Digital Image Processing (CS/ECE 545) Lecture 2: Histograms and Point Operations (Part 1)

Prof Emmanuel Agu

Computer Science Dept.

Worcester Polytechnic Institute (WPI)

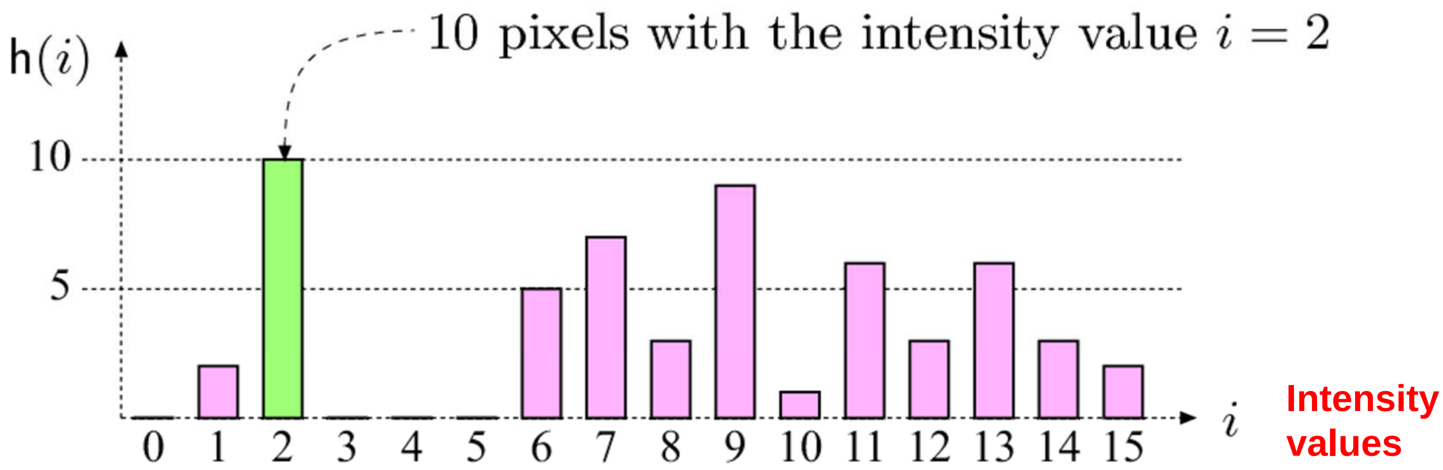
Histograms

- Histograms plots how many times (frequency) each intensity value in image occurs
- Example:
  - Image (left) has 256 distinct gray levels (8 bits)
  - Histogram (right) shows frequency (how many times) each gray level occurs

Histograms

- Many cameras display real time histograms of scene
- Helps avoid taking over-exposed pictures
- Also easier to detect types of processing previously applied to image

Histograms

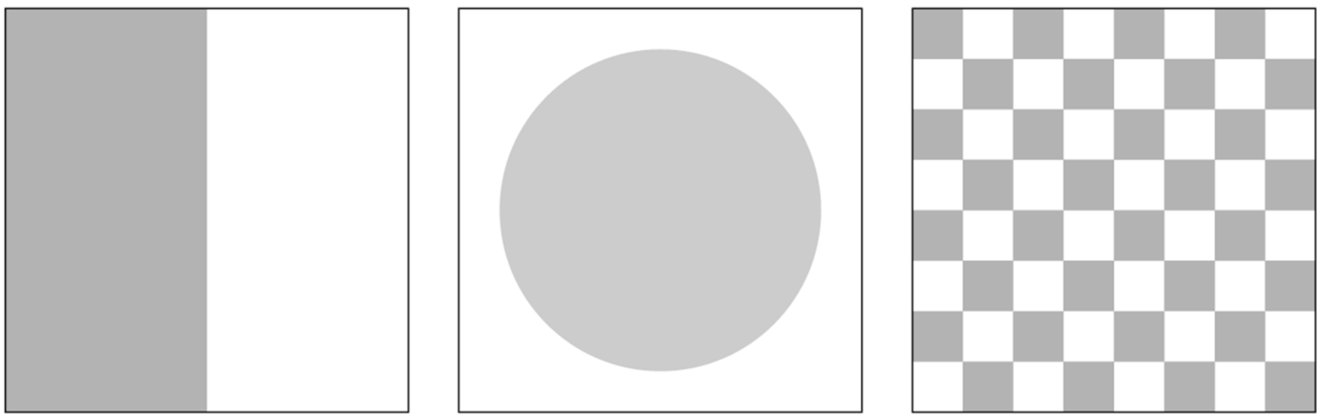


h(i)	0	2	10	0	0	0	5	7	3	9	1	6	3	6	3	2
i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

- E.g. K = 16, 10 pixels have intensity value = 2
- Histograms: only statistical information
- No indication of **location** of pixels

Histograms

- Different images can have **same** histogram
- 3 images below have same histogram



- Half of pixels are gray, half are white
  - Same histogram = same statistics
  - Distribution of intensities could be different
- Can we reconstruct image from histogram? No!

Histograms

- So, a histogram for a grayscale image with intensity values in range

$$I(u, v) \in [0, K-1]$$

would contain exactly K entries

- E.g. 8-bit grayscale image, $K = 2^8 = 256$
- Each histogram entry is defined as:

  h(i) = number of pixels with intensity *I* for all $0 < i < K$.

- E.g: h(255) = number of pixels with intensity = 255
- Formal definition $$\mathsf{h}(i) = \mathrm{card}\{(u, v) \mid I(u, v) = i\}$$

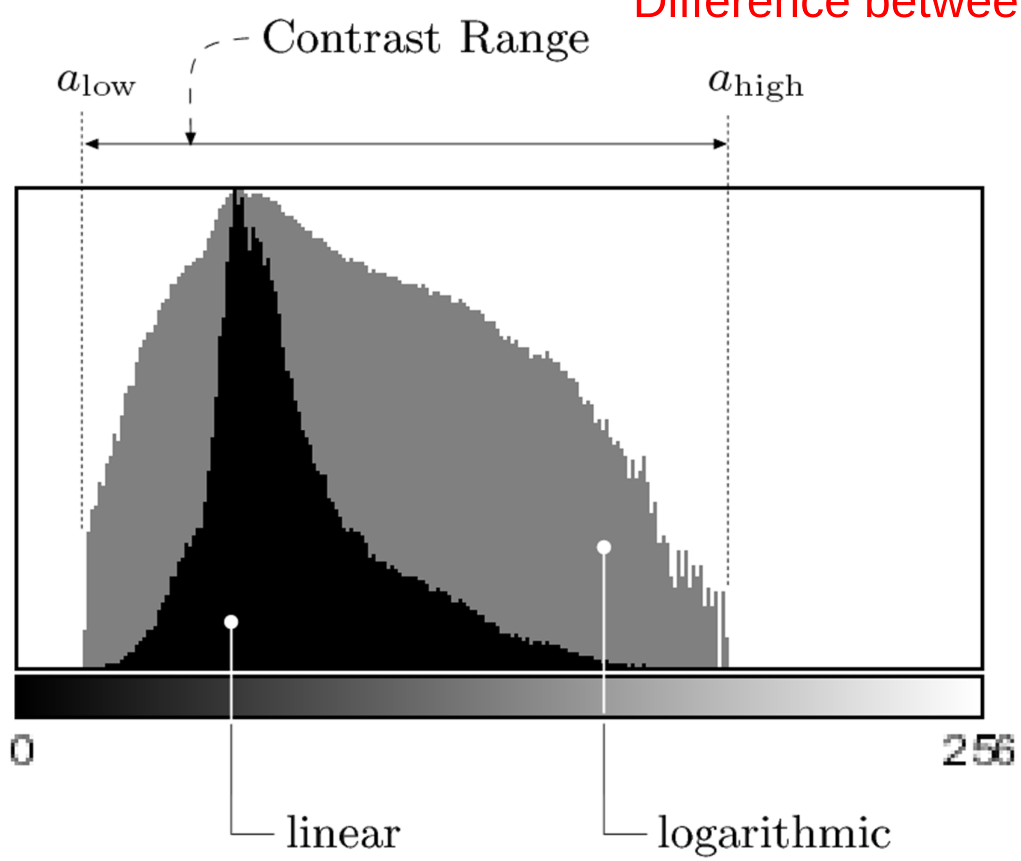
Number (size of set) of pixels

such that

Interpreting Histograms

- Log scale makes low values more visible

Difference between darkest and lightest



Histograms

- Histograms help detect image acquisition issues
- Problems with image can be identified on histogram
  - Over and under exposure
  - Brightness
  - Contrast
  - Dynamic Range
- Point operations can be used to alter histogram. E.g
  - Addition
  - Multiplication
  - Exp and Log
  - Intensity Windowing (Contrast Modification)

Image Brightness

- Brightness of a grayscale image is the **average intensity** of all pixels in image

