CS 4120 Analysis of Algorithms
A term 2018

Practice Final Exam

These problems are sample problems for the final exam, so you may expect similar problems in the final. Do not hand in your solutions. Solutions will be handed out, discussed (and posted on the web) on Tuesday. The final exam is from the material of the whole course, but there will be only one or two problems from the first half. The final exam is a closed book exam, but you may use two sheets of paper (so you may use your midterm sheet) with notes on it. Each problem is worth 20 points.

1. Use the Master Theorem to find the asymptotic solution for the following recurrence: \( T(n) = 5T(\frac{n}{2}) + n^3 \).

2. Use indicator random variables to find the expected number of bins that remain empty when \( m \) balls are distributed into \( n \) bins uniformly at random.

3. Prove or give a counterexample: For any graph \( G \) with distinct positive weights associated with each edge and for any cut of \( G \), the minimum spanning tree of \( G \) contains exactly one edge belonging to the cut.

4. Prove or give a counterexample: Let \( G \) be a flow network with directed edges and positive capacities associated with each edge and let \((S, T)\) be a minimum cut of the network. If we increase the capacity of every edge in \( G \) by 1 the cut \((S, T)\) is still a minimum cut.

5. The longest-cycle problem is the problem of determining a cycle of maximum length in a graph \( G \). Formulate a related decision problem, and show that the decision problem is \( NP \)-complete. You may use any of the \( NP \)-complete problems learned in class for your reduction.