Solutions for Homework 3

1. Exercise 1 on page 184.

\textbf{Solution:}

(a) The state diagram of $M$ is

(b) 

\begin{align*}
i) & \quad [q_0, abaa] \\
& \quad \vdash [q_0, baa] \\
& \quad \vdash [q_1, aa] \\
& \quad \vdash [q_2, a] \\
& \quad \vdash [q_2, \lambda] \\

\vdash [q_0, bbaa] & \quad [q_0, babaa] \quad [q_0, bbbaa] \\
\vdash [q_1, bbaa] & \quad [q_1, babaa] \\
\vdash [q_2, bbaa] & \quad [q_2, bbaa] \\
\vdash [q_0, aba] & \quad [q_0, aba] \\
\vdash [q_0, ba] & \quad [q_0, ba] \\
\vdash [q_1, a] & \quad [q_1, a] \\
\vdash [q_2, \lambda] & \quad [q_2, \lambda]
\end{align*}

(c) The computations in \(i, iii\) and \(iv\) terminate in the accepting state \(q_2\). Therefore the strings \(abaa, bababa\) and \(bbbaa\) are in \(L(M)\).
(d) Two regular expressions describing $L(M)$ are $a^*b^+a^+(ba^*b^+a^+)^*$ and $(a^*b^+a^+b)^*a^*b^+a^+$. (20 points)

2. Exercise 11 on page 185.

**Solution:**

The state diagram of a DFA is

![DFA diagram](image)

(20 points)

3. Design a DFA that accepts the language consisting of the set of those strings over \{a, b, c\} in which the number of a’s plus the number of b’s plus twice the number of c’s is divisible by six.

**Solution:**

The state diagram of a DFA is

![DFA diagram](image)

(20 points)
4. Design an NFA that accepts the following language over the alphabet \{a, b\}:

\[(abc)^*(ab)^*\]

**Solution:**
The state diagram of an NFA is

![NFA Diagram](image)

(20 points)

5. Exercise 36 on page 187.

**Solution:**
(a) \(\lambda - \text{closure}(q_0) = \{q_0, q_2\}\).

(b) The input transition function \(t\) is the following:

<table>
<thead>
<tr>
<th>t</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q_0)</td>
<td>{q_0, q_2}</td>
<td>{q_1, q_2}</td>
<td>{q_1}</td>
</tr>
<tr>
<td>(q_1)</td>
<td>\emptyset</td>
<td>\emptyset</td>
<td>{q_1}</td>
</tr>
<tr>
<td>(q_2)</td>
<td>\emptyset</td>
<td>{q_1, q_2}</td>
<td>\emptyset</td>
</tr>
</tbody>
</table>

(c) The equivalent DFA:
(d) A regular expression is $a^*b^*c^*$. (20 points)