CS 2022/ MA 2201 Discrete Mathematics
A term 2015

Solutions for the Practice Final Exam

These problems are sample problems for the final exam, so you may expect similar problems in the final. Do not hand in your solutions. Solutions will be handed out, discussed (and posted on the web) on Tuesday. The final exam is from the material of the whole course, but there will be only one or two problems from the first half. The final exam is a closed book exam, but you may use two sheets of paper (so you may use your midterm sheet) with notes on it. Each problem is worth 20 points.

1. Is $(\neg q \to \neg p) \equiv (p \to q)$? Justify your response.

   Solution: Yes, check truth table.

2. Prove by induction that $\sum_{i=1}^{n}(2i-1) = n^2$ for all positive integers n.

   Solution:

   Basis Step: It is true for $n = 1$, since 1=1.

   Inductive Step: Assume that it is true for n, so $\sum_{i=1}^{n}(2i-1) = n^2$. Show that it is true for $n+1$, so $\sum_{i=1}^{n+1}(2i-1) = (n+1)^2$. Indeed,

   $$\sum_{i=1}^{n+1}(2i-1) = \sum_{i=1}^{n}(2i-1) + 2(n+1) - 1 = n^2 + 2n + 1 = (n+1)^2.$$

3. The US Senate has 100 members. If the senators consist of 85 men and 15 women, in how many ways can we pick a committee of 10 senators consisting of 5 men and 5 women.

   Solution: There are $\binom{85}{5}$ ways to pick the 5 men, and $\binom{15}{5}$ ways to pick the 5 women. By the product rule , there are $\binom{85}{5}\binom{15}{5}$ ways to pick the committee of 5 men and 5 women.

4. Prove or give a counterexample to the following: For a set A and binary relation R on A, if R is reflexive and symmetric, then R must be transitive as well.

Solution: Counterexample: Consider $A = \{1, 2, 3\}$ and

$$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}.$$

R is symmetric and reflexive, but not transitive.

5. For each of the following , either give an example or prove there are none:

   (a) A simple graph with 6 vertices, whose degrees are 2, 2, 2, 3, 4, 4.

   (b) A simple graph with 8 vertices, whose degrees are 0, 1, 2, 3, 4, 5, 6, 7.

   (c) A simple graph with degrees 1, 2, 2, 3.

   Solution:

   (a) None. It is not possible to have one vertex of odd degree.

   (b) None. It is not possible to have a vertex of degree 7 and a vertex of degree 0.

   (c) A triangle with an edge hanging off.