1. Give a regular expression that represents the set of all non-empty strings over $\Sigma = \{a, b, c\}$ in which all the $a$’s precede the $b$’s, which in turn precede the $c$’s.

Solution:

$$a^+ b^+ c^+ \cup a^+ b^+ c^* \cup a^+ b^* c^+$$

(20 points)

2. Consider the following grammar $G$:

$$S \rightarrow XY$$
$$X \rightarrow aX | bX | a$$
$$Y \rightarrow Ya | Yb | a$$

(a) Give a leftmost derivation of $abaabb$.

(b) Build the derivation tree for the derivation in part (1).

(c) What is $L(G)$?

Solution:

(a) The following is a leftmost derivation of $abaabb$:

$$S \Rightarrow XY
\Rightarrow aXY
\Rightarrow abXY
\Rightarrow abaY
\Rightarrow abaYb
\Rightarrow abaYbb
\Rightarrow abaabb$$

(b)
3. Construct a context-free grammar over the alphabet $\Sigma = \{a, b, c\}$ whose language is 

$$L = \{a^i b^j c^k \mid 0 \leq 2(i + j) \leq k\}.$$ 

**Solution:** The grammar is:

$$S \rightarrow aSc | A$$
$$A \rightarrow bAcc | Ac | \lambda$$

(20 points)

4. Construct two regular grammars, one ambiguous and one unambiguous, that generate the language consisting of the set of strings over $\Sigma = \{a, b\}$ in which the number of $b$’s is divisible by three.

**Solution:**

Unambiguous regular grammar:

$$S \rightarrow aS | bA | \lambda$$
$$A \rightarrow aA | bB$$
$$B \rightarrow aB | bS$$

(c) 

$$L(G) = (a \cup b)^* a a (a \cup b)^*$$

(20 points)
Ambiguous regular grammar:

\[
S \rightarrow aS \mid bA \mid \lambda \\
A \rightarrow aA \mid bB \mid bC \\
B \rightarrow aB \mid bS \\
C \rightarrow aC \mid bS
\]

It is ambiguous because there are two different leftmost derivations for the string \(bbb\):

- \(S \Rightarrow bA \Rightarrow bbB \Rightarrow bbbS \Rightarrow bbb\)
- \(S \Rightarrow bA \Rightarrow bbC \Rightarrow bbbS \Rightarrow bbb\)

(20 points)

5. Design a DFA that accepts the language consisting of the set of those strings over \(\{a, b\}\) that do not contain the substring \(aaa\).

**Solution:**

The state diagram of a DFA is

(20 points)