Solutions of the Sample Problems for the Midterm Exam

1. Give a regular expression that represents the set of strings over \( \Sigma = \{a, b\} \) that contain the substring \( ab \) and the substring \( ba \).

   Solution:
   \[
   \left( a^+b^+a(a \cup b)^+ \right) \cup \left( b^+a^+b(a \cup b)^+ \right)
   \]

   (20 points)

2. Consider the following grammar \( G \):

   \[
   \begin{align*}
   S & \rightarrow SAB | \lambda \\
   A & \rightarrow aA | a \\
   B & \rightarrow bB | \lambda 
   \end{align*}
   \]

   (a) Give a leftmost derivation of \( abbaab \).
   (b) Build the derivation tree for the derivation in part (1).
   (c) What is \( L(G) \)?

   Solution:
(a) The following is a leftmost derivation of $abbaab$:

$$
S \Rightarrow SAB
\Rightarrow SABAB
\Rightarrow ABAB
\Rightarrow aBAB
\Rightarrow abBAB
\Rightarrow abbBAB
\Rightarrow abbAB
\Rightarrow abbaAB
\Rightarrow abbaaB
\Rightarrow abbaabB
\Rightarrow abbaab
$$

(b)

(c)

$$L(G) = a(a \cup b)^* \cup \lambda$$

(20 points)

3. Construct a regular grammar over the alphabet $\Sigma = \{a, b, c, d\}$ whose language is the set of strings that contain exactly two $b$-s.

Solution:

The following is a regular grammar over $\{a, b, c, d\}$ whose language is the set of strings containing exactly two $b$-s:

$$
S \rightarrow aS \mid cS \mid dS \mid bB
\quad B \rightarrow aB \mid cB \mid dB \mid bC
\quad C \rightarrow aC \mid cC \mid dC \mid \lambda
$$
4. Consider the following grammar $G$:

\[
S \rightarrow aSA | \lambda \\
A \rightarrow bA | \lambda
\]

(a) Give a regular expression for $L(G)$.
(b) Is $G$ ambiguous? Explain your answer.

Solution:
(a) The following is a regular expression for $L(G)$:

\[a^{+}b^{*} \cup \lambda\]

(b) Yes the grammar is ambiguous. Here are two different leftmost derivations for the string $aabb$:

\[
S \Rightarrow aSA \\
\Rightarrow aaSAA \\
\Rightarrow aaAA \\
\Rightarrow aabAA \\
\Rightarrow aabbAA \\
\Rightarrow aabbA \\
\Rightarrow aabb
\]

and

\[
S \Rightarrow aSA \\
\Rightarrow aaSAA \\
\Rightarrow aaAA \\
\Rightarrow aaA \\
\Rightarrow aabA \\
\Rightarrow aabbA \\
\Rightarrow aabb
\]

5. Design a DFA that accepts the language consisting of the set of those strings over \{a, b, c\} in which the number of a's plus the number of b's plus twice the number of c's is divisible by six.
Solution:

The state diagram of a DFA is

(20 points)