Practice Midterm Exam

These problems are sample problems for the midterm exam, so you may expect similar problems in the midterm. Do not hand in your solutions. Solutions will be handed out and discussed (and posted on the web) on Monday, the day before the exam. The midterm exam is a closed book exam, but you may use one sheet of paper (written on both sides) with notes on it. Each problem is worth 20 points.

1. Is \((\neg q \lor p) \equiv (q \rightarrow p)\)? Justify your response.

2. Letting the Universe of Discourse be the set of students at WPI, let \(P(x)\) denote the statement “\(x\) likes CS2022/MA2201”. Express, in predicate logic, the sentence “At least two students like CS2022/MA2201, though not everybody likes it”.

3. Suppose that \(|A| = m \geq 1\) and \(|B| = n \geq 1\). What is the most that can be said about the relationship between \(m\) and \(n\) for each of the following to be true?
   
   (a) There is an injection from \(A\) to \(B\).
   
   (b) There is a surjection from \(A\) to \(B\).
   
   (c) There is a bijection from \(A\) to \(B\).

4. (a) Translate the following inference into propositional logic.
   
   If today is Thursday, then I have a test in CS or a test in Econ.
   
   If my Econ professor is sick, then I will not have a test in Econ.
   
   Today is Thursday and my Econ professor is sick. Therefore I have a test in CS.
   
   (b) Is the inference correct? Justify your response.

5. Prove by contradiction the following. For all rational number \(x\) and irrational number \(y\), the sum of \(x\) and \(y\) is irrational.