Introduction to Computer Graphics with WebGL

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Computer Viewing
Projection

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Objectives

- Introduce the mathematics of projection
- Add WebGL projection functions in MV.js

Projections and Normalization

- The default projection in the eye (camera) frame is orthogonal
- For points within the default view volume

$$x_p = x$$
$$y_p = y$$
$$z_p = 0$$

- Most graphics systems use *view normalization*
  - All other views are converted to the default view by transformations that determine the projection matrix
  - Allows use of the same pipeline for all views

Homogeneous Coordinate Representation

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default orthographic projection

$$x_p = x$$
$$y_p = y$$
$$z_p = 0$$
$$w_p = 1$$

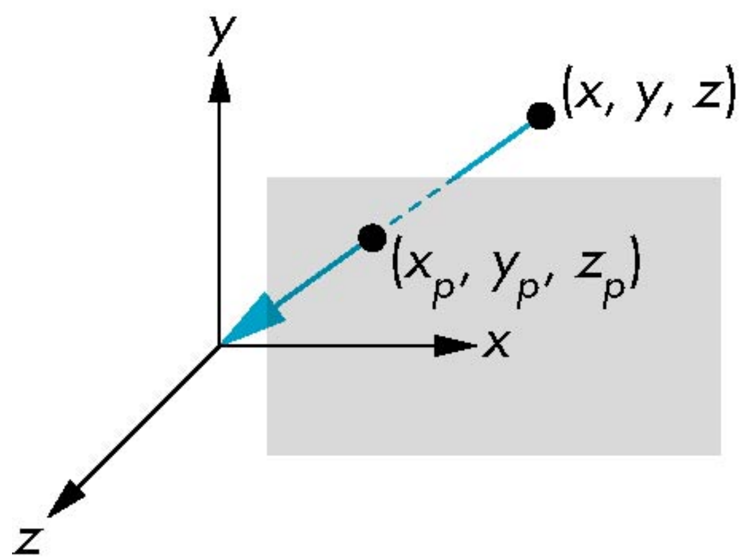
$$\mathbf{p}_p = \mathbf{M}\mathbf{p}$$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In practice, we can let $\mathbf{M} = \mathbf{I}$ and set the z term to zero later