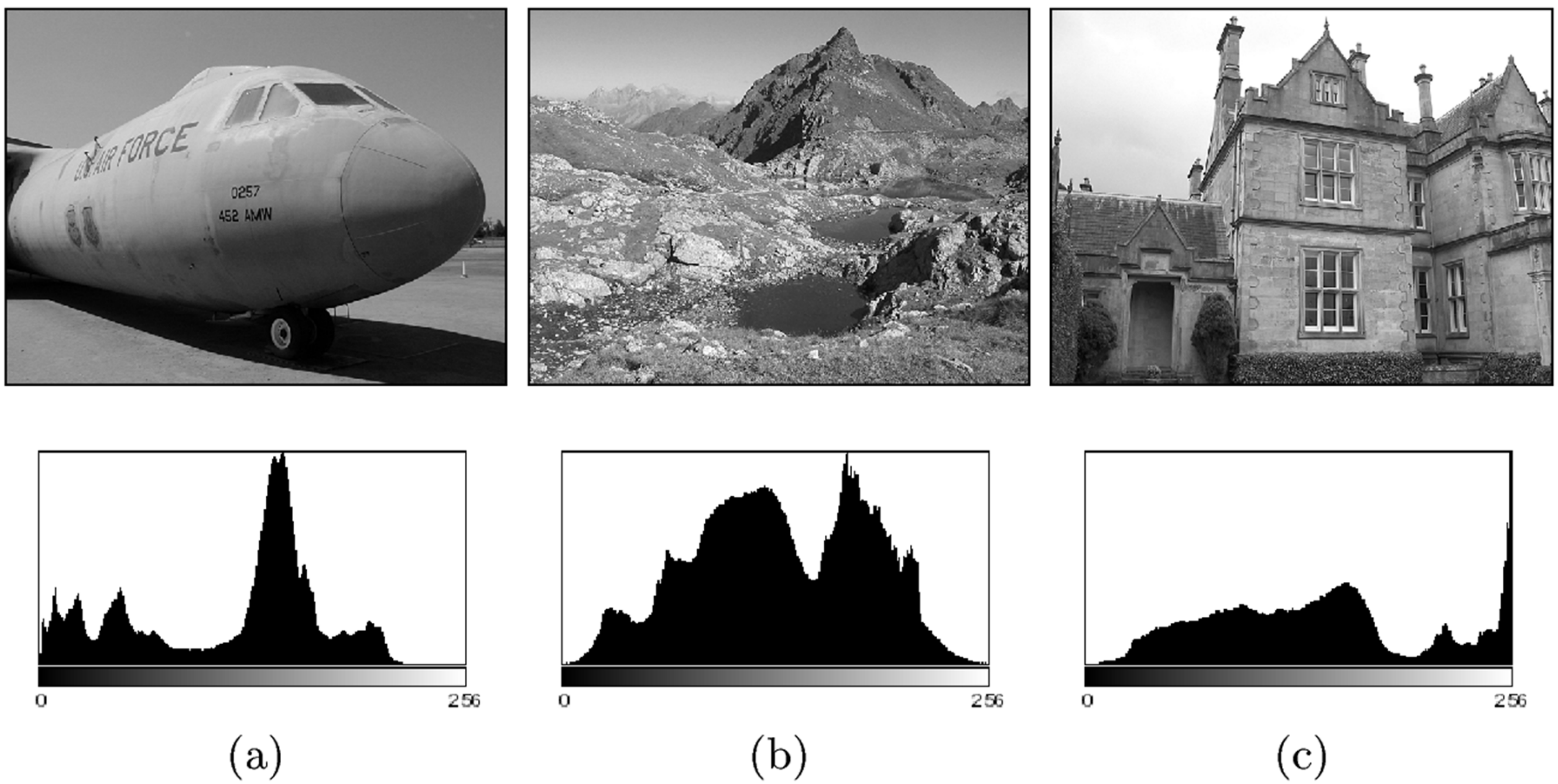
$$B(I) = \frac{1}{wh}\sum_{v=1}^{h}\sum_{u=1}^{w} I(u,v)$$

2. Divide by total number of pixels

1. Sum up all pixel intensities

Detecting Bad Exposure using Histograms

Exposure? Are intensity values spread (good) out or bunched up (bad)



Image

Histogram

Underexposed

Properly Exposed

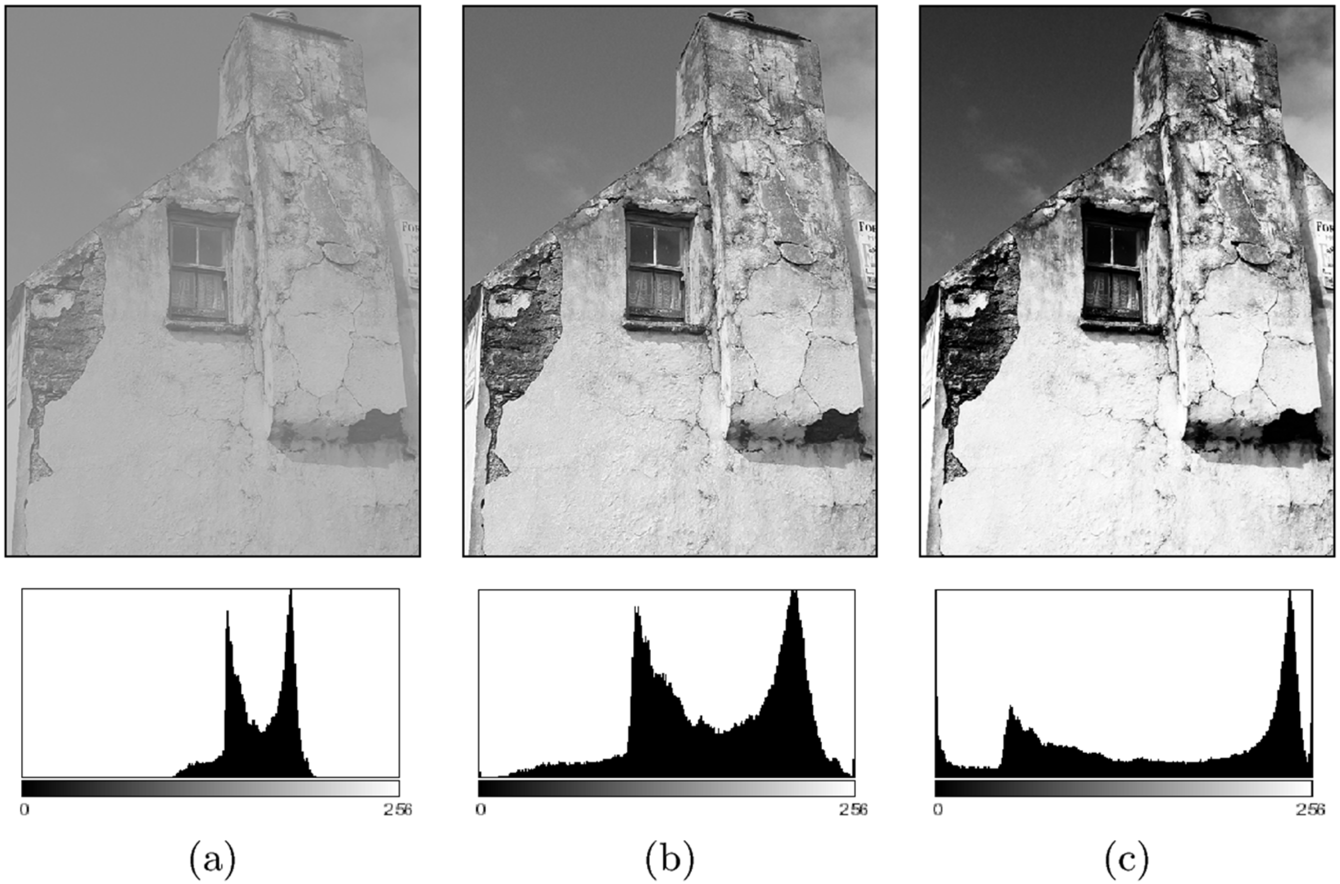
Overexposed

Image Contrast

- The contrast of a grayscale image indicates how easily objects in the image can be distinguished
- **High contrast image:** many distinct intensity values
- **Low contrast:** image uses few intensity values

Histograms and Contrast

Good Contrast? Widely spread intensity values
+ large difference between min and max intensity values



Image

Histogram

Low contrast **Normal contrast** **High contrast**

Contrast Equation?

- Many different equations for contrast exist
- Examples:

$$\text{Contrast} = \frac{\text{Change in Luminance}}{\text{Average Luminance}}$$

- Michalson's equation for contrast

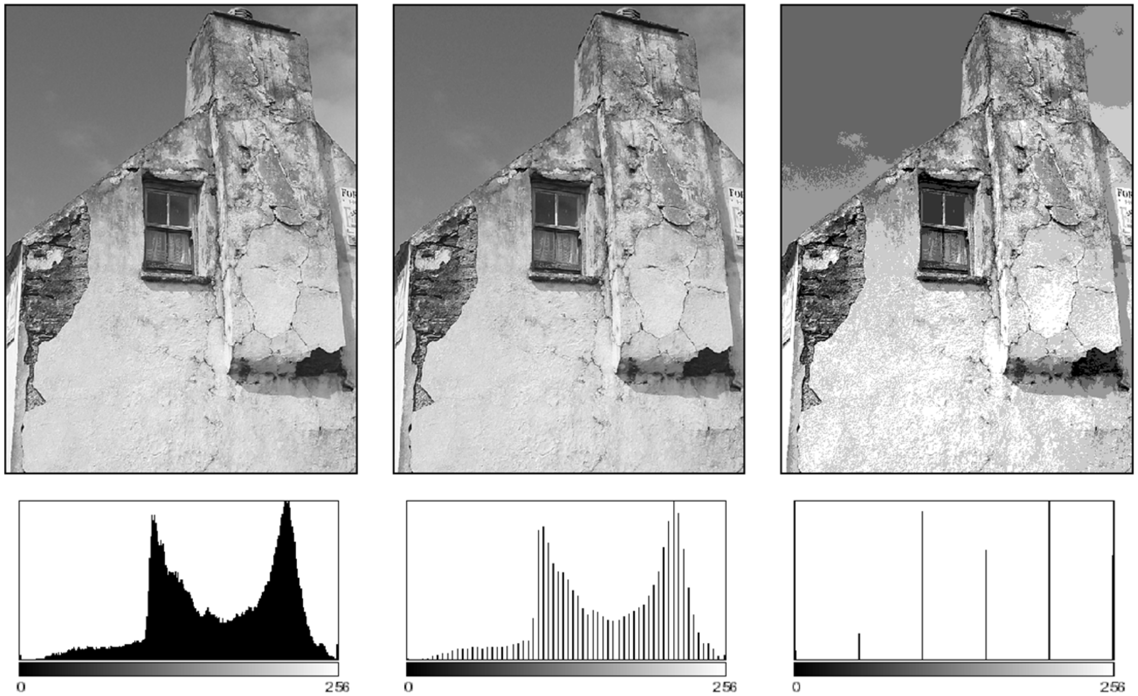
$$C_M(I) = \frac{\max(I) - \min(I)}{\max(I) + \min(I)}$$

Contrast Equation?

