Solutions for the Midterm Exam

1. Give a regular expression that represents the set of strings over $\Sigma = \{a, b\}$ with an even number of $a$’s or an odd number of $b$’s.

Solution:

$$(b^*ab^*a)^*b^* \cup (a^*ba^*b)^*a^*ba^*$$

(20 points)

2. Consider the following grammar $G$:

$$
S \rightarrow XY \\
X \rightarrow aX \mid bX \mid a \\
Y \rightarrow Ya \mid Yb \mid a
$$

(a) Give a leftmost derivation of $abaabb$.

(b) Build the derivation tree for the derivation in part (a).

(c) What is $L(G)$?

Solution:

(a) The following is a leftmost derivation of $abaabb$:

$$S \Rightarrow XY \\
\Rightarrow aXY \\
\Rightarrow abXY \\
\Rightarrow abaY \\
\Rightarrow abaYb \\
\Rightarrow abaYbb \\
\Rightarrow abaabb$$

(b)
3. Construct two regular grammars, one ambiguous and one unambiguous, that generate the language consisting of the set of strings over $\Sigma = \{a, b\}$ in which the number of $a$’s is divisible by three.

Solution:

Unambiguous regular grammar:

$$
S \rightarrow bS \mid aA \mid \lambda \\
A \rightarrow bA \mid aB \\
B \rightarrow bB \mid aS
$$

Ambiguous regular grammar:

$$
S \rightarrow bS \mid aA \mid \lambda \\
A \rightarrow bA \mid aB \mid aC \\
B \rightarrow bB \mid aS \\
C \rightarrow bC \mid aS
$$

It is ambiguous because there are two different leftmost derivations for the string $aaa$:

$$
S \Rightarrow aA \Rightarrow aaB \Rightarrow aaaS \Rightarrow aaa
$$
and

\[
S \Rightarrow aA \\
\Rightarrow aaC \\
\Rightarrow aaaS \\
\Rightarrow aaa
\]

(20 points)

4. Design a DFA that accepts the language consisting of the set of those strings over \( \{a, b\} \) that do not contain the substring \( aaa \).

Solution:
The state diagram of a DFA is

![DFA Diagram](image)

(20 points)

5. Construct the state diagram of a DFA equivalent to the following NFA. What is the language accepted by these machines?
Solution:

The language is \( a^+ b^*(ab^+)^* \)

(20 points)