \(s\) from \(t\). (15 points)
5. What is the least significant decimal digit of $17^{17^{17}}$? (15 points)

6. Exercise 31.7-1 on page 964. (15 points)

7. Problem 34-1 a.,b.,c. on page 1102. (15 points)