Solutions for Homework 4

1. Use the technique from Section 6.1 in the book (i.e. constructing an NFA-λ for a given regular set by following the recursive definition of the regular set) to build the state diagram of an NFA-λ that accepts the language $b(ab)^*b$.

Solution:

2. Exercise 3 on page 217. (There is an error in the book, it should say: The language of the DFA M in Example 5.3.7 consists of...)

Solution:

After deleting the state $[o_a, e_b]$ we get the following expression graph:
Finally after deleting the only remaining state $[o_a, o_b]$ that is not the starting state and the accepting state we get the final expression graph:

Thus in the second figure on page 195 we have

$$u = aa \cup ab(bb)^*ba, \quad v = b \cup ab(bb)^*a, \quad w = a(bb)^*a$$ and

$$x = b \cup a(bb)^*ba,$$

and the regular expression is

$$u^*v(w \cup x(u)^*w)^*.$$
3. Exercise 4 on page 217.

Solution:

(a.)

(b.)
c.)

\[
S \rightarrow aS|bA|aZ \\
A \rightarrow bA|aS|bZ \\
Z \rightarrow \lambda
\]

d.)

\[
S \rightarrow bA|aB \\
A \rightarrow aS|bC \\
B \rightarrow aB|bA|\lambda \\
C \rightarrow aS|bC|\lambda
\]

where \(\{S,Z\} = B\) and \(\{A,Z\} = C\).

e.) \((a \cup b^+a)^*(a \cup b^+b)\).

(20 points)


**Solution:** (with the pumping lemma) Let us assume indirectly that the language \(L = \{ww|w \in \{a,b\}^*\}\) is regular. This implies that \(L\) is accepted by some DFA. Let \(k\) be the number of states of the DFA. By the pumping lemma, every string \(z \in L\) of length \(k\) or more can
be decomposed into substrings $u, v$ and $x$ such that $\text{length}(uv) \leq k$, $\text{length}(v) > 0$ and $uv^ix \in L$ for all $i \geq 0$.

Consider the string $z = a^kb^ka^kb^k$. Clearly $z \in L$ (with $w = a^kb^k$) and $\text{length}(z) \geq k$. Using the pumping lemma we decompose $z$ into substrings $u, v$ and $x$, where $0 < \text{length}(uv) \leq k$. Then $v$ is a substring of the first $a^k$. But in this case $uv^2x$ cannot be in $L$, a contradiction. $L$ is non-regular. (20 points)

5. Exercise 1 on page 247.

Solution:

a.) $L(M) = \{a^ib^j | i \geq j \geq 0\}$.

b.)

\begin{center}
\begin{tikzpicture}
    \node[state] (q0) at (0,0) {$q_0$};
    \node[state] (q1) at (2,0) {$q_1$};
    \node[state] (q2) at (1,-2) {$q_2$};

    \draw[->] (q0) edge [loop above] node {$a$} (q0);
    \draw[->] (q0) edge [above] node {$A\lambda$} (q1);
    \draw[->] (q1) edge [loop above] node {$A\lambda$} (q1);
    \draw[->] (q1) edge [loop below] node {$b\ A\lambda$} (q1);
    \draw[->] (q0) edge [below] node {$b\ A\lambda$} (q2);
    \draw[->] (q2) edge [loop below] node {$b\ A\lambda$} (q2);

    \end{tikzpicture}
\end{center}

c.) Here are all computations for the string $aab$: 
For the string \textit{abb}:

\[
\begin{array}{cccc}
[q_0, aab, \lambda] & [q_0, aab, \lambda] & [q_0, aab, \lambda] & [q_0, aab, \lambda] \\
\vdash [q_0, ab, A] & \vdash [q_1, aab, \lambda] & \vdash [q_0, ab, A] & \vdash [q_0, ab, A] \\
\vdash [q_0, b, AA] & reject & \vdash [q_1, ab, A] & \vdash [q_0, b, AA] \\
\vdash [q_2, \lambda, A] & reject & \vdash [q_1, ab, \lambda] & \vdash [q_1, b, AA] \\
\vdash [q_2, \lambda, \lambda] & reject & \vdash [q_0, b, AA] & reject \\
\vdash [q_2, \lambda, \lambda] & accept & \vdash [q_1, b, A] & reject \\
\vdash [q_1, b, \lambda] & reject & & \\
\end{array}
\]

For the string \textit{aba}:

\[
\begin{array}{cccc}
[q_0, aba, \lambda] & [q_0, aba, \lambda] & [q_0, aba, \lambda] & [q_0, aba, \lambda] \\
\vdash [q_0, ba, A] & \vdash [q_1, aba, \lambda] & \vdash [q_0, ba, A] & \vdash [q_0, ba, A] \\
\vdash [q_2, a, \lambda] & reject & \vdash [q_1, aba, \lambda] & \vdash [q_1, ba, A] \\
\vdash [q_2, a, \lambda] & reject & \vdash [q_0, ba, A] & reject \\
\vdash [q_2, a, \lambda] & reject & \vdash [q_1, ba, A] & reject \\
\vdash [q_2, a, \lambda] & reject & & \\
\end{array}
\]

\textit{d.})

\[
\begin{array}{cccc}
[q_0, aabb, \lambda] & [q_0, aab, \lambda] & [q_0, aab, \lambda] & [q_0, aab, \lambda] \\
\vdash [q_0, abb, A] & \vdash [q_1, aab, \lambda] & \vdash [q_0, ab, A] & \vdash [q_0, ab, A] \\
\vdash [q_0, bb, AA] & \vdash [q_1, ab, A] & \vdash [q_0, b, AA] & \vdash [q_0, b, AA] \\
\vdash [q_2, b, A] & reject & \vdash [q_1, b, AA] & reject \\
\vdash [q_2, b, A] & accept & \vdash [q_1, b, \lambda] & reject \\
\vdash [q_2, b, \lambda] & & & \\
\end{array}
\]

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\[ q_0, aaab, \lambda \]
\[ \vdash q_0, aab, A \]
\[ \vdash q_0, ab, AA \]
\[ \vdash q_0, b, AAA \]
\[ \vdash q_2, \lambda, AA \]
\[ \vdash q_2, \lambda, A \]
\[ \vdash q_2, \lambda, \lambda \]

accept

(20 points)