1. Exercise 24.3-5 on page 663. (15 points)

2. Give a simple example of a directed graph with negative-weight edges (but no negative-weight cycles) for which Dijkstra’s algorithm produces incorrect answers. Where does the proof of correctness break down when negative-weight edges are allowed? (15 points)

3. Run the Ford-Fulkerson algorithm to find the maximum flow in the following flow network: (15 points)

4. Suppose you seek a flow \( f \) in a multiple source, multiple sink network \( G = (V, E) \), where \( V = \{s_1, \ldots, s_m, t_1, \ldots, t_n, v_1, \ldots, v_p\} \) with sources \( s_1, \ldots, s_m \) and sinks \( t_1, \ldots, t_n \) in which each source \( s_i \) produces exactly \( a_i \) units of flow, \( 1 \leq i \leq m \), and each sink consumes exactly \( b_j \) units of flow, \( 1 \leq j \leq n \). Describe an efficient algorithm to test whether flow \( f \) exists. (20 points)

5. Prove or give a counterexample: In any flow network with directed edges and positive capacities associated with each edge and for any minimum cut \((S, T)\) of the network, increasing the capacity of any edge from \( S \) to \( T \) will increase the maximum flow, i.e. the amount of flow the network can carry. (15 points)

6. Problem 34-1 a., b., c. on page 1102. (20 points)