1. Use the technique from Section 6.1 in the book (i.e. constructing an NFA-\(\lambda\) for a given regular set by following the recursive definition of the regular set) to build the state diagram of an NFA-\(\lambda\) that accepts the language \(b(ab)^*b\).

   **Solution:**

   ![State Diagram](image)

   (20 points)

2. Exercise 3 on page 217. (There is an error in the book, it should say: The language of the DFA M in Example 5.3.7 consists of...)

   **Solution:**

   After deleting the state \([o_a, e_b]\) we get the following expression graph:
Finally after deleting the only remaining state \([o_a, o_b]\) that is not the starting state and the accepting state we get the final expression graph:

Thus in the second figure on page 195 we have

\[
    u = aa \cup ab(bb)^*ba, \quad v = b \cup ab(bb)^*a, \quad w = a(bb)^*a \quad \text{and} \quad x = b \cup a(bb)^*ba,
\]

and the regular expression is

\[
    u^*v(w \cup x(u)^*v)^*.
\]
(20 points)

3. Exercise 4 on page 217.

Solution:

a.)

\[ S \]

\[ A \]

\[ Z \]

b.)
c.)

\[
S \to aS \mid bA \mid aZ \\
A \to bA \mid aS \mid bZ \\
Z \to \lambda
\]

d.)

\[
S \to bA \mid aB \\
A \to aS \mid bC \\
B \to aB \mid bA \mid \lambda \\
C \to aS \mid bC \mid \lambda
\]

where \( \{S, Z\} = B \) and \( \{A, Z\} = C \).

e.) \( (a \cup b^+a)^*(a \cup b^+b) \).

(20 points)


**Solution:** (with the pumping lemma) Let us assume indirectly that the language \( L = \{ww \mid w \in \{a, b\}^*\} \) is regular. This implies that \( L \) is accepted by some DFA. Let \( k \) be the number of states of the DFA. By the pumping lemma, every string \( z \in L \) of length \( k \) or more can
be decomposed into substrings $u, v$ and $x$ such that $\text{length}(uv) \leq k$, \text{length}(v) > 0$ and $uv^ix \in L$ for all $i \geq 0$.

Consider the string $z = a^kb^ka^kb^k$. Clearly $z \in L$ (with $w = a^kb^k$) and $\text{length}(z) \geq k$. Using the pumping lemma we decompose $z$ into substrings $u, v$ and $x$, where $0 < \text{length}(uv) \leq k$. Then $v$ is a substring of the first $a^k$. But in this case $uv^2x$ cannot be in $L$, a contradiction. $L$ is non-regular. (20 points)

5. Exercise 1 on page 247.

**Solution:**

a.) $L(M) = \{a^ib^j | i \geq j \geq 0\}$.

b.)

---

\[
\begin{array}{c}
q_0 \\
\xrightarrow{a} lA \\
\xrightarrow{b} q_2 \\
\xrightarrow{A} q_1 \\
\end{array}
\]

---

c.) Here are all computations for the string $aab$:
For the string abb:

\[
\begin{align*}
[q_0, abb, \lambda] & \quad [q_0, abb, \lambda] & \quad [q_0, abb, \lambda] & \quad [q_0, abb, \lambda] \\
\vdash [q_0, ab, A] & \quad \vdash [q_1, abb, \lambda] & \quad \vdash [q_0, ab, A] & \quad \vdash [q_0, ab, A] \\
\vdash [q_0, b, AA] & \quad \vdash [q_1, ab, A] & \quad \vdash [q_0, b, AA] & \quad \vdash [q_0, ab, A] \\
\vdash [q_2, \lambda, A] & \quad \vdash [q_1, ab, \lambda] & \quad \vdash [q_1, b, AA] & \quad \vdash [q_1, b, AA] \\
\vdash [q_2, \lambda, \lambda] & \quad \vdash [q_1, b, \lambda] & \quad \vdash [q_1, b, \lambda] \\
\text{accept} & \quad \text{reject} & \quad \text{reject} & \quad \text{reject}
\end{align*}
\]

For the string aba:

\[
\begin{align*}
[q_0, aba, \lambda] & \quad [q_0, aba, \lambda] & \quad [q_0, aba, \lambda] & \quad [q_0, aba, \lambda] \\
\vdash [q_0, ba, A] & \quad \vdash [q_1, aba, \lambda] & \quad \vdash [q_0, ba, A] & \quad \vdash [q_0, ba, A] \\
\vdash [q_2, a, \lambda] & \quad \vdash [q_1, ab, A] & \quad \vdash [q_1, ba, A] & \quad \vdash [q_1, ba, A] \\
\vdash [q_2, a, \lambda] & \quad \vdash [q_1, ab, \lambda] & \quad \vdash [q_1, ba, \lambda] & \quad \vdash [q_1, ba, \lambda] \\
\text{reject} & \quad \text{reject} & \quad \text{reject} & \quad \text{reject}
\end{align*}
\]

d.)

\[
\begin{align*}
[q_0, aabb, \lambda] & \quad [q_0, aabb, \lambda] & \quad [q_0, aabb, \lambda] & \quad [q_0, aabb, \lambda] \\
\vdash [q_0, abb, A] & \quad \vdash [q_1, aabb, \lambda] & \quad \vdash [q_0, abb, A] & \quad \vdash [q_0, abb, A] \\
\vdash [q_0, bb, AA] & \quad \vdash [q_1, abb, \lambda] & \quad \vdash [q_0, bb, AA] & \quad \vdash [q_0, bb, AA] \\
\vdash [q_2, b, A] & \quad \vdash [q_1, aab, \lambda] & \quad \vdash [q_2, b, A] & \quad \vdash [q_2, a, \lambda] \\
\vdash [q_2, \lambda, \lambda] & \quad \vdash [q_2, a, \lambda] & \quad \vdash [q_2, \lambda, \lambda] & \quad \text{accept}
\end{align*}
\]
\[ q_0, aaab, \lambda \]
\[ \vdash q_0, aab, A \]
\[ \vdash q_0, ab, AA \]
\[ \vdash q_0, b, AAA \]
\[ \vdash q_2, \lambda, AA \]
\[ \vdash q_2, \lambda, A \]
\[ \vdash q_2, \lambda, \lambda \]

accept

(20 points)