CS 3133 Foundations of Computer Science
C term 2012

Solutions for Homework 1

1. Exercise 30 on page 60.

   Solution:

   The set of strings over $\{a, b\}$ that do not begin with the substring aaa is represented by

   $$(\lambda \cup \boldsymbol{a} \cup \boldsymbol{aa})(\lambda \cup \boldsymbol{b}^{+}(\boldsymbol{a} \cup \boldsymbol{b})^{*})$$

   (15 points)

2. Exercise 34 on page 61.

   Solution: The set of even length strings over $\{a, b\}$ is represented by

   $$((\boldsymbol{a} \cup \boldsymbol{b})(\boldsymbol{a} \cup \boldsymbol{b}))^{*}$$

   and the set of odd length strings is represented by

   $$(\boldsymbol{a} \cup \boldsymbol{b})((\boldsymbol{a} \cup \boldsymbol{b})(\boldsymbol{a} \cup \boldsymbol{b}))^{*}$$

   Therefore the set of odd length strings over $\{a, b\}$ which contain bb is represented by

   $$((\boldsymbol{a} \cup \boldsymbol{b})(\boldsymbol{a} \cup \boldsymbol{b}))^{*}\boldsymbol{bb}(\boldsymbol{a} \cup \boldsymbol{b})((\boldsymbol{a} \cup \boldsymbol{b})(\boldsymbol{a} \cup \boldsymbol{b}))^{*} \bigcup$$
   $$(\boldsymbol{a} \cup \boldsymbol{b})((\boldsymbol{a} \cup \boldsymbol{b})(\boldsymbol{a} \cup \boldsymbol{b}))^{*}\boldsymbol{bb}((\boldsymbol{a} \cup \boldsymbol{b})(\boldsymbol{a} \cup \boldsymbol{b}))^{*}$$

   (15 points)

3. Exercise 39.d on page 61.

   Solution:

   $$(\boldsymbol{a} \cup \boldsymbol{b})^{*} = (\boldsymbol{a} \cup \boldsymbol{ba^{*}})^{*} = (\boldsymbol{a^{*}} \cup \boldsymbol{ba^{*}})^{*}$$

   Here in the first equality we have used the third regular expression identity in 12. of Table 2.1, and in the second equality we have used the first identity in 12. (15 points)

4. Let Σ be an alphabet, and u, v, w regular expressions over Σ. Are the following regular expression identities true?

   (a) $u \cup (vw) = (u \cup v)(u \cup w)$.

   (b) $u^*(v \cup w) = u^*v \cup u^*w$.

   If yes, explain why, if no, give a counterexample.

   Solution:

   (a) False. For example if $\Sigma = \{a, b, c\}$ and $u = \boldsymbol{a}, v = \boldsymbol{b}, w = \boldsymbol{c}$, then the string ac is contained in $(u \cup v)(u \cup w)$ but it is not contained in $u \cup (vw)$.

   (b) True. This is just regular expression identity 9. of Table 2.1 with u^* playing the role of u. (20 points)

5. Exercise 44 on page 39.

   Solution:

   The set of ancestors of a node x in a tree is defined recursively in the following way:

   basis: x is an ancestor of itself.

   recursive step: If y is an ancestor of x and z is the parent of y (if there is any) then z is also an ancestor of x.

   closure: y is ancestor of x only if it can be obtained by finitely many applications of the recursive step. (15 points)

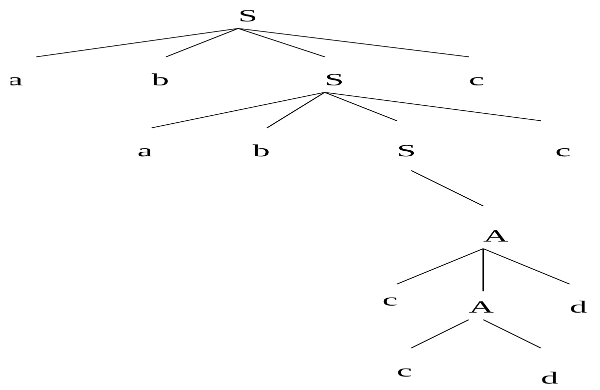
6. Exercise 1 on page 97.

   Solution:

   (a) The derivation of $ababccddcc$ and the applied rules are the following:

$$\begin{array}{llll} S & \Rightarrow abSc & \quad & S \to abSc \\ & \Rightarrow ababScc & & S \to abSc \\ & \Rightarrow ababAcc & & S \to A \\ & \Rightarrow ababcAdcc & & A \to cAd \\ & \Rightarrow ababccddcc & & A \to cd \end{array}$$

   (b)



(c) $L(G) = \{(ab)^n c^m d^m c^n | n \geq 0, m > 0\}$. (20 points)