- These equations work well for simple images with 2 luminances (i.e. uniform foreground and background)
- Does not work well for complex scenes with many luminances or if min and max intensities are small

Histograms and Dynamic Range

- **Dynamic Range:** Number of distinct pixels in image



(a) High Dynamic Range

(b) Low Dynamic Range (64 intensities)

(c) Extremely low Dynamic Range (6 intensity values)

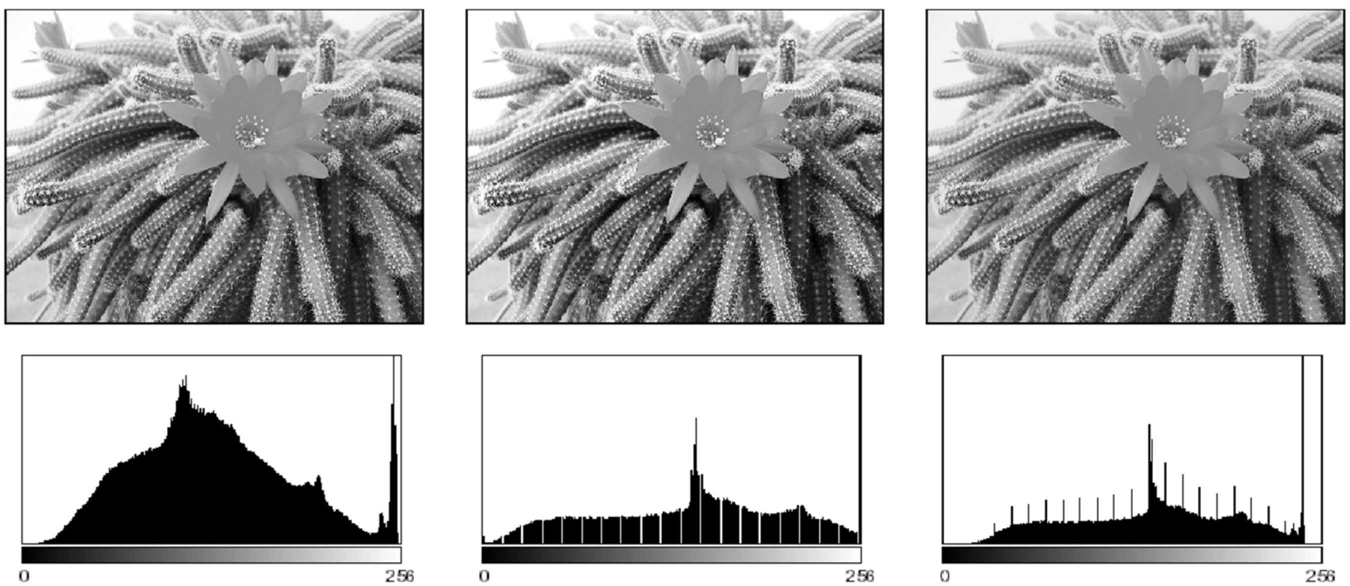
- Difficult to increase image dynamic range (e.g. interpolation)
- HDR (12-14 bits) capture typical, then down-sample

High Dynamic Range Imaging

- **High dynamic range** means very bright and very dark parts in a single image (many distinct values)
- Dynamic range in photographed scene may exceed number of available bits to represent pixels
- Solution:
  - Capture multiple images at different exposures
  - Combine them using image processing

Detecting Image Defects using Histograms

- No “best” histogram shape, depends on application
- Image defects
  - **Saturation:** scene illumination values outside the sensor’s range are set to its min or max values => results in spike at ends of histogram
  - **Spikes and Gaps in manipulated images** (not original). Why?

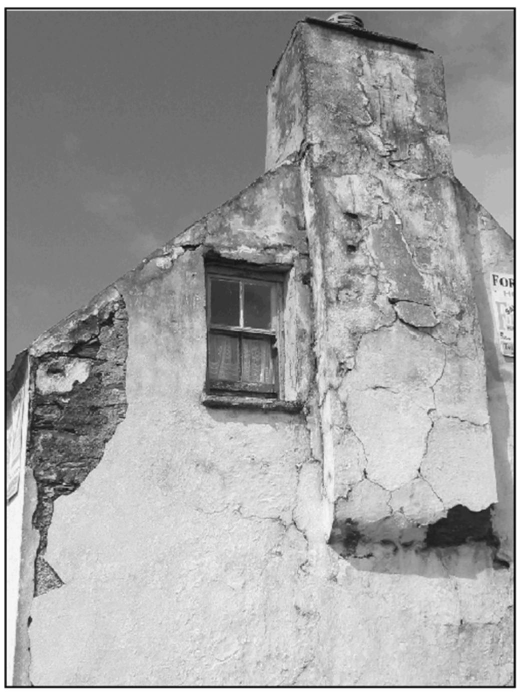


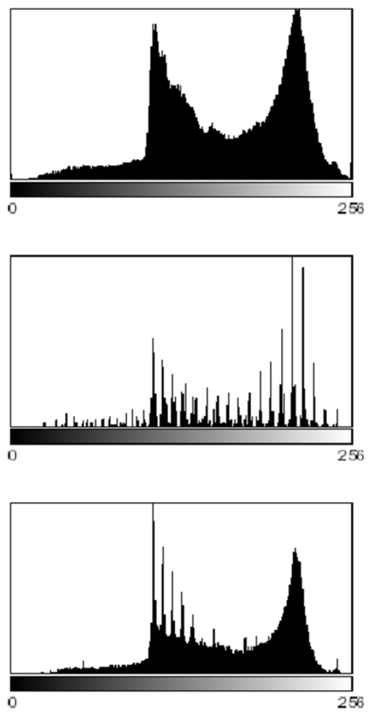
(a) (b) (c)

Image Defects: Effect of Image Compression

- Histograms show impact of image compression
- Example: in GIF compression, dynamic range is reduced to only few intensities (quantization)

Original Image





(a) **Original Histogram**

(b) **Histogram after GIF conversion**

(c) **Fix? Scaling image by 50% and Interpolating values recreates some lost colors**
But GIF artifacts still visible

Effect of Image Compression

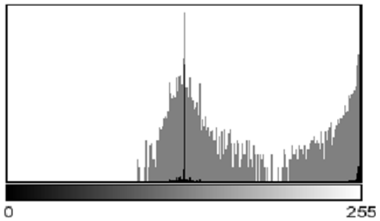
- Example: Effect of JPEG compression on line graphics
- JPEG compression designed for color images

(a)

(b)

Original histogram has only 2 intensities (gray and white)

(c)



(d)

JPEG image appears dirty, fuzzy and blurred

Its Histogram contains gray values not in original

Computing Histograms

```
public class Compute_Histogram implements PlugInFilter {

    public int setup(String arg, ImagePlus img) {
        return DOES_8G + NO_CHANGES;
    }

    public void run(ImageProcessor ip) {
        int[] H = new int[256]; // histogram array
        int w = ip.getWidth();
        int h = ip.getHeight();

        for (int v = 0; v < h; v++) {
            for (int u = 0; u < w; u++) {
                int i = ip.getPixel(u,v);
                H[i] = H[i] + 1;
            }
        }
        ... //histogram H[] can now be used
    }

} // end of class Compute_Histogram
```

Receives 8-bit image, Will not change it

Create array to store histogram computed

Get width and height of image

Iterate through image pixels, add each intensity to appropriate histogram bin

ImageJ Histogram Function

- ImageJ has a histogram function (**`getHistogram( )`**)
- Prior program can be simplified if we use it

```
public void run(ImageProcessor ip) {
    int[] H = ip.getHistogram();
    ... // histogram H[] can now be used
}
```

Returns histogram as an array of integers

Large Histograms: Binning

- High resolution image can yield very large histogram
- Example: 32-bit image = 2^{32} = 4,294,967,296 columns
- Such a large histogram impractical to display
- Solution? Binning!
  - Combine **ranges of intensity values** into histogram columns

So, given the image $I : \Omega \rightarrow [0, K-1]$, the binned histogram for I is the function

$$h(i) = \text{card}\{(u, v) |\ a_i \leq I(u, v) < a_{i+1}\},$$

where $0 = a_0 < a_1 < \ldots < a_B = K.$

Number (size of set) of pixels

such that

Pixel's intensity is between a_i and a_{i+1}

Calculating Bin Size

- Typically use equal sized bins
- Bin size? $\frac{\textit{Number of distinct values in image}}{\textit{Number of bins}}$
- Example: To create 256 bins from 14-bit image

$$Bin\ size = \frac{2^{14}}{256} = 64$$

$$\begin{array}{lclcl}
\mathsf{h}(0) & \leftarrow & 0 \leq I(u,v) < & 64 \\
\mathsf{h}(1) & \leftarrow & 64 \leq I(u,v) < & 128 \\
\mathsf{h}(2) & \leftarrow & 128 \leq I(u,v) < & 192 \\
\vdots & & \vdots \quad\quad \vdots & \vdots \\
\mathsf{h}(j) & \leftarrow & a_j \leq I(u,v) < & a_{j+1} \\
\vdots & & \vdots \quad\quad \vdots & \vdots \\
\mathsf{h}(255) & \leftarrow & 16320 \leq I(u,v) < & 16384
\end{array}$$

Binned Histogram

- To calculate which bin a pixel's intensity belongs to

$$\frac{I(u,v)}{k_B} = \frac{I(u,v)}{K/B} = I(u,v) \cdot \frac{B}{K}$$

- Previous example, B = 256, K = 2^{14} = 16384

```
int[] binnedHistogram(ImageProcessor ip) {
  int K = 256; // number of intensity values
  int B = 32; // size of histogram, must be defined
  int[] H = new int[B]; // histogram array
  int w = ip.getWidth();
  int h = ip.getHeight();

  for (int v = 0; v < h; v++) {
    for (int u = 0; u < w; u++) {
      int a = ip.getPixel(u, v);
      int i = a * B / K; // integer operations only!
      H[i] = H[i] + 1;
    }
  }
  // return binned histogram
  return H;
}
```

