Sample Problems for the Midterm Exam

These problems are sample problems for the midterm exam, so you may expect similar problems in the midterm. Do not hand in your solutions. Solutions will be handed out (and posted on the web) on Monday, the day before the exam. The midterm exam is a closed book exam, but you may use one sheet of paper (written on both sides) with notes on it.

1. Give a regular expression that represents the set of strings over $\Sigma = \{a, b\}$ that contain the substring $ab$ and the substring $ba$. (20 points)

2. Consider the following grammar $G$:

\[
S \rightarrow SAB|\lambda \\
A \rightarrow aA|a \\
B \rightarrow bB|\lambda
\]

(a) Give a leftmost derivation of $abbaab$.

(b) Build the derivation tree for the derivation in part (a).

(c) What is $L(G)$? (20 points)

3. Construct a regular grammar over the alphabet $\Sigma = \{a, b, c, d\}$ whose language is the set of strings that contain exactly two $b$-s. (20 points)

4. Consider the following grammar $G$:

\[
S \rightarrow aSA|\lambda \\
A \rightarrow bA|\lambda
\]

(a) Give a regular expression for $L(G)$.

(b) Is $G$ ambiguous? Explain your answer.
5. Design a DFA that accepts the language consisting of the set of those strings over \( \{a, b, c\} \) in which the number of \( a \)'s plus the number of \( b \)'s plus twice the number of \( c \)'s is divisible by six. (20 points)

(20 points)