



**CS 563 Advanced Topics in  
Computer Graphics**  
*Microfacet BRDFs and Scattering*

by Emmanuel Agu

- BRDFs have evolved historically
- 1970's: Empirical models
  - Phong's illumination model
- 1980s:
  - Physically based models
  - Microfacet models (e.g. Cook Torrance model)
- 1990's
  - Physically-based appearance models of specific effects (materials, weathering, dust, etc)
- Early 2000's
  - Measurement & acquisition of static materials/lights (wood, translucence, etc)
- Late 2000's
  - Last week: Measurement & acquisition of time-varying BRDFs (ripening, etc)

# Physically-Based Shading Models

- Phong model produces pretty pictures
- **Cons:** empirical fudge ( $\cos^n\phi$ ), plastic look
- First physically-based models were based on microfacet Theory
- Classic: Cook-Torrance shading model (TOGS 1982)

# Cook-Torrance Shading Model

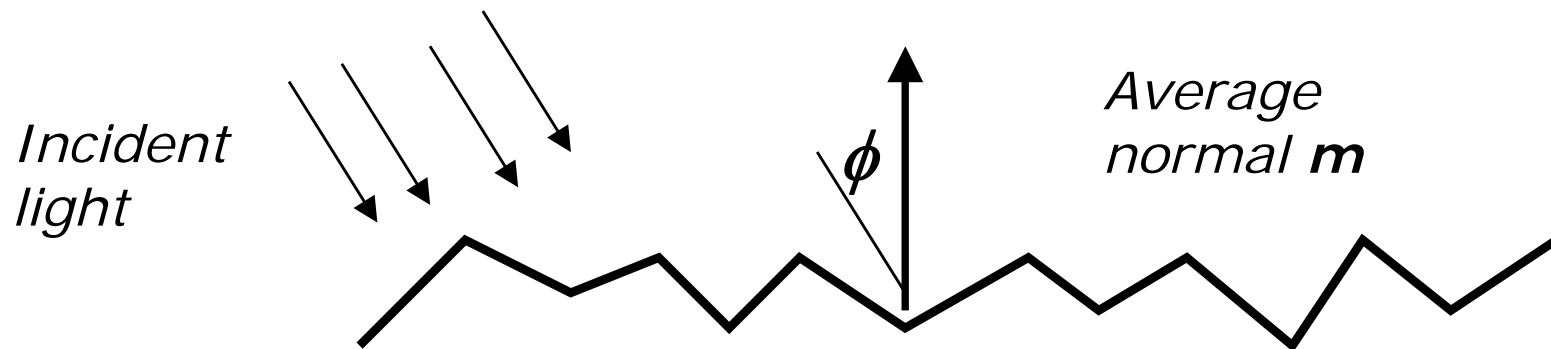
- Similar ambient and diffuse terms to Phong
- More complex specular component than  $(\cos^n \phi)$
- Define new specular term

$$\cos^n \phi \rightarrow \frac{F(\phi, \eta)DG}{(\mathbf{m} \cdot \mathbf{v})}$$

- where
  - D - Distribution term
  - G – Geometric term
  - F – Fresnel term
- Let's explain each term

# Distribution Term, D

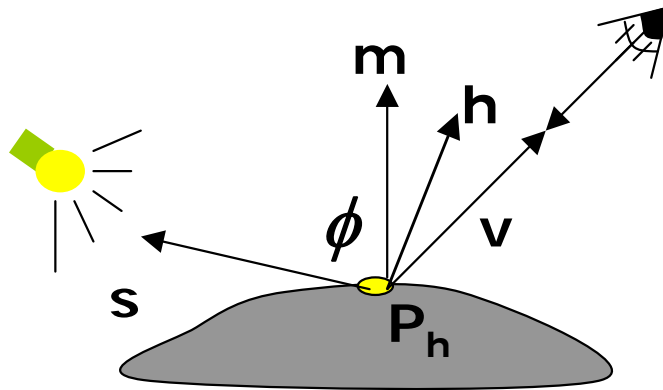
- Surface composed of **microfacets**: small V-shaped grooves