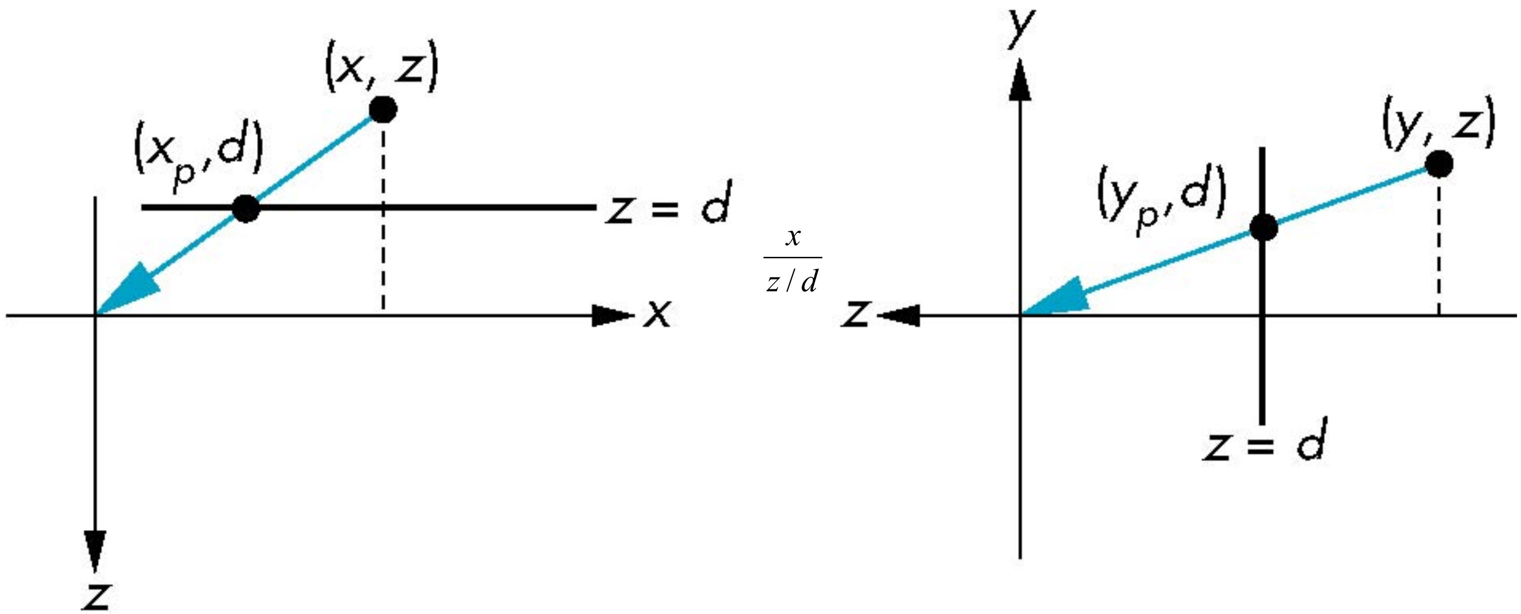
Simple Perspective

- Center of projection at the origin
- Projection plane $z = d,\ d < 0$



Perspective Equations

Consider top and side views



$$x_p = \frac{x}{z/d} \qquad y_p = \frac{y}{z/d} \qquad z_p = d$$

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Homogeneous Coordinate Form

consider $\mathbf{q} = \mathbf{M}\mathbf{p}$ where $\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$

$$\mathbf{q} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad \Rightarrow \quad \mathbf{p} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

Perspective Division

- However $w \neq 1$, so we must divide by w to return from homogeneous coordinates
- This *perspective division* yields

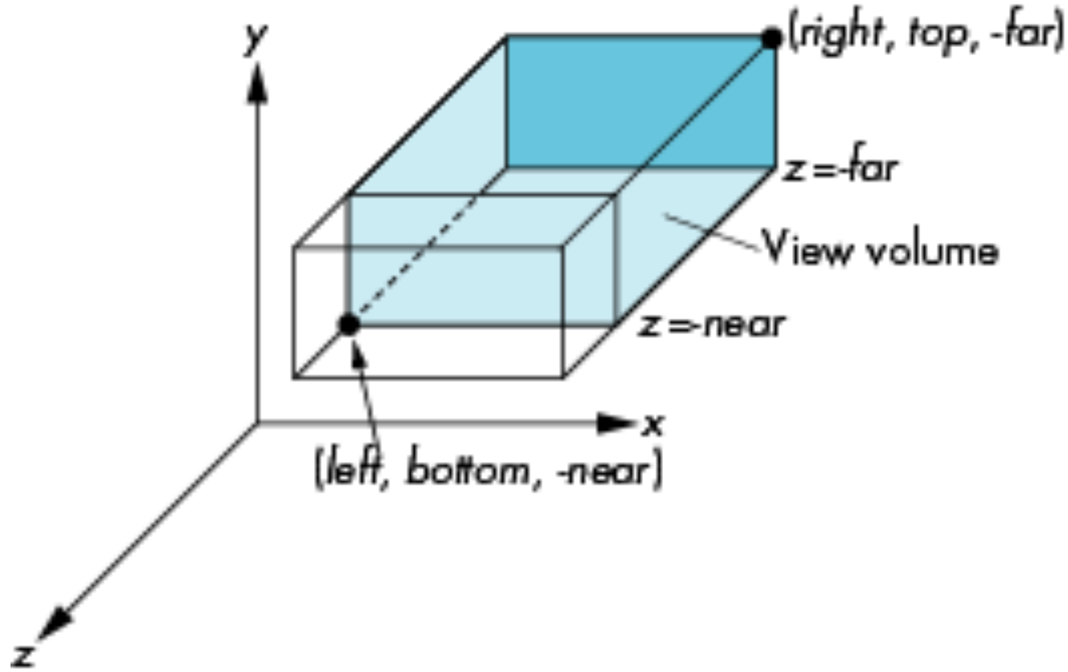
$$x_p = \frac{x}{z/d} \qquad y_p = \frac{y}{z/d} \qquad z_p = d$$

the desired perspective equations

- We will consider the corresponding clipping volume with mat.h functions that are equivalent to deprecated OpenGL functions

