Solutions of the Sample Problems for the Midterm Exam

1. Give a regular expression that represents the set of strings over \( \Sigma = \{a, b\} \) that contain the substring \( ab \) and the substring \( ba \).

   Solution:
   \[
   (a^+b^+a(a \cup b)^*) \cup (b^+a^+b(a \cup b)^*)
   \]

   (20 points)

2. Consider the following grammar \( G \):

   \[
   S \rightarrow SAB|\lambda \\
   A \rightarrow aA|a \\
   B \rightarrow bB|\lambda
   \]

   (a) Give a leftmost derivation of \( abbaab \).

   (b) Build the derivation tree for the derivation in part (1).

   (c) What is \( L(G) \)?

   Solution:
(a) The following is a leftmost derivation of \textit{abbaab}:

\[
\begin{align*}
S & \Rightarrow SAB \\
   & \Rightarrow SABAB \\
   & \Rightarrow ABAB \\
   & \Rightarrow aBAB \\
   & \Rightarrow abBAB \\
   & \Rightarrow abbBAB \\
   & \Rightarrow abbAB \\
   & \Rightarrow abbaAB \\
   & \Rightarrow abbaaB \\
   & \Rightarrow abbaabB \\
   & \Rightarrow abbaab
\end{align*}
\]

(b)

(c)

\[
L(G) = a(a \cup b)^* \cup \lambda
\]

(20 points)

3. Construct a regular grammar over the alphabet \(\Sigma = \{a, b, c, d\}\) whose language is the set of strings that contain exactly two \(b\)-s.

\textbf{Solution:}

The following is a \textit{regular} grammar over \(\{a, b, c, d\}\) whose language is the set of strings containing exactly two \(b\)-s:

\[
\begin{align*}
S & \rightarrow aS \mid cS \mid dS \mid bB \\
B & \rightarrow aB \mid cB \mid dB \mid bC \\
C & \rightarrow aC \mid cC \mid dC \mid \lambda
\end{align*}
\]
4. Consider the following grammar $G$:

$$
S \rightarrow aSA | \lambda \\
A \rightarrow bA | \lambda
$$

(a) Give a regular expression for $L(G)$.

(b) Is $G$ ambiguous? Explain your answer.

Solution:

(a) The following is a regular expression for $L(G)$:

$$a^+b^* \cup \lambda$$

(b) Yes the grammar is ambiguous. Here are two different leftmost derivations for the string $aabb$:

$S \Rightarrow aSA$
$\Rightarrow aaSAA$
$\Rightarrow aaAA$
$\Rightarrow aabAA$
$\Rightarrow aabAA$
$\Rightarrow aabbA$
$\Rightarrow aabb$

and

$S \Rightarrow aSA$
$\Rightarrow aaSAA$
$\Rightarrow aaAA$
$\Rightarrow aaA$
$\Rightarrow aabA$
$\Rightarrow aabbA$
$\Rightarrow aabb$

5. Design a DFA that accepts the language consisting of the set of those strings over $\{a, b, c\}$ in which the number of $a$'s plus the number of $b$'s plus twice the number of $c$'s is divisible by six.
Solution:
The state diagram of a DFA is

(20 points)