Solutions for Homework 4

1. Use the technique from Section 6.1 in the book (i.e. constructing an NFA-\(\lambda\) for a given regular set by following the recursive definition of the regular set) to build the state diagram of an NFA-\(\lambda\) that accepts the language \(b(ab)^*b\).

Solution:

\[\begin{align*}
&\xrightarrow{b} b &\xrightarrow{\lambda} a &\xrightarrow{\lambda} &\xrightarrow{b} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{b} \circlosum \\
&\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} &\xrightarrow{\lambda} \nonumber
\end{align*}\]

(20 points)

2. Exercise 3 on page 217. (There is an error in the book, it should say: The language of the DFA M in Example 5.3.7 consists of...)

Solution:

After deleting the state \([a, e]\) we get the following expression graph:
Finally after deleting the only remaining state \([o_a, o_b]\) that is not the starting state and the accepting state we get the final expression graph:

Thus in the second figure on page 195 we have

\[ u = aa \cup ab(bb)^*ba, \quad v = b \cup ab(bb)^*a, \quad w = a(bb)^*a \quad \text{and} \quad x = b \cup a(bb)^*ba, \]

and the regular expression is

\[ u^*v(w \cup x(u)^*w)^*. \]
(20 points)

3. Exercise 4 on page 217.

Solution:

a.)

b.)
c.)

\[
S \rightarrow aS|bA|aZ \\
A \rightarrow bA|aS|bZ \\
Z \rightarrow \lambda
\]

d.)

\[
S \rightarrow bA|aB \\
A \rightarrow aS|bC \\
B \rightarrow aB|bA|\lambda \\
C \rightarrow aS|bC|\lambda
\]

where \( \{S, Z\} = B \) and \( \{A, Z\} = C \).

e.) \((a \cup b^+a)^*(a \cup b^+b)\).

(20 points)


**Solution:** (with the pumping lemma) Let us assume indirectly that the language \( L = \{a^n b^m | n < m\} \) is regular. This implies that \( L \) is accepted by some DFA. Let \( k \) be the number of states of the DFA. By the pumping lemma, every string \( z \in L \) of length \( k \) or more can
be decomposed into substrings $u, v$ and $x$ such that $\text{length}(uv) \leq k$, $\text{length}(v) > 0$ and $uv^ix \in L$ for all $i \geq 0$.

Consider the string $z = a^kb^{k+1}$. Clearly $z \in L$ and $\text{length}(z) \geq k$. Using the pumping lemma we decompose $z$ into substrings $u, v$ and $x$, where $0 < \text{length}(uv) \leq k$. Then $v$ is a substring of the first $a^k$. But in this case $uv^2x$ cannot be in $L$, since we have at least as many $a$’s as $b$’s, a contradiction. $L$ is non-regular. (20 points)

5. Exercise 1 on page 247.

**Solution:**

a.) $L(M) = \{a^ib^j | i \geq j \geq 0\}$.

b.)

![Diagram](attachment:diagram.png)

c.) Here are all computations for the string $aab$: 

5
For the string $abb$:

\[
\begin{align*}
[q_0, aab, \lambda] &\vdash [q_0, ab, A] & [q_0, aab, \lambda] &\vdash [q_0, ab, A] & [q_0, aab, \lambda] &\vdash [q_0, ab, A] \\
[q_0, b, AA] &\vdash [q_1, aab, \lambda] & [q_0, b, AA] &\vdash [q_1, ab, A] & [q_0, b, AA] &\vdash [q_0, b, AA] \\
[q_2, \lambda, A] &\vdash [q_0, bb, A] & reject & [q_1, ab, \lambda] & [q_1, ab, A] &\vdash [q_1, b, A] \\
[q_2, \lambda, \lambda] &\vdash [q_0, bb, A] & reject & [q_1, b, \lambda] & [q_1, b, \lambda] &\vdash [q_1, b, \lambda] \\
accept & [q_0, bb, A] & reject & [q_1, bb, \lambda] & [q_1, bb, \lambda] &\vdash [q_1, bb, \lambda] \\
\end{align*}
\]

For the string $aba$:

\[
\begin{align*}
[q_0, aba, \lambda] &\vdash [q_0, ba, A] & [q_0, aba, \lambda] &\vdash [q_0, ba, A] & [q_0, aba, \lambda] &\vdash [q_0, ba, A] \\
[q_0, ba, A] &\vdash [q_0, bb, AA] & [q_1, aba, \lambda] &\vdash [q_1, ba, A] & [q_1, aba, \lambda] &\vdash [q_1, ba, A] \\
[q_2, a, \lambda] &\vdash [q_1, aba, \lambda] & reject & [q_1, ba, A] & [q_1, ba, \lambda] &\vdash [q_1, ba, \lambda] \\
[q_2, a, \lambda] &\vdash [q_0, bb, A] & reject & [q_2, b, A] & [q_2, b, A] &\vdash [q_2, b, A] \\
\end{align*}
\]

d.)

\[
\begin{align*}
[q_0, aabb, \lambda] &\vdash [q_0, ab, A] \\
[q_0, aabb, \lambda] &\vdash [q_0, ab, A] \\
[q_0, b, AA] &\vdash [q_0, bb, AA] \\
[q_0, b, AA] &\vdash [q_0, b, AA] \\
[q_2, b, A] &\vdash [q_2, b, A] \\
[q_2, \lambda, \lambda] &\vdash [q_2, \lambda, \lambda] \\
accept & [q_2, \lambda, \lambda] \\
\end{align*}
\]
\[
\begin{align*}
[q_0, aaab, \lambda] &\vdash [q_0, aab, A] \\
&\vdash [q_0, ab, AA] \\
&\vdash [q_0, b, AAA] \\
&\vdash [q_2, \lambda, AA] \\
&\vdash [q_2, \lambda, A] \\
&\vdash [q_2, \lambda, \lambda] \\
\text{accept}
\end{align*}
\]

(20 points)