1. Give a regular expression that represents the set of all strings over \( \Sigma = \{a, b\} \) in which every \( a \) is either immediately preceded or immediately followed by \( b \), for example \( baab, aba \) and \( b \).

Solution:

\[
(ab \cup ba \cup aba \cup b)^*
\]

or

\[
((a \cup \lambda)b(a \cup \lambda))^*
\]

(20 points)

2. Consider the following grammar \( G \):

\[
S \rightarrow XY \\
X \rightarrow Xa \mid Xb \mid a \\
Y \rightarrow aY \mid bY \mid b
\]

(a) Give a leftmost derivation of \( abaabb \).

(b) Build the derivation tree for the derivation in part (a).

(c) What is \( L(G) \)?

Solution:

(a) The following is a leftmost derivation of \( abaabb \):

\[
S \Rightarrow XY \\
\Rightarrow XbY \\
\Rightarrow XabY \\
\Rightarrow XaabY \\
\Rightarrow XbaabY \\
\Rightarrow abaabY \\
\Rightarrow abaabb
\]

(20 points)
(b) \[ L(G) = a(a \cup b)^*b \]

(20 points)

3. Construct two regular grammars, one ambiguous and one unambiguous, that generate the language determined in the previous problem 2(c).

Solution:

Unambiguous regular grammar:

\[
\begin{align*}
S & \rightarrow aA \\
A & \rightarrow aA \mid bA \mid b
\end{align*}
\]

Ambiguous regular grammar:

\[
\begin{align*}
S & \rightarrow aA \mid aB \\
A & \rightarrow aA \mid bA \mid b \\
B & \rightarrow aB \mid bB \mid b
\end{align*}
\]

It is ambiguous because there are two different leftmost derivations for the string \(ab\):

\[
\begin{align*}
S & \Rightarrow aA \\
& \Rightarrow ab
\end{align*}
\]

and

\[
\begin{align*}
S & \Rightarrow aB \\
& \Rightarrow ab
\end{align*}
\]

(20 points)
4. Design a DFA that accepts the language determined in problem 2(c).
   (20 points)

   **Solution:**
   
   The state diagram of a DFA is

   ![State Diagram](image)

5. Construct the state diagram of a DFA equivalent to the following NFA by using the subset construction method. What is the language accepted by these machines? (20 points)

   ![State Diagram](image)

   **Solution:**
Here the states correspond to the following subsets: $A = \{ q_0 \}$, $B = \{ q_0, q_1 \}$, $C = \{ q_0, q_1, q_2 \}$ and $D = \{ q_0, q_1, q_2, q_3 \}$. The language is $(a \cup b)^*aaa$. 