Create array to store histogram computed

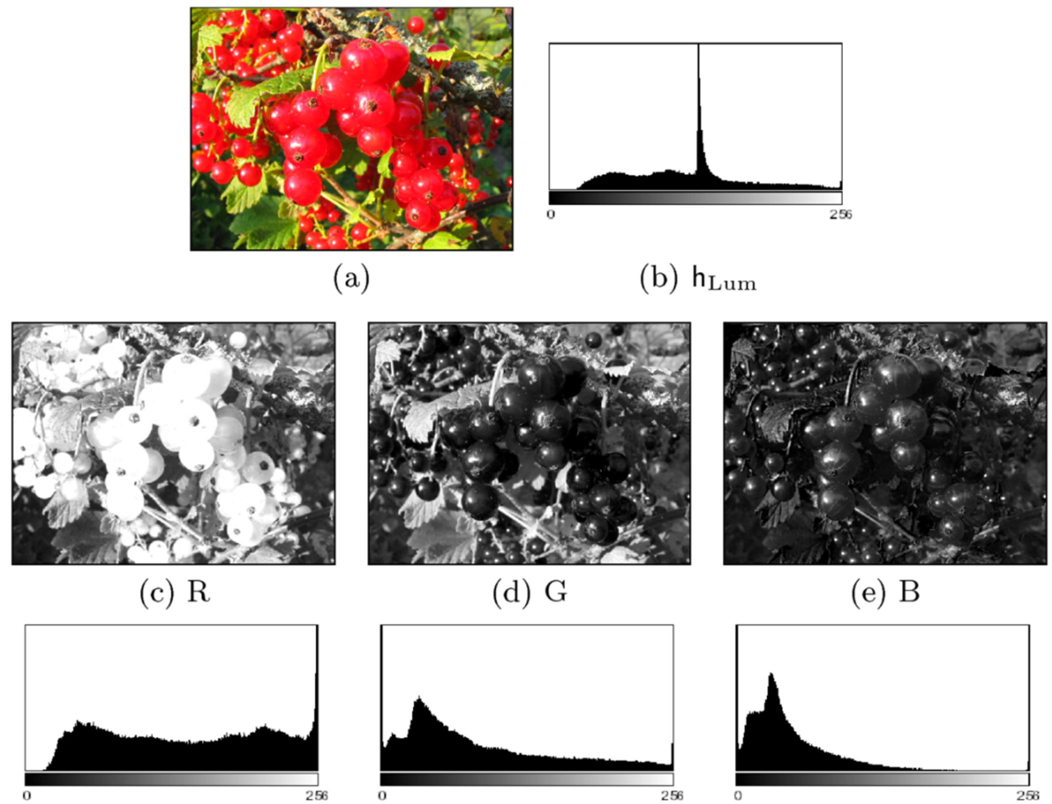
Calculate which bin to add pixel's intensity

Increment corresponding histogram

Color Image Histograms

Two types:

1. **Intensity histogram:**
   - Convert color image to gray scale
   - Display histogram of gray scale
2. **Individual Color Channel Histograms:** 3 histograms (R,G,B)



(a) (b) h_{Lum}

(c) R (d) G (e) B

(f) h_R (g) h_G (h) h_B

Color Image Histograms

- Both types of histograms provide useful information about lighting, contrast, dynamic range and saturation effects
- No information about the actual color distribution!
- Images with totally different RGB colors can have same R, G and B histograms
- Solution to this ambiguity is the **Combined Color Histogram**.
  - More on this later

Cumulative Histogram

- Useful for certain operations (e.g. histogram equalization) later
- Analogous to the **Cumulative Density Function (CDF)**
- Definition:

$$\mathsf{H}(i) = \sum_{j=0}^{i} \mathsf{h}(j) \quad \text{for} \;\; 0 \le i < K$$

- Recursive definition

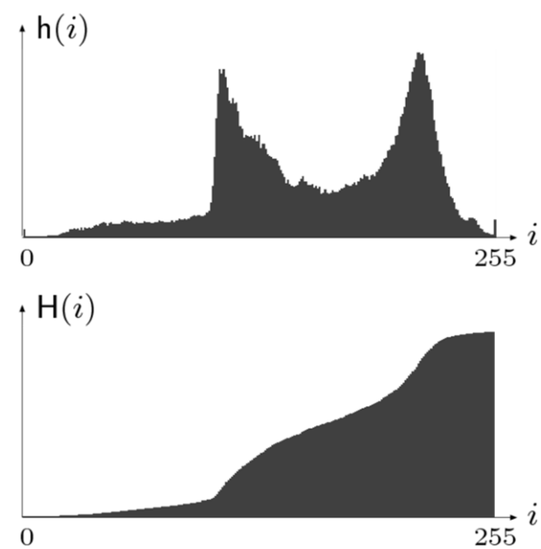
$$\mathsf{H}(i) = \begin{cases} \mathsf{h}(0) & \text{for} \;\; i = 0 \\ \mathsf{H}(i-1) + \mathsf{h}(i) & \text{for} \;\; 0 < i < K \end{cases}$$

- Monotonically increasing

$$\mathsf{H}(K-1) = \sum_{j=0}^{K-1} \mathsf{h}(j) = M \cdot N$$

Last entry of Cum. histogram

Total number of pixels in image



Point Operations

- Point operations changes a pixel's intensity value according to some function (don't care about pixel's neighbor)

$$a' \leftarrow f(a)$$
$$I'(u,v) \leftarrow f\big(I(u,v)\big)$$

- Also called a **homogeneous operation**
- New pixel intensity **depends on**
  - Pixel's previous intensity *I(u,v)*
  - Mapping function *f*( )
- **Does not depend on**
  - Pixel's location *(u,v)*
  - Intensities of neighboring pixels

Some Homogeneous Point Operations

- Addition (Changes brightness)

$$f(p) = p + k \qquad \text{E.g.} \quad f_{\text{bright}}(p) = p + 10$$

- Multiplication (Stretches/shrinks image contrast range)

$$f(p) = k \times p \qquad \text{E.g.} \quad f_{\text{contrast}}(p) = p \text{ x } 1.5$$

- Real-valued functions

$$\exp(x), \log(x), (1/x), x^k, \text{ etc.}$$

- Quantizing pixel values
- Global thresholding
- Gamma correction

Point Operation Pseudocode

- **Input:** Image with pixel intensities *I(u,v)* defined on [1 .. *w*] x [1 .. *H*]
- **Output:** Image with pixel intensities *I'(u,v)*

for v = 1 .. *h*

    for u = 1 .. *w*

        set *I(u, v) = f (I(u,v))*

Non-Homogeneous Point Operation

- New pixel value depends on:
  - Old value + **pixel's location *(u,v)***

$$a' \leftarrow g(a, u, v)$$

$$I'(u, v) \leftarrow g\left(I(u, v), u, v\right)$$

Clamping

- Deals with pixel values outside displayable range
  - If (a > 255)   a = 255;
  - If (a < 0) a = 0;
- Function below will **clamp** (force) all values to fall within range [*a*,*b*]

$$f(p) = \begin{cases} a & \text{if } p < a \\ p & \text{if } a \leq p \leq b \\ b & \text{if } p > b \end{cases}$$

Example: Modify Intensity and Clamp

- Point operation: increase image contrast by 50% then clamp values above 255

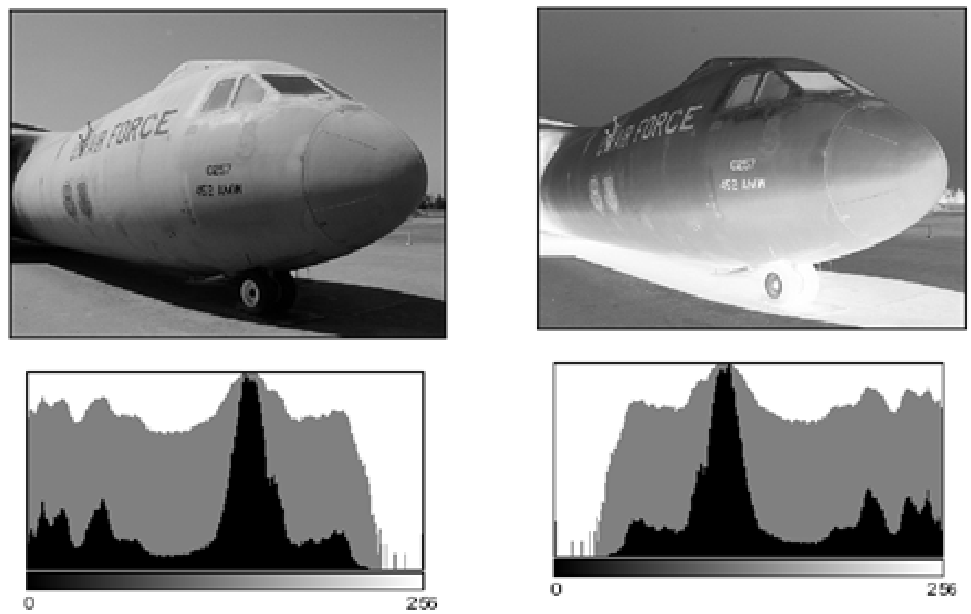
```java
public void run(ImageProcessor ip) {
  int w = ip.getWidth();
  int h = ip.getHeight();

  for (int v = 0; v < h; v++) {
    for (int u = 0; u < w; u++) {
      int a = (int) (ip.get(u, v) * 1.5 + 0.5);
      if (a > 255)
        a = 255;    // clamp to maximum value
      ip.set(u, v, a);
    }
  }
}
```