- Only grooves facing mirror directions reflect light
- D term expresses groove directions
- Note: several grooves occur at each hit point
- Technically, D expresses direction of aggregates (distribution)
- E.g. half of grooves at hit point face 30 degrees, etc

# Cook-Torrance Shading Model

- Can define mirror direction using Blinn's halfway vector,  $\mathbf{h} = \mathbf{s} + \mathbf{v}$
- Only microfacets with  $\mathbf{V}$  normal pointing in  $\mathbf{h}$  direction contributes



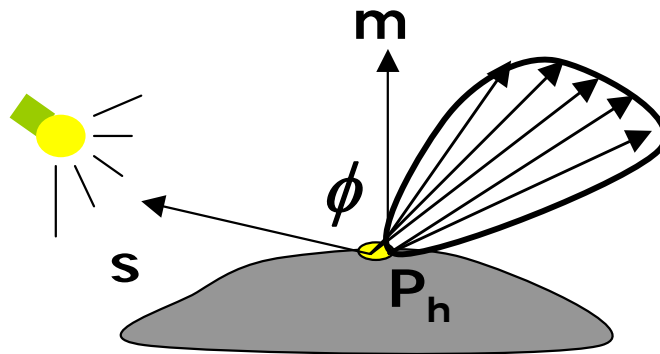
- Define angle  $\delta$  as deviation of  $\mathbf{h}$  from surface normal
- $D(\delta)$  is fraction of microfacets facing angle  $\delta$
- Can actually plug old Phong cosine ( $\cos^n \phi$ ), in as  $D$
- More widely used is Beckmann distribution

$$D(\delta) = \frac{1}{4\mathbf{m}^2 \cos^4(\delta)} e^{-\left(\frac{\tan(\delta)}{\mathbf{m}}\right)^2}$$

- Where  $\mathbf{m}$  expresses roughness of surface

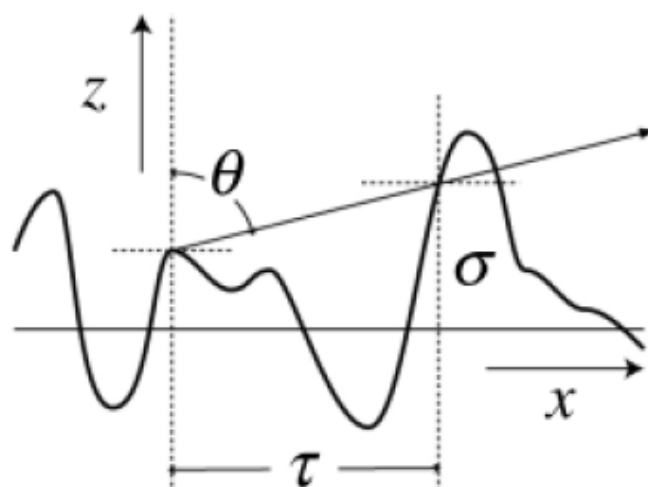
# Cook-Torrance Shading Model

- $m$  is Root-mean-square (RMS) value of slope of V-groove
- Basically,  $m$  expresses slope of V-groove
- $m = 0.2$  for nearly smooth
- $m = 0.6$  for very rough



# Microfacet Slope

- Slope



- Beckmann Distribution of Microfacet Slope

$$D(\alpha) = \frac{1}{\sqrt{\pi} m^2 \cos^2 \alpha} e^{-\frac{\tan^2 \alpha}{m^2}} \quad m = \frac{2\sigma}{\tau}$$

