1. Use the technique from Section 6.1 in the book (i.e. constructing an NFA-$\lambda$ for a given regular set by following the recursive definition of the regular set) to build the state diagram of an NFA-$\lambda$ that accepts the language $b(ab)^*b$.

Solution:

![State Diagram](image)

(20 points)

2. Exercise 3 on page 217. (There is an error in the book, it should say: The language of the DFA M in Example 5.3.7 consists of...)

Solution:

After deleting the state $[o_a, e_b]$ we get the following expression graph:
Finally after deleting the only remaining state \([o_a, o_b]\) that is not the starting state and the accepting state we get the final expression graph:

Thus in the second figure on page 195 we have

\[ u = aa \cup ab(bb)^*ba, \quad v = b \cup ab(bb)^*a, \quad w = a(bb)^*a \quad \text{and} \quad x = b \cup a(bb)^*ba, \]

and the regular expression is

\[ u^*v(w \cup x(u)^*w)^*. \]
(20 points)

3. Exercise 4 on page 217.

Solution:

a.)

b.)
c.)

\[ \begin{align*}
S & \rightarrow aS|bA|aZ \\
A & \rightarrow bA|aS|bZ \\
Z & \rightarrow \lambda
\end{align*} \]

d.)

\[ \begin{align*}
S & \rightarrow bA|aB \\
A & \rightarrow aS|bC \\
B & \rightarrow aB|bA|\lambda \\
C & \rightarrow aS|bC|\lambda
\end{align*} \]

where \( \{S, Z\} = B \) and \( \{A, Z\} = C \).

e.) \((a \cup b^+a)^*(a \cup b^+b)\).

(20 points)


**Solution:** (with the pumping lemma) Let us assume indirectly that the language \( L = \{a^n b^m | n < m \} \) is regular. This implies that \( L \) is accepted by some DFA. Let \( k \) be the number of states of the DFA. By the pumping lemma, every string \( z \in L \) of length \( k \) or more can
be decomposed into substrings $u,v$ and $x$ such that $\text{length}(uv) \leq k$, $\text{length}(v) > 0$ and $uv^ix \in L$ for all $i \geq 0$.

Consider the string $z = a^kb^{k+1}$. Clearly $z \in L$ and $\text{length}(z) \geq k$. Using the pumping lemma we decompose $z$ into substrings $u,v$ and $x$, where $0 < \text{length}(uv) \leq k$. Then $v$ is a substring of the first $a^k$. But in this case $uv^2x$ cannot be in $L$, since we have at least as many $a$'s as $b$'s, a contradiction. $L$ is non-regular. (20 points)

5. Exercise 1 on page 247.

Solution:

a.) $L(M) = \{a^ib^j|i \geq j \geq 0\}$.

b.)

```
\begin{center}
\begin{tikzpicture}
\node[state, initial] (q0) {$q_0$};
\node[state, accepting, right of=q0] (q1) {$q_1$};
\node[state, below of=q0] (q2) {$q_2$};
\draw[->] (q0) -- node[above] {$a$} (q1);
\draw[->] (q0) -- node[below] {$b$} (q2);
\draw[->] (q1) -- node[above] {$\lambda$} (q0);
\draw[->] (q2) -- node[below] {$b$} (q0);
\end{tikzpicture}
\end{center}
```

c.) Here are all computations for the string $aab$:
For the string `abb`:

\[
\begin{align*}
[q_0, aabb, \lambda] & \vdash [q_0, abb, \lambda] \vdash [q_0, ab, A] \vdash [q_0, b, AA] \vdash [q_2, \lambda, \lambda] \text{ reject} \\
[q_0, abb, \lambda] & \vdash [q_1, abb, \lambda] \vdash [q_0, ab, A] \vdash [q_0, b, AA] \vdash [q_1, b, A] \text{ reject} \\
[q_0, abb, \lambda] & \vdash [q_1, b, A] \vdash [q_1, bb, \lambda] \text{ reject} \\
[q_0, abb, \lambda] & \vdash [q_1, bb, \lambda] \text{ reject}
\end{align*}
\]

For the string `aba`:

\[
\begin{align*}
[q_0, aba, \lambda] & \vdash [q_0, aba, \lambda] \vdash [q_0, ba, A] \vdash [q_0, a, \lambda] \text{ reject} \\
[q_0, aba, \lambda] & \vdash [q_1, aba, \lambda] \vdash [q_0, ba, A] \vdash [q_0, a, \lambda] \text{ reject} \\
[q_0, aba, \lambda] & \vdash [q_1, ba, A] \vdash [q_1, ba, \lambda] \text{ reject} \\
[q_0, aba, \lambda] & \vdash [q_1, ba, \lambda] \text{ reject}
\end{align*}
\]

d.)

\[
\begin{align*}
[q_0, aabb, \lambda] & \vdash [q_0, abb, \lambda] \vdash [q_0, ab, A] \vdash [q_0, b, AA] \vdash [q_2, \lambda, \lambda] \text{ accept}
\end{align*}
\]
\[ \begin{align*}
[q_0, aaab, \lambda] \\
\vdash [q_0, aab, A] \\
\vdash [q_0, ab, AA] \\
\vdash [q_0, b, AAA] \\
\vdash [q_2, \lambda, AA] \\
\vdash [q_2, \lambda, A] \\
\vdash [q_2, \lambda, \lambda] \\
\text{accept}
\end{align*} \]

(20 points)