Increase contrast by 50%

Inverting Images

$$f_{\text{invert}}(a) = -a + a_{\text{max}} = a_{\text{max}} - a$$

- 2 steps
  1. Multiple intensity by -1
  2. Add constant (e.g. a_{max}) to put result in range $[0, a_{max}]$
- Implemented as ImageJ method `invert( )`



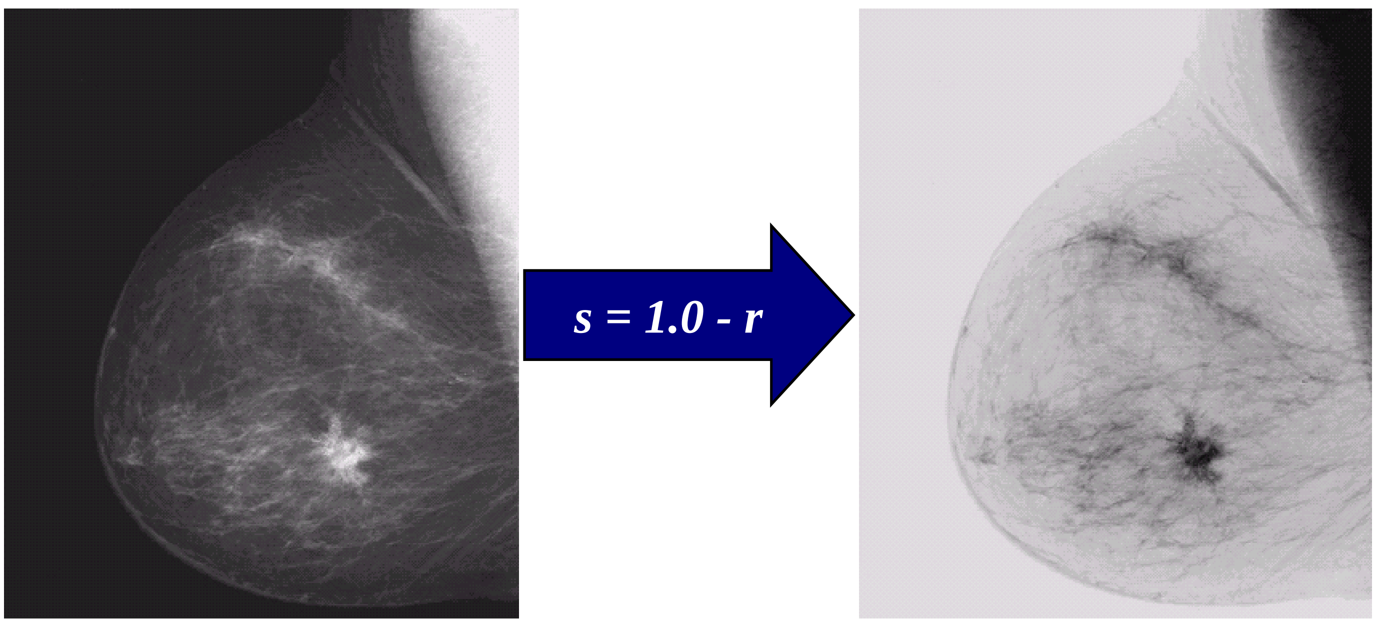
(a) (c)

Original Inverted Image

Image Negatives (Inverted Images)

- Image negatives useful for enhancing white or grey detail embedded in dark regions of an image
  - Note how much clearer the tissue is in the negative image of the mammogram below

Original Image



Negative Image

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Thresholding

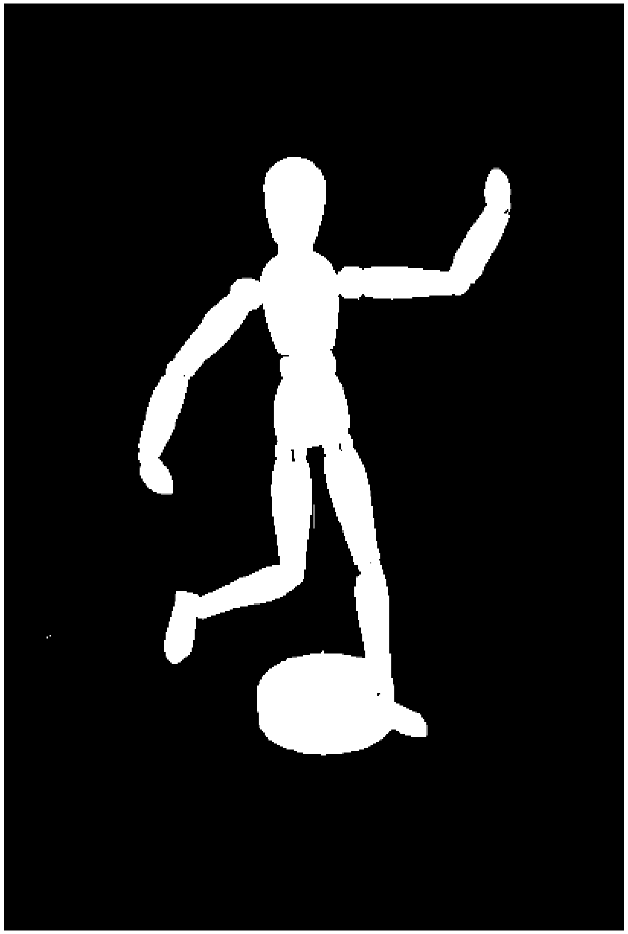
- Input values below **threshold** a_{th} set to a_0
- Input values above **threshold** a_{th} set to a_1

$$f_{\text{threshold}}(a) = \begin{cases} a_0 & \text{for} \quad a < a_{\text{th}} \\ a_1 & \text{for} \quad a \geq a_{\text{th}} \end{cases}$$

- Converts grayscale image to binary image (binarization) if
  - $a_0 = 0$
  - $a_1 = 1$
- Implemented as imageJ method `threshold( )`

Thresholding Example

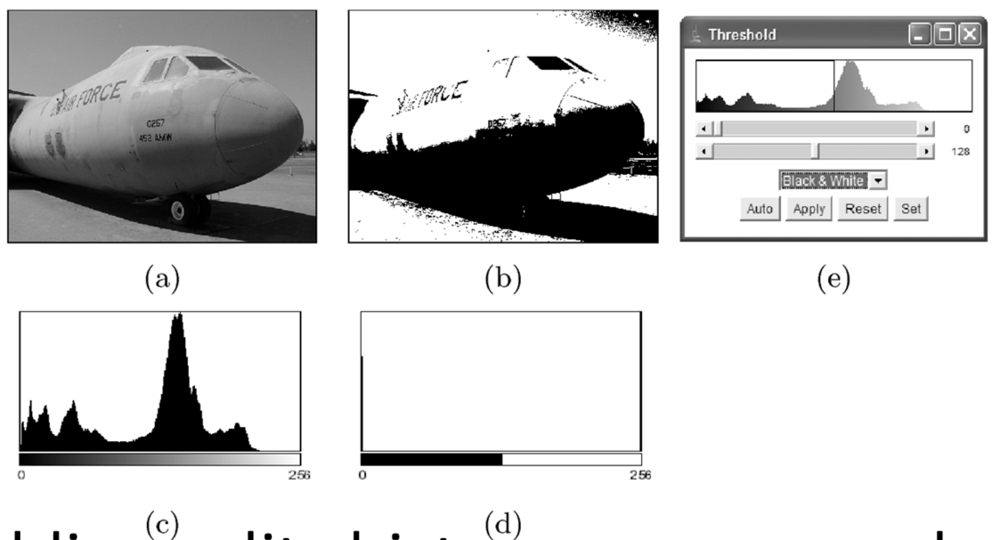
Original Image



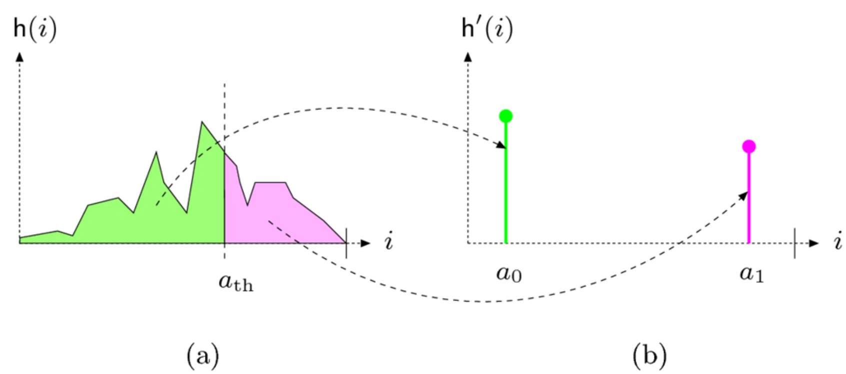
Thresholded Image

Thresholding and Histograms

- Example with a_{th} = 128

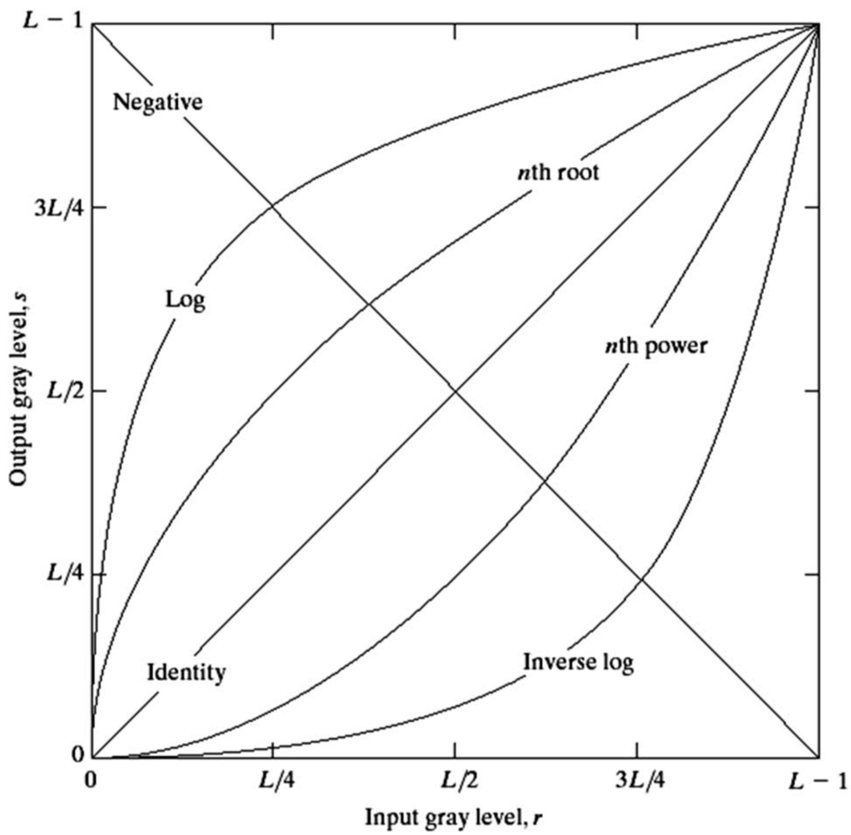


- Thresholding splits histogram, merges halves into a_0 a_1



Basic Grey Level Transformations

- 3 most common gray level transformation:
  - Linear
    - Negative/Identity
  - Logarithmic
    - Log/Inverse log
  - Power law
    - n^{th} power/n^{th} root

