

CS 2022/ MA 2201 Discrete Mathematics
A term 2011

Solutions for the Practice Midterm Exam

These problems are sample problems for the midterm exam, so you may expect similar problems in the midterm. Do not hand in your solutions. Solutions will be handed out, discussed (and posted on the web) on Monday, the day before the exam. The midterm exam is a closed book exam, but you may use one sheet of paper (written on both sides) with notes on it. Each problem is worth 20 points.

1. Is $(\neg q \vee p) \equiv (q \rightarrow p)$? Justify your response.

Solution: Yes, here is the truth table.

p	q	$\neg q \vee p$	$q \rightarrow p$
T	T	T	T
T	F	T	T
F	T	F	F
F	F	T	T

2. Letting the Universe of Discourse be the set of students at WPI, let $P(x)$ denote the statement “ x likes CS2022/MA2201”. Express, in predicate logic, the sentence “At least two students like CS2022/MA2201, though not everybody likes it”.

Solution:

$$\exists x \exists y (P(x) \wedge P(y) \wedge (x \neq y)) \wedge (\neg \forall x P(x)).$$

3. Suppose that $|A| = m \geq 1$ and $|B| = n \geq 1$. What is the most that can be said about the relationship between m and n for each of the following to be true?
- (a) There is an injection from A to B .
 - (b) There is a surjection from A to B .

(c) There is a bijection from A to B .

Solution: (a) $m \leq n$, (b) $m \geq n$, (c) $m = n$.

4. (a) Translate the following inference into propositional logic.
If today is Thursday, then I have a test in CS or a test in Econ.
If my Econ professor is sick, then I will not have a test in Econ.
Today is Thursday and my Econ professor is sick. Therefore I have a test in CS.

(b) Is the inference correct? Justify your response.

Solution:

(a) t - Today is Thursday, c - I have a test in CS, e - I have a test in Econ, s - My econ professor is sick. The argument is:

$$\begin{array}{l} t \rightarrow (c \vee e) \\ s \rightarrow \neg e \\ t \wedge s \\ \hline c \end{array}$$

(b) Yes. From $t \wedge s$ we can conclude t and s (simplification). From t and $t \rightarrow (c \vee e)$ we can conclude $c \vee e$ (modus ponens). From $c \vee e$ and $\neg e$ we can conclude c (disjunctive syllogism).

5. Prove by contradiction the following. For all rational number x and irrational number y , the sum of x and y is irrational.

Solution: Assume that this statement is not true. From this assumption we have to get a contradiction. Since this statement is not true we must have a rational number x and an irrational number y such that $x + y$ is rational, say $x + y = \frac{p}{q}$ where p and q are integers. Since x is rational we have $x = \frac{a}{b}$ for some integers a and b . Then $y = \frac{p}{q} - \frac{a}{b} = \frac{pb - aq}{bq}$ a contradiction, since y is irrational, it cannot be the fraction of two integers.