Beckmann

# Other Microfacet Distributions

- Some popular distributions

■ <b>Blinn</b>	$D_1(\alpha) = \cos^{c_1} \alpha$	$c_1 = \frac{\ln 2}{\ln \cos \beta}$
■ <b>Torrance-Sparrow</b>	$D_2(\alpha) = e^{-(c_2 \alpha)^2}$	$c_2 = \frac{\sqrt{2}}{\beta}$
■ <b>Trowbridge-Reitz</b>	$D_3(\alpha) = \frac{c_3^2}{(1 - c_3^2) \cos^2 \alpha - 1}$	$c_3 = \left( \frac{\cos^2 \beta - 1}{\cos^2 \beta - \sqrt{2}} \right)^{1/2}$

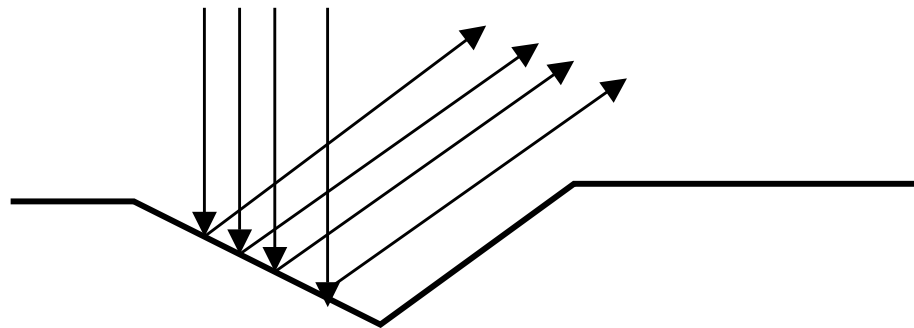
# Self-Shadowing

- Geometric Term,  $G$



## Geometric Term, G

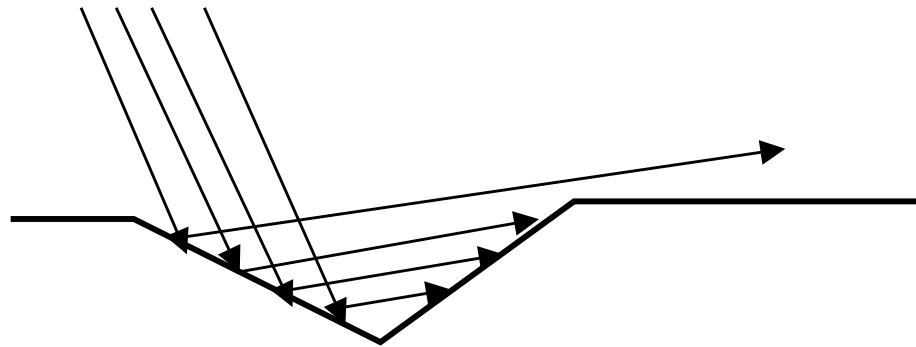
- Surface may be so rough that edges block groove interior from light
- This is known as **shadowing** or **masking**
- Geometric term G accounts for this
- Break G into 3 cases:
- **G, case a:** No self-shadowing



- Mathematically,  $G = 1$

# Geometric Term, G

- **G, case b:** No blocking of incident light, partial blocking of exiting light (**masking**)



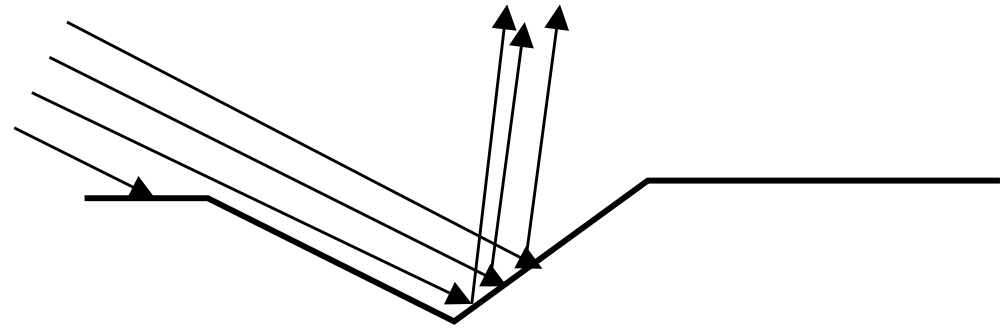
- Mathematically,

$$G_m = \frac{2(\mathbf{m} \cdot \mathbf{h})(\mathbf{m} \cdot \mathbf{h})}{\mathbf{h} \cdot \mathbf{s}}$$