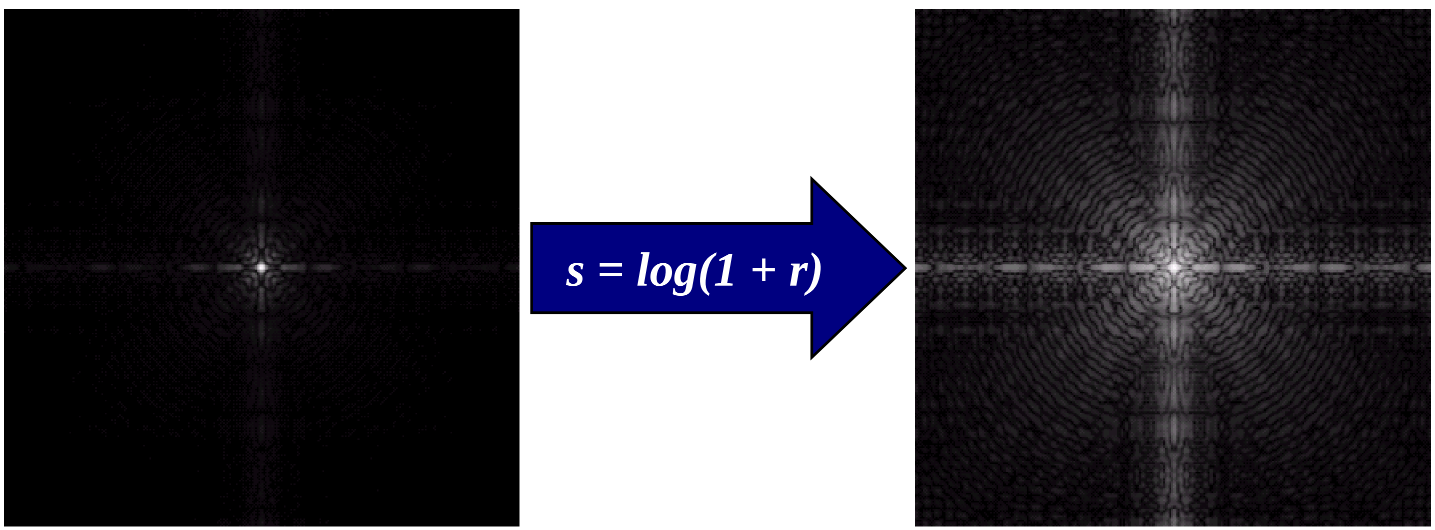


Logarithmic Transformations

- Maps narrow range of input levels => wider range of output values
- Inverse log transformation does opposite transformation
- The general form of the log transformation is

New pixel value → $s = c * log(1 + r)$ ← Old pixel value

- Log transformation of Fourier transform shows more detail



Power Law Transformations

- Power law transformations have the form

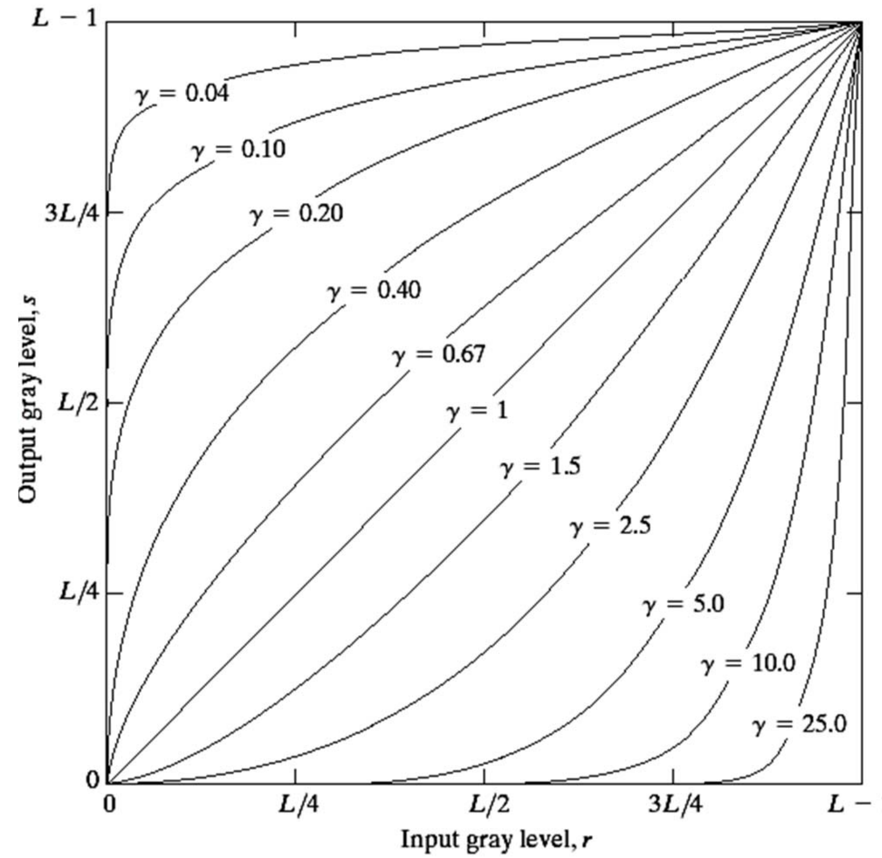
$$s = c * r^{\gamma}$$

Power

New pixel value

Constant

Old pixel value



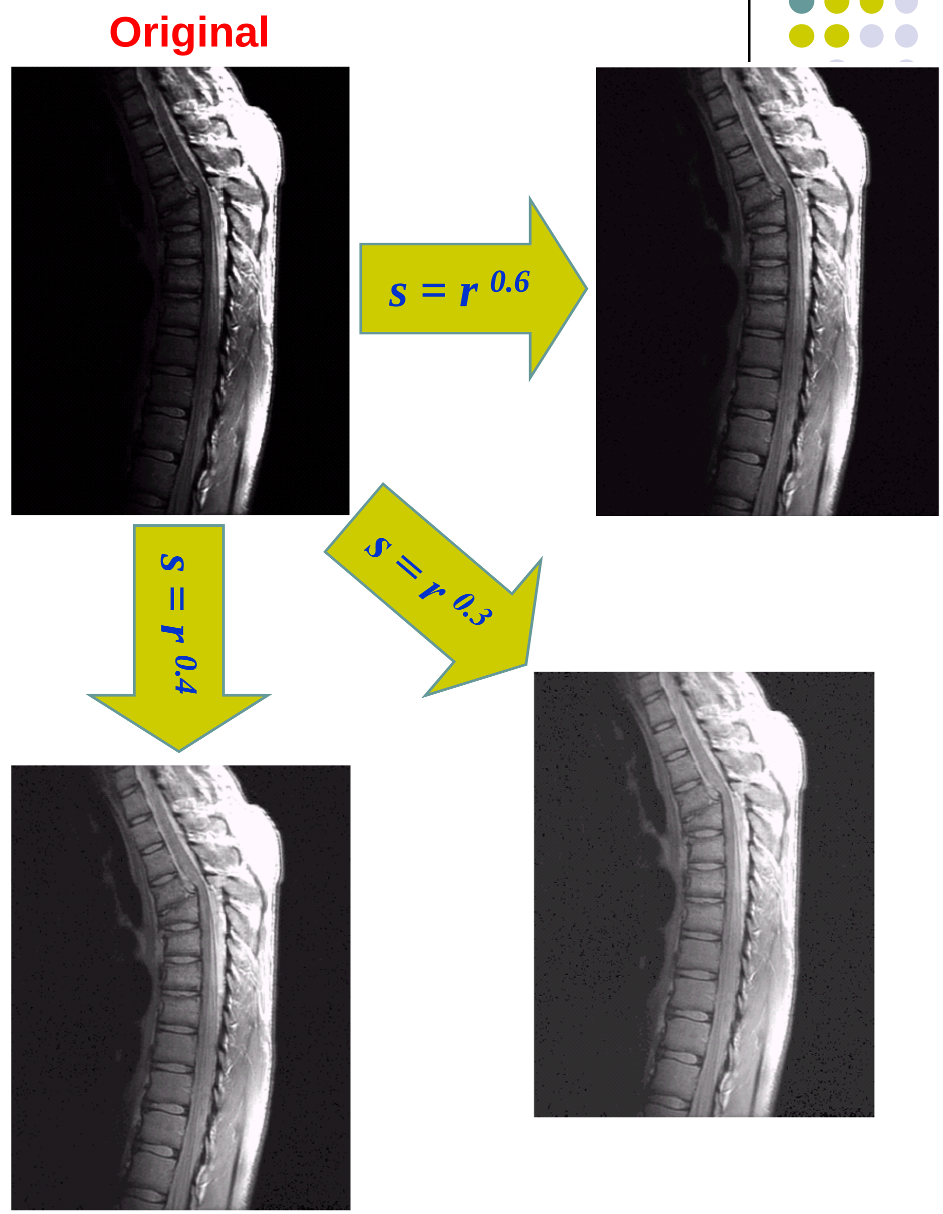
- Map narrow range of dark input values into wider range of output values or vice versa
- Varying γ gives a whole family of curves

