

CS 2022/ MA 2201 Discrete Mathematics
A term 2011

Solutions for Homework 4

1. Exercise 14 on page 330.

Solution:

Basis Step: For $n = 1$ we get $1 \cdot 2^1 = (1 - 1)2^{1+1} + 2$, which is true statement $2 = 2$.

Inductive Step: Let us assume the inductive hypothesis, namely that

$$\sum_{k=1}^n k \cdot 2^k = (n - 1)2^{n+1} + 2,$$

and then from this we have to prove that

$$\sum_{k=1}^{n+1} k \cdot 2^k = n \cdot 2^{n+2} + 2.$$

Indeed, by using the inductive hypothesis we get

$$\begin{aligned} \sum_{k=1}^{n+1} k \cdot 2^k &= \left(\sum_{k=1}^n k \cdot 2^k \right) + (n+1)2^{n+1} = (n-1)2^{n+1} + 2 + (n+1)2^{n+1} = \\ &= 2n \cdot 2^{n+1} + 2 = n \cdot 2^{n+2} + 2, \end{aligned}$$

as desired. (15 points)

2. Exercise 24 a-d.) on page 396.

Solution: It is useful to note first that there are exactly 9000 numbers in this range.

a.) Every ninth number is divisible by 9, so the answer is one ninth of 9000 or 1000.

b.) Every other number is even, so the answer is one half of 9000 or 4500.

c.) We can reason from left to right. There are 9 choices for the first (left-most) digit (since it cannot be 0), then 9 choices for the second digit (since it cannot equal the first digit but it can be 0), then, in a similar way, 8 choices for the third digit, and 7 choices for the right-most digit. Therefore there are $9 \cdot 9 \cdot 8 \cdot 7 = 4536$ such numbers.

d.) Every third number is divisible by 3, so one third of 9000 or 3000 numbers in this range are divisible by 3. The remaining 6000 are not. (15 points)

3. Exercise 46 on page 397.

Solution: (a) We first place the bride in any of the 6 positions. Then, from left to right in the remaining positions, we choose the other 5 people to be in the picture; this can be done in $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 15,120$ ways. Therefore, the answer is $6 \cdot 15,120 = 90,720$.

(b) We first place the bride in any of the 6 positions, and then place the groom in any of the remaining 5 positions. Then, from left to right in the remaining positions, we choose the other 4 people to be in the picture; this can be done in $8 \cdot 7 \cdot 6 \cdot 5 = 1,680$ ways. Therefore, the answer is $6 \cdot 5 \cdot 1,680 = 50,400$.

(c) From part (a) there are 90,720 ways for the bride to be in the picture. There are (from part (b)) 50,400 ways for both the bride and groom to be in the picture. Therefore there are $90,720 - 50,400 = 40,320$ ways for just the bride to be in the picture. Symmetrically, there are 40,320 ways for just the groom to be in the picture. Therefore, the answer is $40,320 + 40,320 = 80,640$.

(15 points)

4. Exercise 26 on page 406.

Solution: Let the people be A, B, C, D and E . Suppose the following pairs are friends: $(A, B), (B, C), (C, D), (D, E)$ and (E, A) . The other 5 pairs are enemies. In this example, there are no three mutual friends and no three mutual enemies. (20 points)

5. Exercise 26 on page 414.

Solution:

(a) $C(13, 10) = \binom{13}{10} = 286.$

(b) $P(13, 10) = 13 \cdot 12 \cdot \dots \cdot 4.$

(c)

- First solution: There is only one way to choose the 10 players without choosing a woman, since there are exactly 10 men. Therefore using part a.) there are $286-1=285$ ways to choose the players if at least one of them must be a woman.
- Second Solution: $C(3, 1) \cdot C(10, 9) + C(3, 2) \cdot C(10, 8) + C(3, 3) \cdot C(10, 7) = \binom{3}{1} \cdot \binom{10}{9} + \binom{3}{2} \cdot \binom{10}{8} + \binom{3}{3} \cdot \binom{10}{7} = 285.$

(20 points)

6. Exercise 8 on page 421.

Solution: By the binomial theorem the coefficient is

$$\binom{17}{9} 3^8 2^9.$$

(15 points)