# Geometric Term, G

- G, case c: Partial blocking of incident light, no blocking of exiting light (**shadowing**)
- Mathematically,

$$G_m = \frac{2(\mathbf{m} \cdot \mathbf{h})(\mathbf{m} \cdot \mathbf{h})}{\mathbf{h} \cdot \mathbf{s}}$$



- G term is minimum of 3 cases, hence

$$G = (1, G_m, G_s)$$

# Fresnel Term, F

- So, again recall that specular term

$$spec = \frac{F(\phi, \eta) DG}{(\mathbf{m} \cdot \mathbf{v})}$$

- Microfacets are not perfect mirrors
- $F(\phi, \eta)$  term gives fraction of incident light reflected (angle-dependent)
- $\phi$  is incident angle,  $\eta$  is refractive index of material

$$F = \frac{1}{2} \frac{(g - c)^2}{(g + c)^2} \left\{ 1 + \left( \frac{c(g + c) - 1}{c(g - c) - 1} \right)^2 \right\}$$

- where  $c = \cos(\phi) = \mathbf{m} \cdot \mathbf{s}$  and  $g^2 = \eta^2 + c^2 + 1$

# Fresnel Term, F

- Combining expressions

$$spec = \frac{F(\phi, \eta)DG}{(\mathbf{m} \cdot \mathbf{v})}$$

- In above expression for F, could simply use  $FDG$
- Why divide by  $\mathbf{m} \cdot \mathbf{v}$ ?
- Accounts for why when eye is close to surface, more microfacets are seen per solid angle than when eye is close to normal

# Fresnel Term, $F$

- Required that  $k_d + k_s = 1$
- For spec, we need  $F(\phi, \eta)$
- Usually,  $F(0, \eta)$  is available from tables (Terlouwian)
- Inserting  $\phi = 0, c = 1$  in expression for  $F$

$$F = \frac{(\eta - 1)^2}{(\eta + 1)^2}$$

- And

$$\eta = \frac{1 + \sqrt{F_0}}{1 - \sqrt{F_0}}$$

- So, use tabulated  $F(0, \eta)$  values to calculate  $\eta$
- Then use calculated  $\eta$  in original equation for  $F$

# Some Fresnel Values, $F(0)$

- At incident angle 0

Material	Fresnel Value (R,G,B)
Water	0.02, 0.02, 0.02
Plastic	0.05, 0.05, 0.05
Glass	0.08, 0.08, 0.08
Diamond	0.17, 0.17, 0.17
Copper	0.95, 0.64, 0.54
Aluminum	0.91, 0.92, 0.92

- Schlick approximation to get arbitrary F

$$F(\theta) = F(0) + (1 - F(0))(1 - \cos\theta)^5$$

- Oren-Nayar – Lambertian not specular
- Aishikhminn-Shirley – Grooves not v-shaped.  
Other Shapes
- BRDF viewer
- Microfacet generator