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Power Law Example

- Magnetic Resonance (MR) image of fractured human spine

- Different power values highlight different details



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

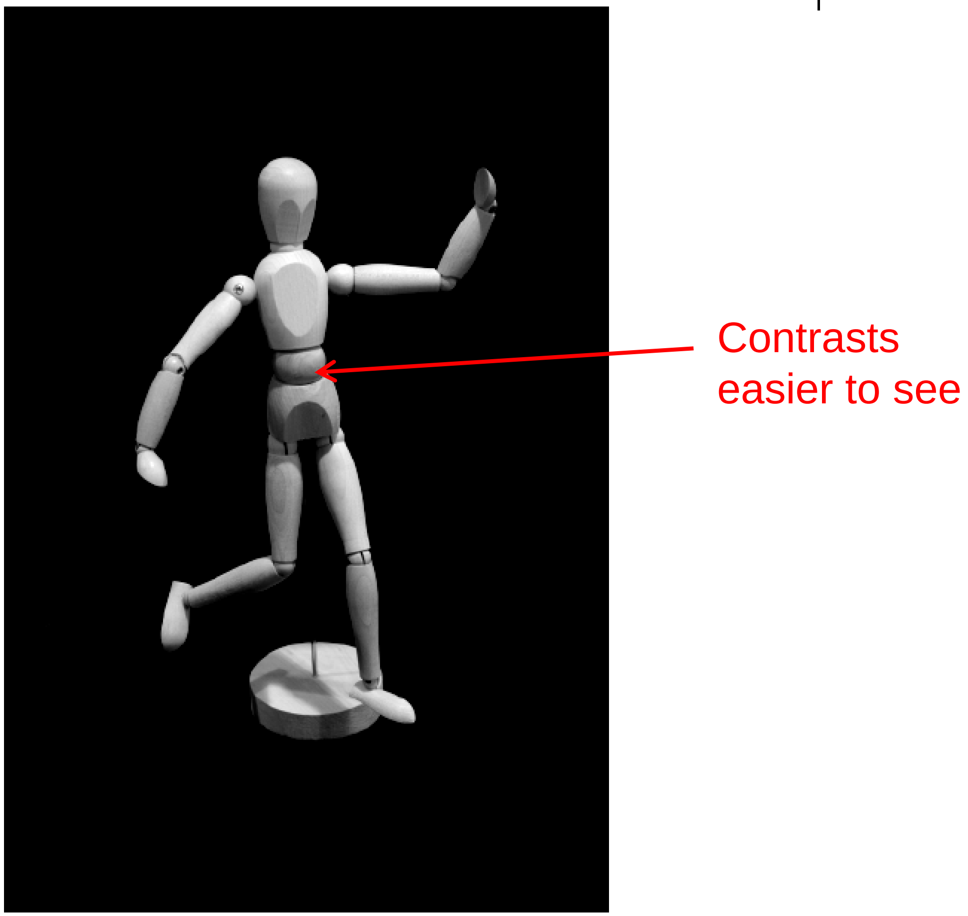
Intensity Windowing

- A clamp operation, then linearly stretching image intensities to fill possible range
- To window an image in [*a*,*b*] with max intensity ***M***

$$f(p) = \begin{cases} 0 & \text{if } \; p < a \\ M \times \frac{p-a}{b-a} & \text{if } \; a \le p \le b \\ M & \text{if } \; p > b \end{cases}$$

Intensity Windowing Example

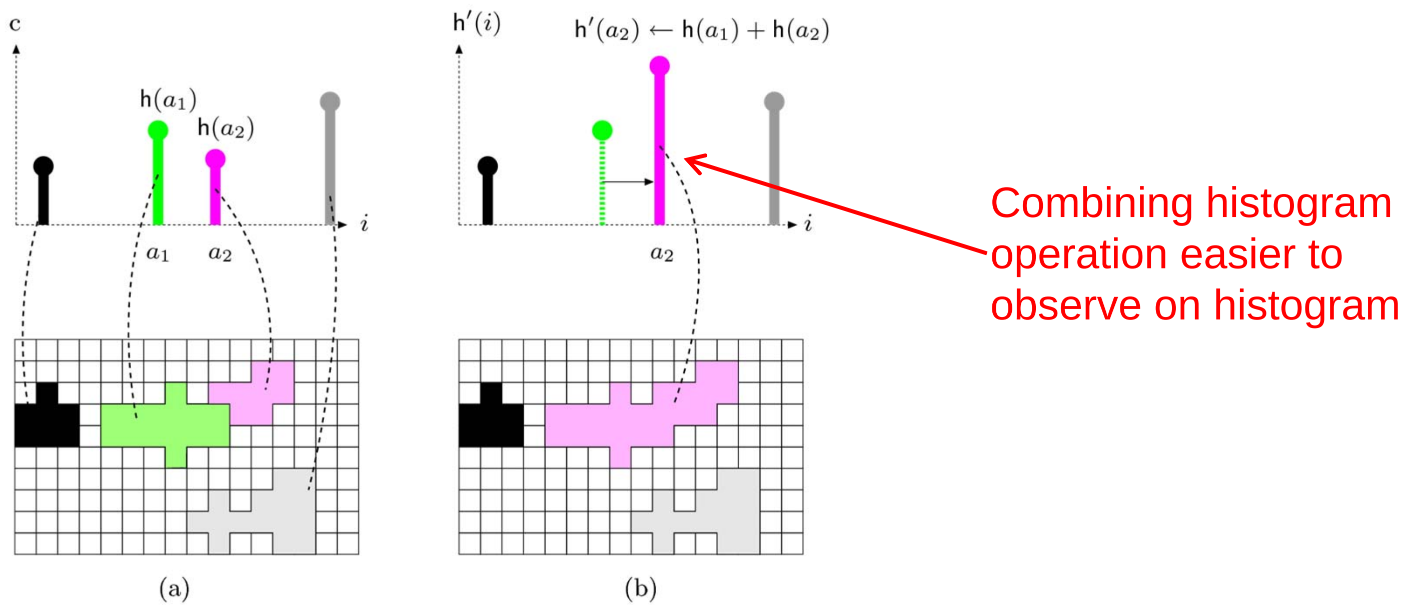
Original Image



Windowed Image

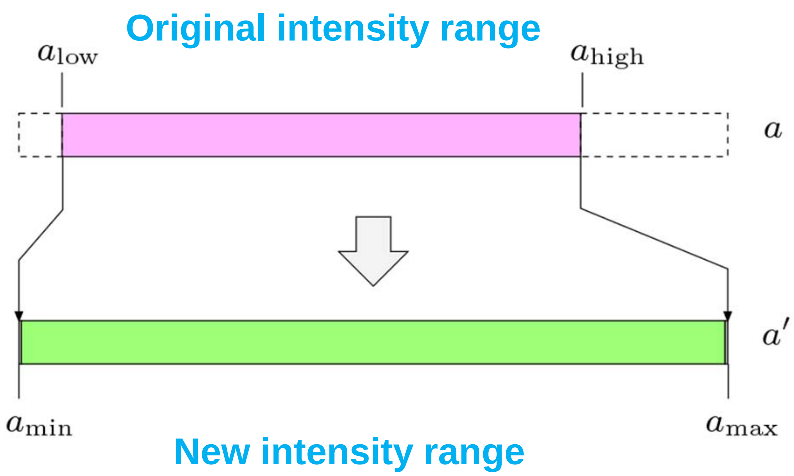
Point Operations and Histograms

- Effect of some point operations easier to observe on histograms
  - Increasing brightness
  - Raising contrast
  - Inverting image
- Point operations only shift, merge histogram entries
- Operations that merge histogram bins are irreversible



Automatic Contrast Adjustment

- Point operation that modifies pixel intensities such that available range of values is fully covered
- Algorithm:
  - Find high and lowest pixel intensities a_{low}, a_{high}
  - Linear stretching of intensity range

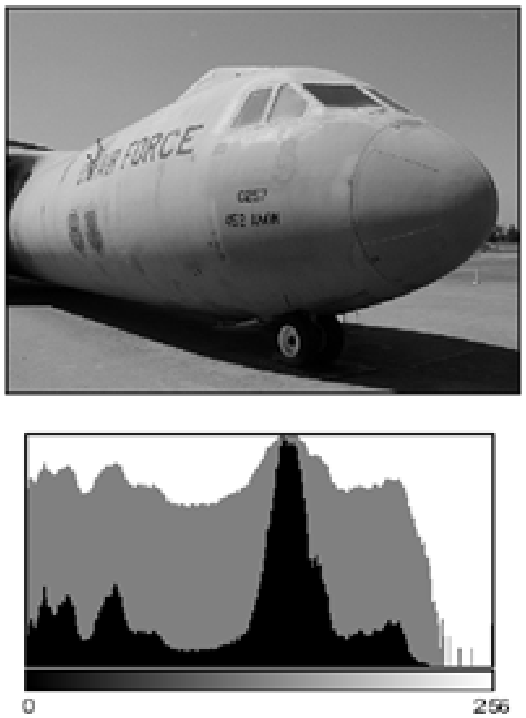


$$f_{ac}(a) = a_{min} + (a - a_{low}) \cdot \frac{a_{max} - a_{min}}{a_{high} - a_{low}}$$

