Solutions for the Midterm Exam

1. Give a regular expression that represents the set of all strings over $\Sigma = \{a, b\}$ in which every $b$ is either immediately preceded or immediately followed by $a$, for example as in $abba$, $bab$ and $a$.

   Solution:
   \[
   (ba \cup ab \cup bab \cup a)^* \\
   \text{or} \\
   ((b \cup \lambda)a(b \cup \lambda))^*
   \]

   (20 points)

2. Consider the following grammar $G$:

   \[
   S \rightarrow XY \\
   X \rightarrow Xa \mid Xb \mid b \\
   Y \rightarrow aY \mid bY \mid a
   \]

   (a) Give a leftmost derivation of $bbaaba$.

   (b) Build the derivation tree for the derivation in part (a).

   (c) What is $L(G)$?

   Solution:

   (a) The following is a leftmost derivation of $bbaaba$:

   \[
   S \Rightarrow XY \\
   \Rightarrow XbY \\
   \Rightarrow XabY \\
   \Rightarrow XaabY \\
   \Rightarrow XbaabY \\
   \Rightarrow bbaabY \\
   \Rightarrow bbaaba
   \]
L(G) = b(a ∪ b)*a

3. Construct two regular grammars, one ambiguous and one unambiguous, that generate the language determined in the previous problem 2(c).

Solution:
Unambiguous regular grammar:

\[
\begin{align*}
S & \rightarrow bA \\
A & \rightarrow aA | bA | a
\end{align*}
\]

Ambiguous regular grammar:

\[
\begin{align*}
S & \rightarrow bA | bB \\
A & \rightarrow aA | bA | a \\
B & \rightarrow aB | bB | a
\end{align*}
\]

It is ambiguous because there are two different leftmost derivations for the string \(ba\):

\[
\begin{align*}
S & \Rightarrow bA \\
& \Rightarrow ba
\end{align*}
\]

and

\[
\begin{align*}
S & \Rightarrow bB \\
& \Rightarrow ba
\end{align*}
\]

(20 points)
4. Design a DFA that accepts the language determined in problem 2(c). (20 points)

Solution:

The state diagram of a DFA is

![DFA Diagram]

5. Construct the state diagram of a DFA equivalent to the following NFA by using the subset construction method. What is the language accepted by these machines? (20 points)

Solution:

![NFA Diagram and DFA Diagram]
Here the states correspond to the following subsets: $A = \{q_0\}$, $B = \{q_0, q_1\}$, $C = \{q_0, q_1, q_2\}$ and $D = \{q_0, q_2\}$. The language is: $(a \cup b)^* bb(a \cup b)^*$. 