# Subsurface Scattering in liquids and solids

- Examples



Jade



Marble



Skin

- Next week: scattering in atmosphere, clouds, gases, etc

# More Examples...



**Leaves**

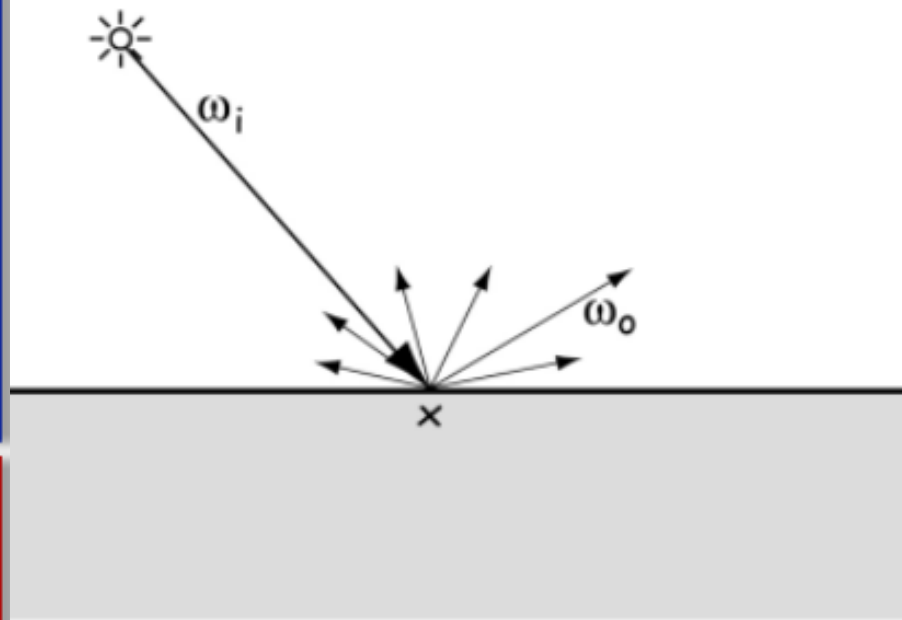


**Hair**



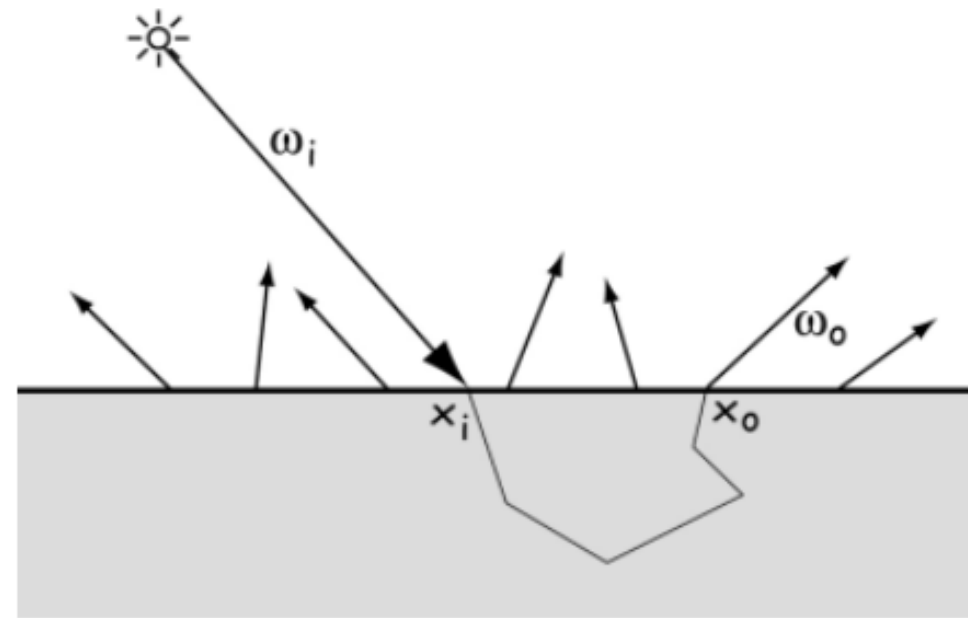
**Milk**

# Subsurface Scattering



$$f_r(x, \omega_i, \omega_o) \equiv \frac{dL_r(x, \omega_o)}{dE_i(x, \omega_i)}$$

