CS 3133 Foundations of Computer Science
A term 2016

Solutions for Homework 3

1. Exercise 1 on page 184.

Solution:
(a) The state diagram of $M$ is

(b)

\begin{align*}
\text{i) } [q_0, abaa] & \\vdash [q_0, baa] \\
& \\vdash [q_1, aa] \\
& \\vdash [q_2, a] \\
& \\vdash [q_2, \lambda] \\
\text{ii) } [q_0, bbbabb] & \\vdash [q_1, bbabb] \\
& \\vdash [q_1, babb] \\
& \\vdash [q_1, abb] \\
& \\vdash [q_2, bb] \\
& \\vdash [q_0, b] \\
& \\vdash [q_1, \lambda] \\
\text{iii) } [q_0, bababa] & \\vdash [q_1, ababa] \\
& \\vdash [q_2, bab] \\
& \\vdash [q_0, aba] \\
& \\vdash [q_0, ba] \\
& \\vdash [q_1, a] \\
& \\vdash [q_2, \lambda] \\
\text{iv) } [q_0, bbbaa] & \\vdash [q_1, bbbaa] \\
& \\vdash [q_1, baa] \\
& \\vdash [q_1, aa] \\
& \\vdash [q_2, a] \\
& \\vdash [q_2, \lambda] \\
\end{align*}

(c) The computations in i, iii and iv terminate in the accepting state $q_2$. Therefore the strings $abaa$, $bababa$ and $bbbaa$ are in $L(M)$. 

1
(d) Two regular expressions describing $L(M)$ are $a^*b^+a^+\left((ba^*b^+a^+)\right)^*$ and $(a^*b^+a+b)^*a^*b^+a^+$. (20 points)

2. Exercise 11 on page 185.

Solution:

The state diagram of a DFA is

![State Diagram](image)

(20 points)

3. Design a DFA that accepts the language consisting of the set of those strings over $\{a, b, c\}$ in which the number of a’s plus the number of b’s plus twice the number of c’s is divisible by six.

Solution:

The state diagram of a DFA is

![State Diagram](image)

(20 points)
4. Design an NFA that accepts the following language over the alphabet \( \{a, b\} \):
\[
(abc)^* (ab)^*
\]

**Solution:**
The state diagram of an NFA is

![NFA Diagram](image)

(20 points)

5. Exercise 36 on page 187.

**Solution:**
(a) \( \lambda - \text{closure}(q_0) = \{q_0, q_2\} \).
(b) The input transition function \( t \) is the following:

<table>
<thead>
<tr>
<th>t</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>q_0</td>
<td>{q_0, q_2}</td>
<td>{q_1, q_2}</td>
<td>{q_1}</td>
</tr>
<tr>
<td>q_1</td>
<td>\emptyset</td>
<td>\emptyset</td>
<td>{q_1}</td>
</tr>
<tr>
<td>q_2</td>
<td>\emptyset</td>
<td>{q_1, q_2}</td>
<td>\emptyset</td>
</tr>
</tbody>
</table>

(c) The equivalent DFA:
(d) A regular expression is $a^*b^*c^*$. (20 points)