

CS 2022/ MA 2201 Discrete Mathematics
A term 2011

Solutions for Homework 2

1. Exercise 22 on page 126.

Solution: Yes. The union of all the sets in the power set of the set S is just the set itself. Therefore, if $P(A) = P(B)$, then $A = B$. (20 points)

2. Exercise 24 on page 136.

Solution: By using a membership table:

A	B	C	$(A - C) - (B - C)$	$(A - B) - C$
1	1	1	0	0
1	1	0	0	0
1	0	1	0	0
1	0	0	1	1
0	1	1	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

(20 points)

3. Exercise 20 on page 153.

Solution:

(a) $f(n) = n + 1$

(b) $f(n) = \lfloor \frac{n}{2} \rfloor$

(c) $f(n) = \begin{cases} n + 1 & \text{if } n \text{ is even} \\ n - 1 & \text{if } n \text{ is odd} \end{cases}$

Thus we have $f(0) = 1, f(1) = 0, f(2) = 3, f(3) = 2$, etc.

(d) $f(n) = 2022$

(20 points)

4. Exercise 32 on page 169.

Solution:

(a)

$$\sum_{j=0}^8 (1 + (-1)^j) = \sum_{j=0}^8 1 + \sum_{j=0}^8 (-1)^j = 9 + 1 = 10$$

(b)

$$\sum_{j=0}^8 (3^j - 2^j) = \frac{3^9 - 1}{2} - (2^9 - 1) = 9330$$

(c)

$$\begin{aligned} \sum_{j=0}^8 (2 \times 3^j + 3 \times 2^j) &= 2 \sum_{j=0}^8 3^j + 3 \sum_{j=0}^8 2^j = 2 \frac{3^9 - 1}{2} + 3(2^9 - 1) = \\ &= 19682 + 1533 = 21215 \end{aligned}$$

(d)

$$\sum_{j=0}^8 (2^{j+1} - 2^j) = 2^9 - 1 = 511$$

(20 points)

5. Show that the set of integers that are *not* multiples of 3 is a countable set.

Solution:

Here is a bijection between the set of positive integers and the set of integers that are not multiples of 3:

$$f(n) = \begin{cases} 3k + 1 & \text{if } n = 4k + 1 \\ -(3k + 1) & \text{if } n = 4k + 2 \\ 3k + 2 & \text{if } n = 4k + 3 \\ -(3k + 2) & \text{if } n = 4k + 4 \end{cases}$$

Thus $f(1) = 1, f(2) = -1, f(3) = 2, f(4) = -2$, etc. we get all the integers that are not multiples of 3 exactly once. (20 points)