Solutions of the Sample Problems for the Midterm Exam

1. Give a regular expression that represents the set of strings over $\Sigma = \{a, b\}$ that contain the substring $ab$ and the substring $ba$.

   Solution:
   \[
   (a^+b^+a(a \cup b)^*) \cup (b^+a^+b(a \cup b)^*)
   \]

   (20 points)

2. Consider the following grammar $G$:

   $S \rightarrow SAB | \lambda$
   $A \rightarrow aA | a$
   $B \rightarrow bB | \lambda$

   (a) Give a leftmost derivation of $abbaab$.
   (b) Build the derivation tree for the derivation in part (1).
   (c) What is $L(G)$?

   Solution:
(a) The following is a leftmost derivation of \textit{abbaab}:

\[
S \Rightarrow SAB \\
\Rightarrow SABAB \\
\Rightarrow ABAB \\
\Rightarrow aBAB \\
\Rightarrow abBAB \\
\Rightarrow abbBAB \\
\Rightarrow abbAB \\
\Rightarrow abbaAB \\
\Rightarrow abbaaaB \\
\Rightarrow abbaabB \\
\Rightarrow abbaab 
\]

(b)

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S
A
B
a
b

L(G) = a(a \cup b)^* \cup \lambda
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(20 points)

3. Construct a regular grammar over the alphabet \( \Sigma = \{a, b, c, d\} \) whose language is the set of strings that contain exactly two \( b \)-s.

\textbf{Solution:}

The following is a \textit{regular} grammar over \( \{a, b, c, d\} \) whose language is the set of strings containing exactly two \( b \)-s:

\[
S \rightarrow aS \mid cS \mid dS \mid bB \\
B \rightarrow aB \mid cB \mid dB \mid bC \\
C \rightarrow aC \mid cC \mid dC \mid \lambda
\]
4. Consider the following grammar $G$:

$$S \rightarrow aSA | \lambda$$
$$A \rightarrow bA | \lambda$$

(a) Give a regular expression for $L(G)$.
(b) Is $G$ ambiguous? Explain your answer.

**Solution:**

(a) The following is a regular expression for $L(G)$:

$$a^+b^* \cup \lambda$$

(b) Yes the grammar is ambiguous. Here are two different leftmost derivations for the string $aabb$:

- $S \Rightarrow aSA$
  $\Rightarrow aaSAA$
  $\Rightarrow aaAA$
  $\Rightarrow aabAA$
  $\Rightarrow aabbAA$
  $\Rightarrow aabbA$
  $\Rightarrow aabb$

- $S \Rightarrow aSA$
  $\Rightarrow aaSAA$
  $\Rightarrow aaAA$
  $\Rightarrow aaA$
  $\Rightarrow aabA$
  $\Rightarrow aabbA$
  $\Rightarrow aabb$

(20 points)

5. Design a DFA that accepts the language consisting of the set of those strings over $\{a, b, c\}$ in which the number of $a$’s plus the number of $b$’s plus twice the number of $c$’s is divisible by six.
Solution:
The state diagram of a DFA is

(20 points)