WebGL Orthogonal Viewing

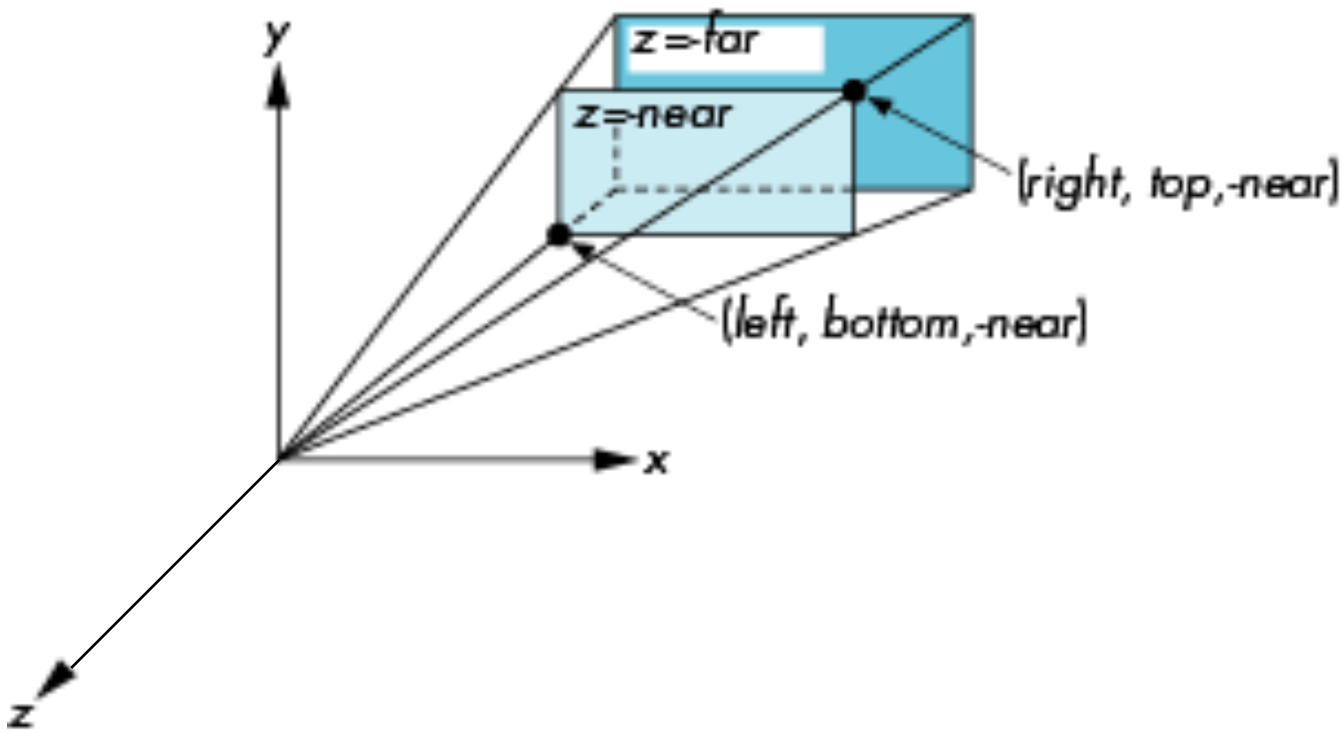
`ortho(left,right,bottom,top,near,far)`



`near` and **`far`** measured <u>from</u> camera

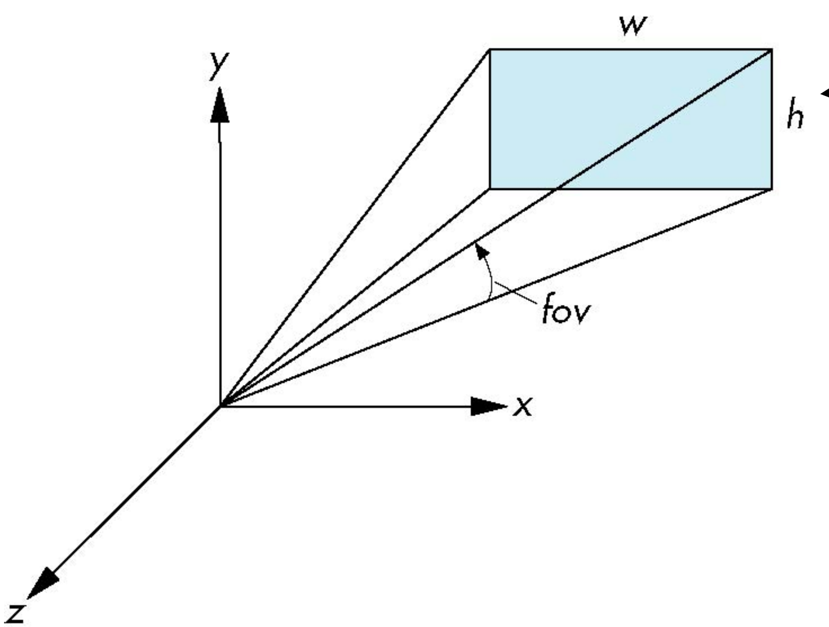
WebGL Perspective

`frustum(left,right,bottom,top,near,far)`



Using Field of View

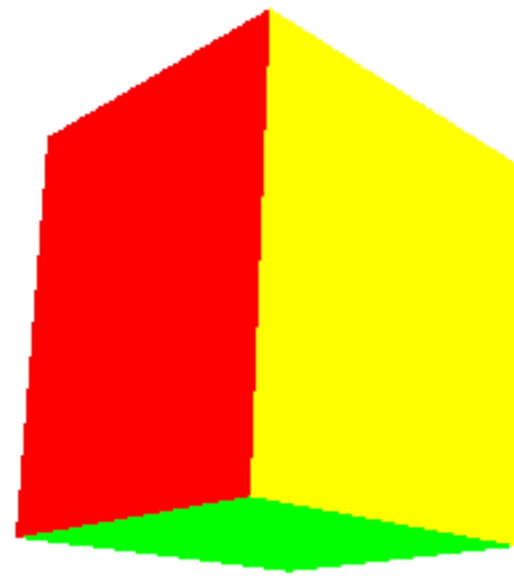
- With **`frustum`** it is often difficult to get the desired view
- **`perpective(fovy, aspect, near, far)`** often provides a better interface

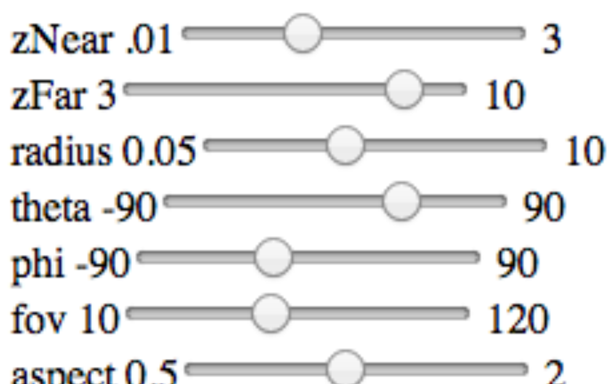


`aspect = w/h`

Computing Matrices

- Compute in JS file, send to vertex shader with gl.uniformMatrix4fv
- Dynamic: update in render() or shader





persepctive2.js

```
var render = function(){
   gl.clear( gl.COLOR_BUFFER_BIT | gl.DEPTH_BUFFER_BIT);
    eye = vec3(radius*Math.sin(theta)*Math.cos(phi),
       radius*Math.sin(theta)*Math.sin(phi), radius*Math.cos(theta));
   modelViewMatrix = lookAt(eye, at , up);
   projectionMatrix = perspective(fovy, aspect, near, far);
   gl.uniformMatrix4fv( modelViewMatrixLoc, false,
        flatten(modelViewMatrix) );
   gl.uniformMatrix4fv( projectionMatrixLoc, false,
       flatten(projectionMatrix) );
   gl.drawArrays( gl.TRIANGLES, 0, NumVertices );
   requestAnimFrame(render);
}
```

vertex shader

```
attribute  vec4 vPosition;
attribute  vec4 vColor;
varying vec4 fColor;
uniform mat4 modelViewMatrix;
uniform mat4 projectionMatrix;

void main() {
   gl_Position = projectionMatrix*modelViewMatrix*vPosition;
   fColor = vColor;
}
```