Reflection

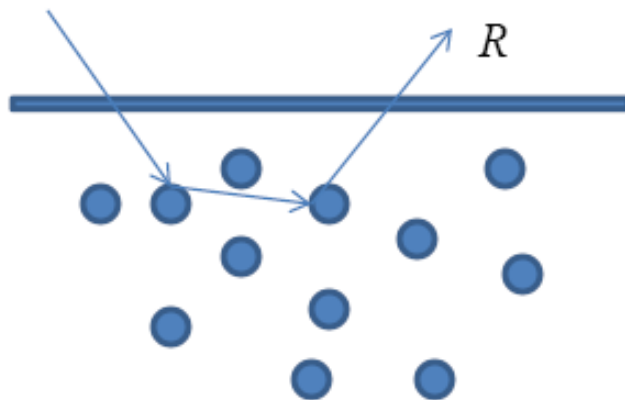


$$S(x_i, \omega_i; x_o, \omega_o) \equiv \frac{dL_r(x_o, \omega_o)}{d\Phi_i(x_i, \omega_i)}$$

Subsurface Scattering

# Modeling Subsurface Scattering

- Multiple layers that reflect or transmit light
- Origin: Kubelka-Munk theory for modeling scattering of paint pigments
- $K$ , fraction of light absorbed
- $S$ , fraction of light scattered



$$R = 1 + \frac{K}{S} - \sqrt{\frac{K}{S} + 2\frac{K}{S}}$$

# Dorsey and Hanrahan

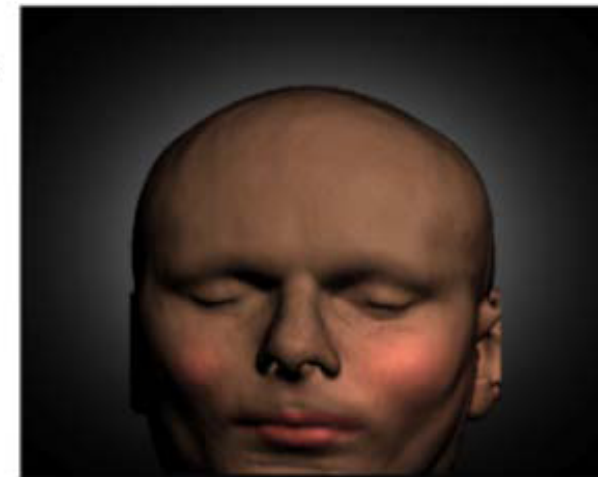
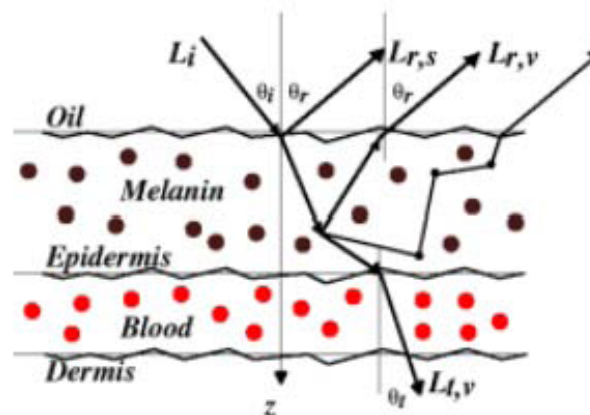
- Kubelka Munk extended by Dorsey, Hanrahan
- Multiple layers of scattering
- Considering material of thickness  $d$

