1. Give a regular expression that represents the set of strings over $\Sigma = \{a, b\}$ that do not contain the substring $aa$.

Solution:

$$b^*(ab^+)^*(\lambda \cup a)$$

(20 points)

2. Consider the following grammar $G$:

$$S \rightarrow XYZ$$
$$X \rightarrow Xa | Xb | a$$
$$Y \rightarrow aY | bY | b$$
$$Z \rightarrow aZ | bZ | a$$

(a) Give a leftmost derivation of $abaabba$.

(b) Build the derivation tree for the derivation in part (a).

(c) What is $L(G)$?

Solution:

(a) The following is a leftmost derivation of $abaabba$:

$$S \Rightarrow XYZ$$
$$\Rightarrow XbYZ$$
$$\Rightarrow XabYZ$$
$$\Rightarrow XaabYZ$$
$$\Rightarrow XbaaabYZ$$
$$\Rightarrow abaabYZ$$
$$\Rightarrow abaabbZ$$
$$\Rightarrow abaabba$$
$L(G) = a(a \cup b)^* b(a \cup b)^* a$

3. Construct two regular grammars, one ambiguous and one unambiguous, that generate the language determined in the previous problem 2(c).

**Solution:**

Unambiguous regular grammar:

$$
S \rightarrow aA \\
A \rightarrow aA \mid bB \\
B \rightarrow aB \mid bB \mid a
$$

Ambiguous regular grammar:

$$
S \rightarrow aA \mid aC \\
A \rightarrow aA \mid bA \mid bB \\
B \rightarrow aB \mid bB \mid a \\
C \rightarrow aC \mid bC \mid bB
$$

It is ambiguous because there are two different leftmost derivations for the string $aba$:

$$
S \Rightarrow aA \\
\Rightarrow abB \\
\Rightarrow aba
$$
and

\[ S \Rightarrow aC \]
\[ \Rightarrow abB \]
\[ \Rightarrow aba \]

(20 points)

4. Design an NFA that accepts the language determined in problem 2(c).
(20 points)

**Solution:**
The state diagram of an NFA is

![NFA Diagram](image)

5. Construct the state diagram of a DFA equivalent to the following NFA by using the subset construction method. What is the language accepted by these machines? (20 points)

**Solution:**

![DFA Diagram](image)
Here the states correspond to the following subsets: $A = \{q_0\}$, $B = \{q_0, q_1\}$, $C = \{q_0, q_1, q_2\}$ and $D = \{q_0, q_1, q_2, q_3\}$. The language is $(a \cup b)^*aaa$. 