

CS 3133 Foundations of Computer Science
C term 2012

Solutions for the Midterm Exam

1. Give a regular expression that represents the set of strings over $\Sigma = \{a, b\}$ with an even number of a 's or an odd number of b 's.

Solution:

$$(b^*ab^*a)^*b^* \cup (a^*ba^*b)^*a^*ba^*$$

(20 points)

2. Consider the following grammar G :

$$\begin{aligned} S &\rightarrow XY \\ X &\rightarrow aX \mid bX \mid a \\ Y &\rightarrow Ya \mid Yb \mid a \end{aligned}$$

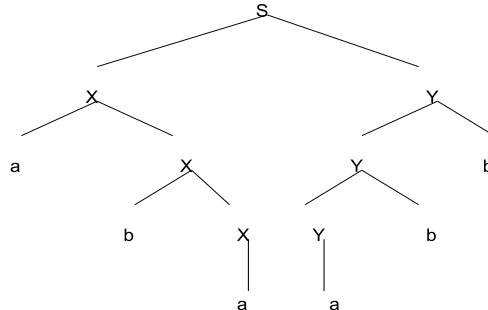
- (a) Give a leftmost derivation of $abaabb$.
(b) Build the derivation tree for the derivation in part (1).
(c) What is $L(G)$?

Solution:

- (a) The following is a leftmost derivation of $abaabb$:

$$\begin{aligned} S &\Rightarrow XY \\ &\Rightarrow aXY \\ &\Rightarrow abXY \\ &\Rightarrow abaY \\ &\Rightarrow abaYb \\ &\Rightarrow abaYbb \\ &\Rightarrow abaabb \end{aligned}$$

- (b)



(c)

$$L(G) = (a \cup b)^* aa (a \cup b)^*$$

(20 points)

3. Construct a context-free grammar over the alphabet $\Sigma = \{a, b, c\}$ whose language is

$$L = \{a^i b^j c^k \mid 0 \leq i + k \leq j\}.$$

Solution: The grammar is:

$$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow aAb \mid Ab \mid \lambda \\ B &\rightarrow bBc \mid \lambda \end{aligned}$$

(20 points)

4. Construct two regular grammars, one ambiguous and one unambiguous, that generate the language $a^+ b^* a^+$.

Solution:

Unambiguous grammar:

$$\begin{aligned} S &\rightarrow aA \mid aB \\ A &\rightarrow aA \mid a \\ B &\rightarrow aB \mid bC \\ C &\rightarrow bC \mid aD \\ D &\rightarrow aD \mid \lambda \end{aligned}$$

This is unambiguous, we start the derivation with $S \rightarrow aA$ to get the strings with no b 's and we start with $S \rightarrow aB$ to get the strings with at least one b .

Ambiguous grammar:

$$\begin{aligned} S &\rightarrow aS \mid aA \\ A &\rightarrow bA \mid aB \mid aC \\ B &\rightarrow aB \mid \lambda \\ C &\rightarrow aC \mid \lambda \end{aligned}$$

It is ambiguous because there are two different leftmost derivations for the string aa :

$$\begin{aligned} S &\Rightarrow aA \\ &\Rightarrow aaB \\ &\Rightarrow aa \end{aligned}$$

and

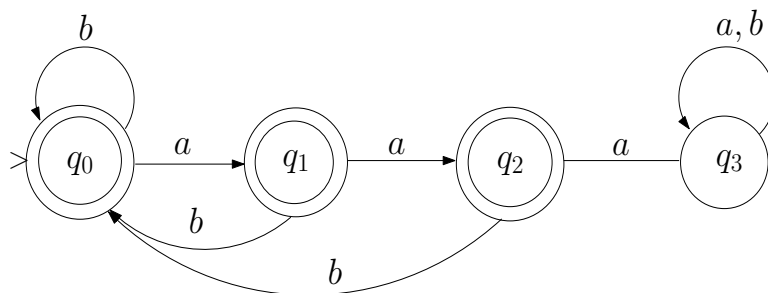
$$\begin{aligned} S &\Rightarrow aA \\ &\Rightarrow aaC \\ &\Rightarrow aa \end{aligned}$$

(20 points)

5. Design a DFA that accepts the language consisting of the set of those strings over $\{a, b\}$ that do not contain the substring aaa .

Solution:

The state diagram of a DFA is



(20 points)