$$R = \frac{\sinh(bSd)}{a \sinh(bSd) + b \cosh(bSd)}$$

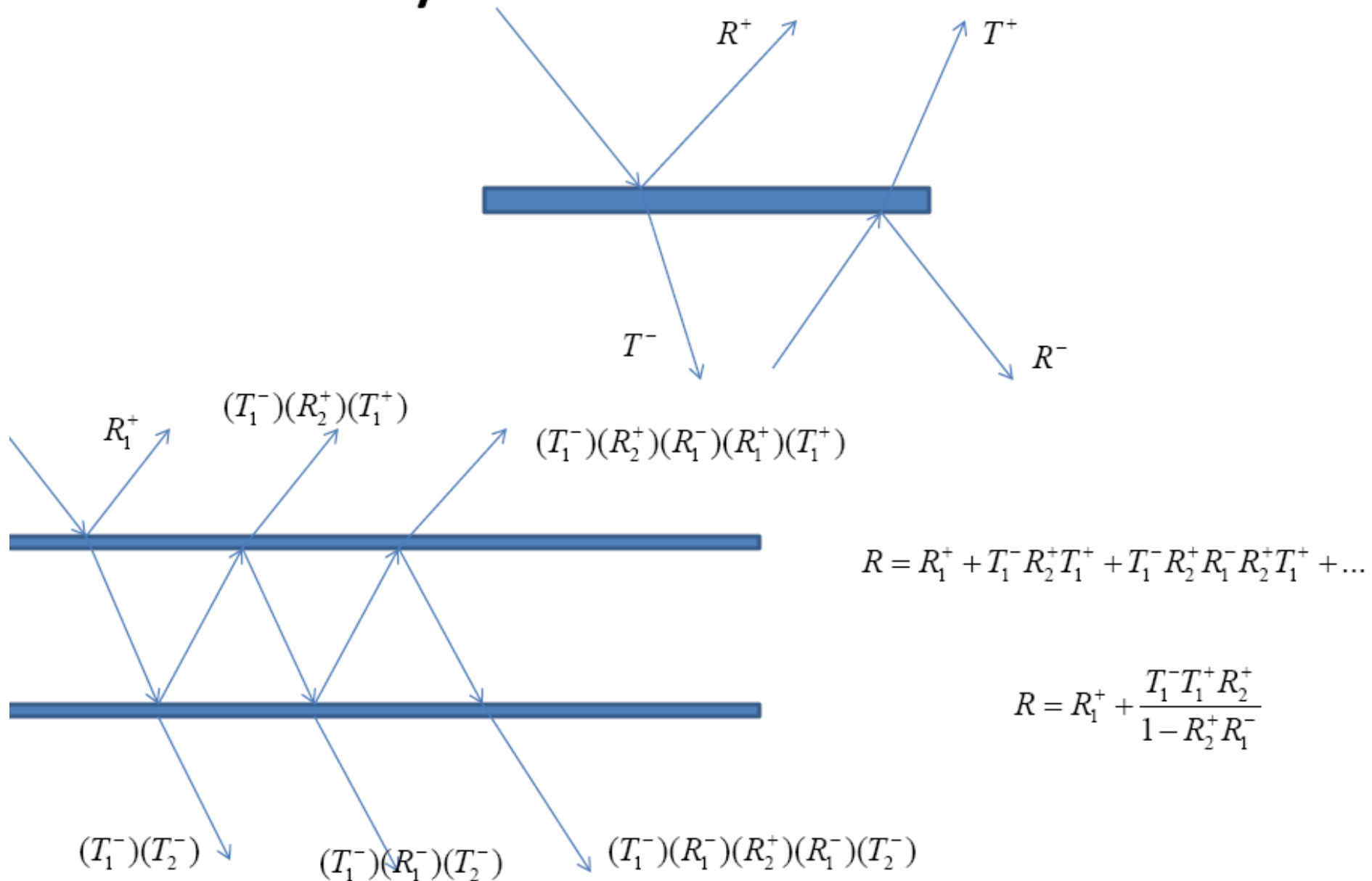
$$T = \frac{b}{a \sinh(bSd) + b \cosh(bSd)}$$

$$a = \frac{S + K}{S}$$

$$b = \sqrt{a^2 - 1}$$



# Dorsey and Hanrahan Model



# References

- Pat Hanrahan, CS 348B slides, 2009
- Hill and Kelley, Computer Graphics using OpenGL (3<sup>rd</sup> edition)
- Matt Pharr, Greg Humphreys "Physically Based Rendering", Chapter 13
- Dorsey and Rushmeier, Modeling of Digital Materials
- Akenine-Moller, Haines and Hoffman, Real Time Rendering, 3<sup>rd</sup> edition