Solutions for Homework 3

1. Exercise 1 on page 184.

Solution:
(a) The state diagram of $M$ is

(b)

\[
\begin{align*}
\text{i) } & [q_0, \text{abaa}] \\
& \vdash [q_0, \text{baa}] \\
& \vdash [q_1, \text{aa}] \\
& \vdash [q_2, a] \\
& \vdash [q_2, \lambda] \\
\text{ii) } & [q_0, \text{bbabaa}] \\
& \vdash [q_1, \text{bbbaa}] \\
& \vdash [q_1, \text{babb}] \\
& \vdash [q_1, \text{ab}] \\
& \vdash [q_1, \lambda] \\
\end{align*}
\]

\[
\begin{align*}
\text{iii) } & [q_0, \text{bababa}] \\
& \vdash [q_1, \text{ababa}] \\
& \vdash [q_2, \text{aba}] \\
& \vdash [q_0, \text{aba}] \\
& \vdash [q_0, \text{ba}] \\
& \vdash [q_1, \text{a}] \\
& \vdash [q_2, \lambda] \\
\text{iv) } & [q_0, \text{bbbaa}] \\
& \vdash [q_1, \text{bbbaa}] \\
& \vdash [q_1, \text{baa}] \\
& \vdash [q_1, \text{aa}] \\
& \vdash [q_2, \text{a}] \\
& \vdash [q_2, \lambda]
\end{align*}
\]

(c) The computations in i, iii and iv terminate in the accepting state $q_2$. Therefore the strings $\text{abaa}$, $\text{bababa}$ and $\text{bbbaa}$ are in $L(M)$. 

(d) Two regular expressions describing $L(M)$ are $a^*b^+a^+(ba^*b^+a^+)^*$ and $(a^*b^+a^+b)^*a^*b^+a^+$. (20 points)

2. Exercise 11 on page 185.
   **Solution:**
   The state diagram of a DFA is
   
   ![State Diagram](image)
   
   (20 points)

3. Design a DFA that accepts the language consisting of the set of those strings over $\{a, b, c\}$ in which the number of $a$’s plus the number of $b$’s plus twice the number of $c$’s is divisible by six.
   **Solution:**
   The state diagram of a DFA is
   
   ![State Diagram](image)
   
   (20 points)
4. Design an NFA that accepts the following language over the alphabet 
\{a, b\}:

\[(abc)^*(ab)^*\]

**Solution:**

The state diagram of an NFA is

![NFA Diagram]

(20 points)

5. Exercise 36 on page 187.

**Solution:**

(a) $\lambda - \text{closure}(q_0) = \{q_0, q_2\}$.

(b) The input transition function $t$ is the following:

<table>
<thead>
<tr>
<th>$t$</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0$</td>
<td>${q_0, q_2}$</td>
<td>${q_1, q_2}$</td>
<td>${q_1}$</td>
</tr>
<tr>
<td>$q_1$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>${q_1}$</td>
</tr>
<tr>
<td>$q_2$</td>
<td>$\emptyset$</td>
<td>${q_1, q_2}$</td>
<td>$\emptyset$</td>
</tr>
</tbody>
</table>

(c) The equivalent DFA:
(d) A regular expression is \(a^*b^*c^*.\) (20 points)