

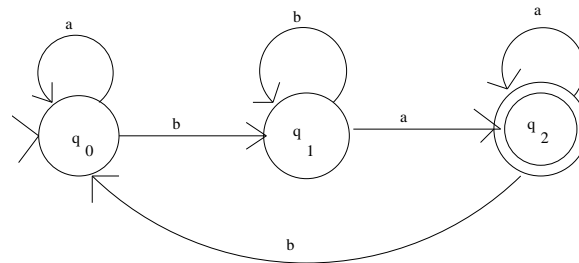
CS 3133 Foundations of Computer Science  
C term 2012

**Solutions for Homework 3**

1. Exercise 1 on page 184.

**Solution:**

(a) The state diagram of  $M$  is



(b)

*i)*

- $[q_0, abaa]$
- $\vdash [q_0, baa]$
- $\vdash [q_1, aa]$
- $\vdash [q_2, a]$
- $\vdash [q_2, \lambda]$

*ii)*

- $[q_0, bbbabb]$
- $\vdash [q_1, bbabb]$
- $\vdash [q_1, babb]$
- $\vdash [q_1, abb]$
- $\vdash [q_2, bb]$
- $\vdash [q_0, b]$
- $\vdash [q_1, \lambda]$

*iii)*

- $[q_0, bababa]$
- $\vdash [q_1, ababa]$
- $\vdash [q_2, baba]$
- $\vdash [q_0, aba]$
- $\vdash [q_0, ba]$
- $\vdash [q_1, a]$
- $\vdash [q_2, \lambda]$

*iv)*

- $[q_0, bbbaa]$
- $\vdash [q_1, bbbaa]$
- $\vdash [q_1, baa]$
- $\vdash [q_1, aa]$
- $\vdash [q_2, a]$
- $\vdash [q_2, \lambda]$

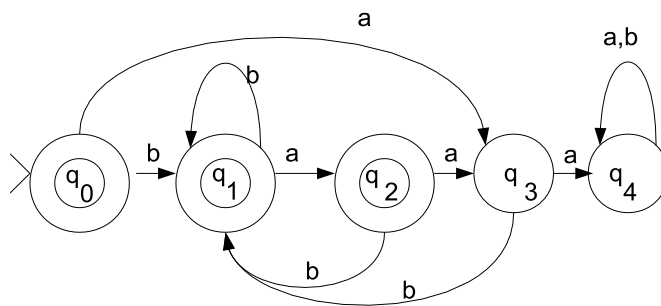
(c) The computations in *i*, *iii* and *iv* terminate in the accepting state  $q_2$ . Therefore the strings  $abaa$ ,  $bababa$  and  $bbbaa$  are in  $L(M)$ .

(d) Two regular expressions describing  $L(M)$  are  $a^*b^+a^+(ba^*b^+a^+)^*$  and  $(a^*b^+a^+b)^*a^*b^+a^+$ . (20 points)

2. Exercise 12 on page 185.

**Solution:**

The state diagram of a DFA is

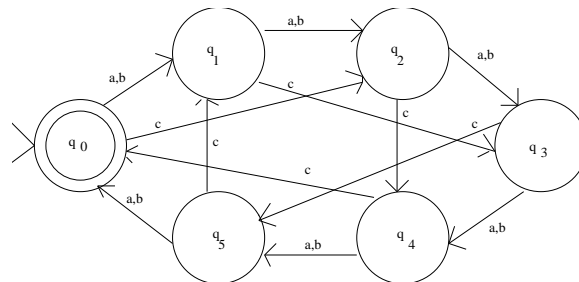


(20 points)

3. Design a DFA that accepts the language consisting of the set of those strings over  $\{a, b, c\}$  in which the number of  $a$ 's plus the number of  $b$ 's plus twice the number of  $c$ 's is divisible by six.

**Solution:**

The state diagram of a DFA is



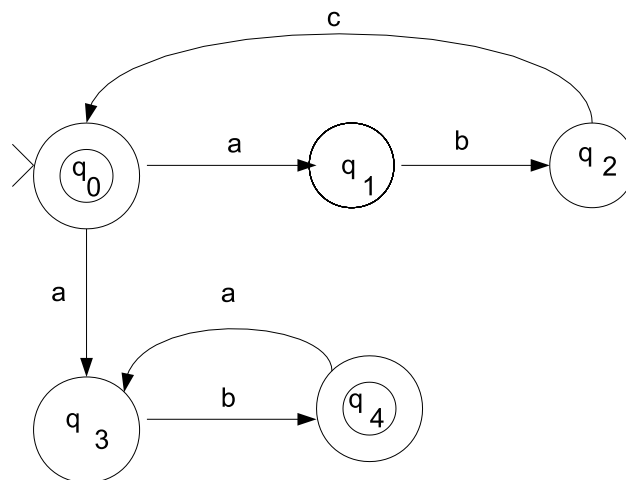
(20 points)

4. Design an NFA that accepts the following language over the alphabet  $\{a, b\}$ :

$$(abc)^*(ab)^*$$

**Solution:**

The state diagram of an NFA is



(20 points)

5. Exercise 36 on page 187.

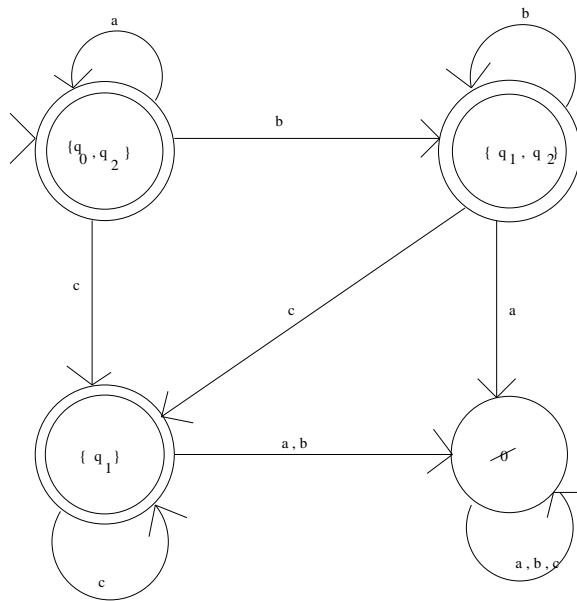
**Solution:**

(a)  $\lambda - closure(q_0) = \{q_0, q_2\}$ .

(b) The input transition function  $t$  is the following:

t	a	b	c
q <sub>0</sub>	{q <sub>0</sub> , q <sub>2</sub> }	{q <sub>1</sub> , q <sub>2</sub> }	{q <sub>1</sub> }
q <sub>1</sub>	∅	∅	{q <sub>1</sub> }
q <sub>2</sub>	∅	{q <sub>1</sub> , q <sub>2</sub> }	∅

(c) The equivalent DFA:



(d) A regular expression is  $a^*b^*c^*$ . (20 points)