If a_{min} = 0 and a_{max} = 255

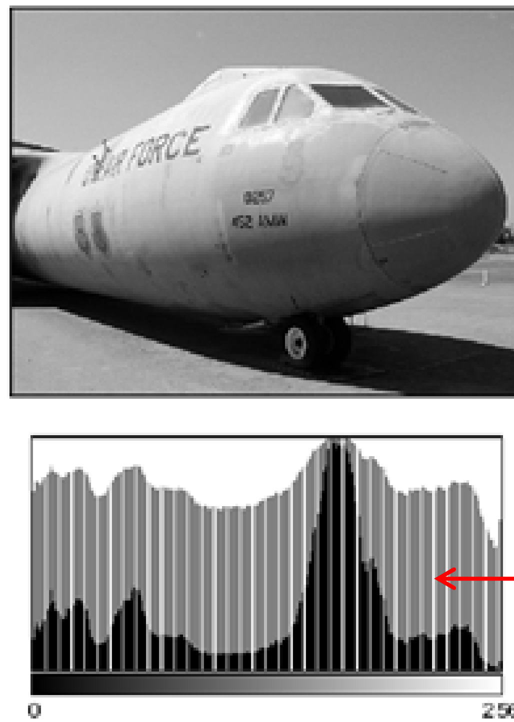
$$f_{ac}(a) = (a - a_{low}) \cdot \frac{255}{a_{high} - a_{low}}$$

Effects of Automatic Contrast Adjustment



(a)

Original



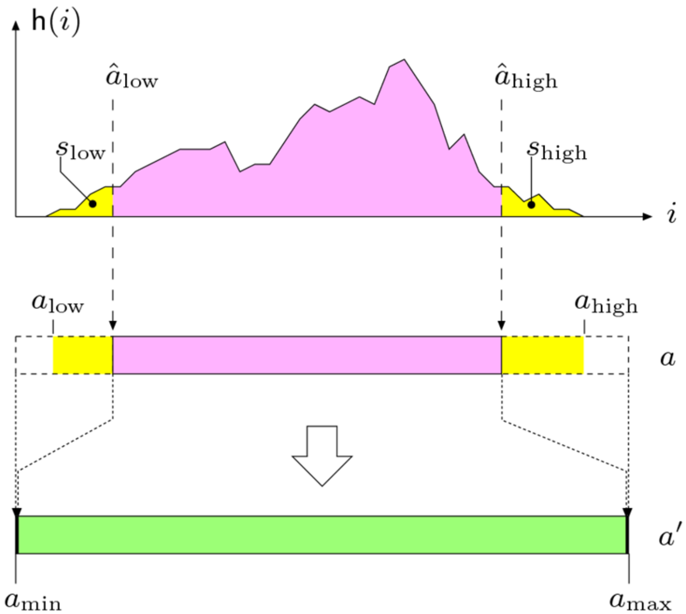
(b)

Result of automatic Contrast Adjustment

Linearly stretching range causes gaps in histogram

Modified Contrast Adjustment

- Better to map only certain range of values
- Get rid of tails (usually noise) based on predefined percentiles ($s_{\text{low}}, s_{\text{high}}$)



$$\hat{a}_{\text{low}} = \min\{\, i \mid \mathsf{H}(i) \geq M \cdot N \cdot s_{\text{low}} \}$$

$$\hat{a}_{\text{high}} = \max\{\, i \mid \mathsf{H}(i) \leq M \cdot N \cdot (1 - s_{\text{high}}) \}$$

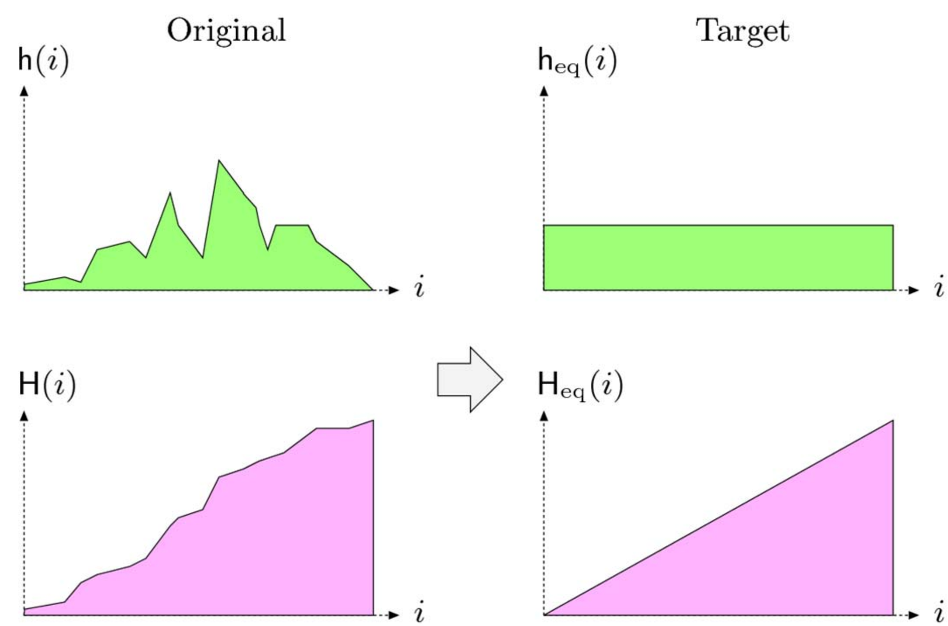
$$f_{\text{mac}}(a) = \begin{cases} a_{\text{min}} & \text{for } a \leq \hat{a}_{\text{low}} \\ a_{\text{min}} + (a - \hat{a}_{\text{low}}) \cdot \dfrac{a_{\text{max}} - a_{\text{min}}}{\hat{a}_{\text{high}} - \hat{a}_{\text{low}}} & \text{for } \hat{a}_{\text{low}} < a < \hat{a}_{\text{high}} \\ a_{\text{max}} & \text{for } a \geq \hat{a}_{\text{high}} \end{cases}$$

Histogram Equalization

- Adjust 2 different images to make their histograms (intensity distributions) similar
- Apply a point operation that changes histogram of modified image into **uniform distribution**

Histogram

Cumulative Histogram



Histogram Equalization

Spreading out the frequencies in an image (or equalizing the image) is a simple way to improve dark or washed out images

Can be expressed as a transformation of histogram

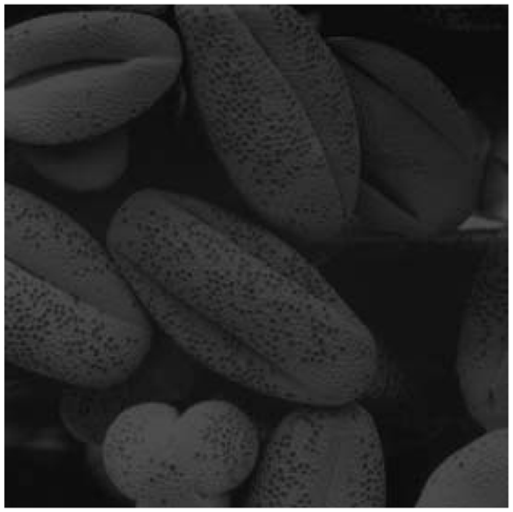
- r_k: input intensity
- s_k: processed intensity
- k: the intensity range (e.g 0.0 – 1.0)

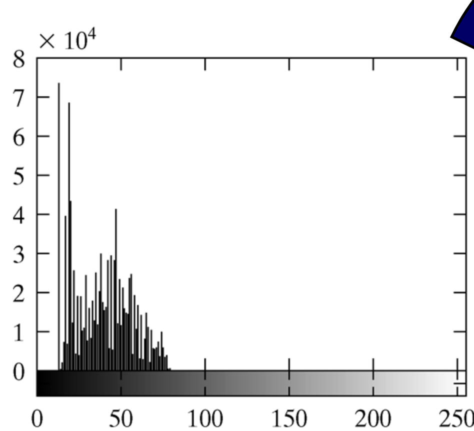
processed intensity → $$s_k = T(r_k)$$ ← input intensity

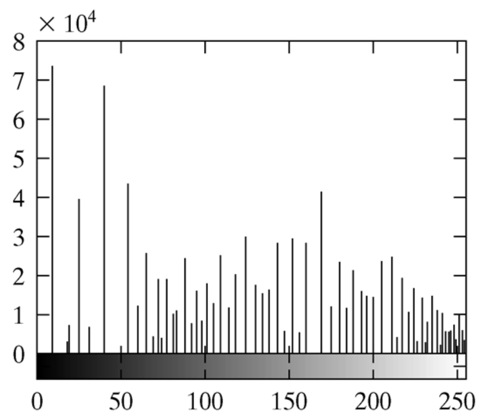
↑ Intensity range (e.g 0 – 255)

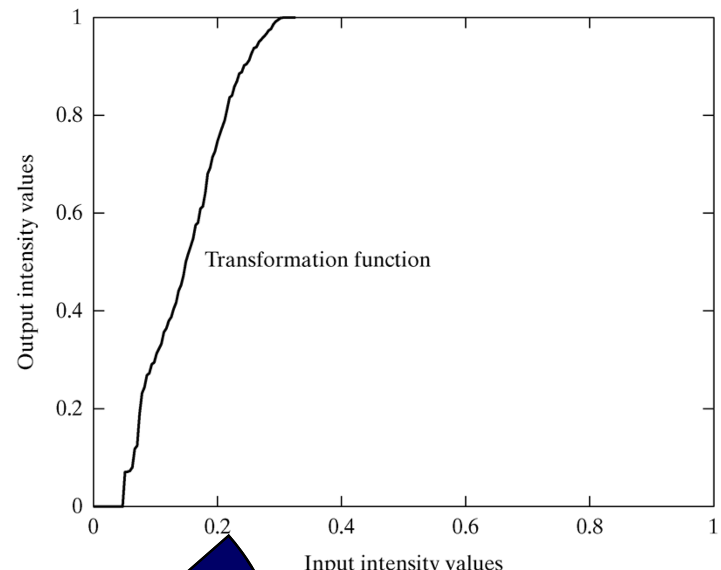
Equalization Transformation Function

Images taken from Gonzalez & Woods, Digital Image Processing (2002)





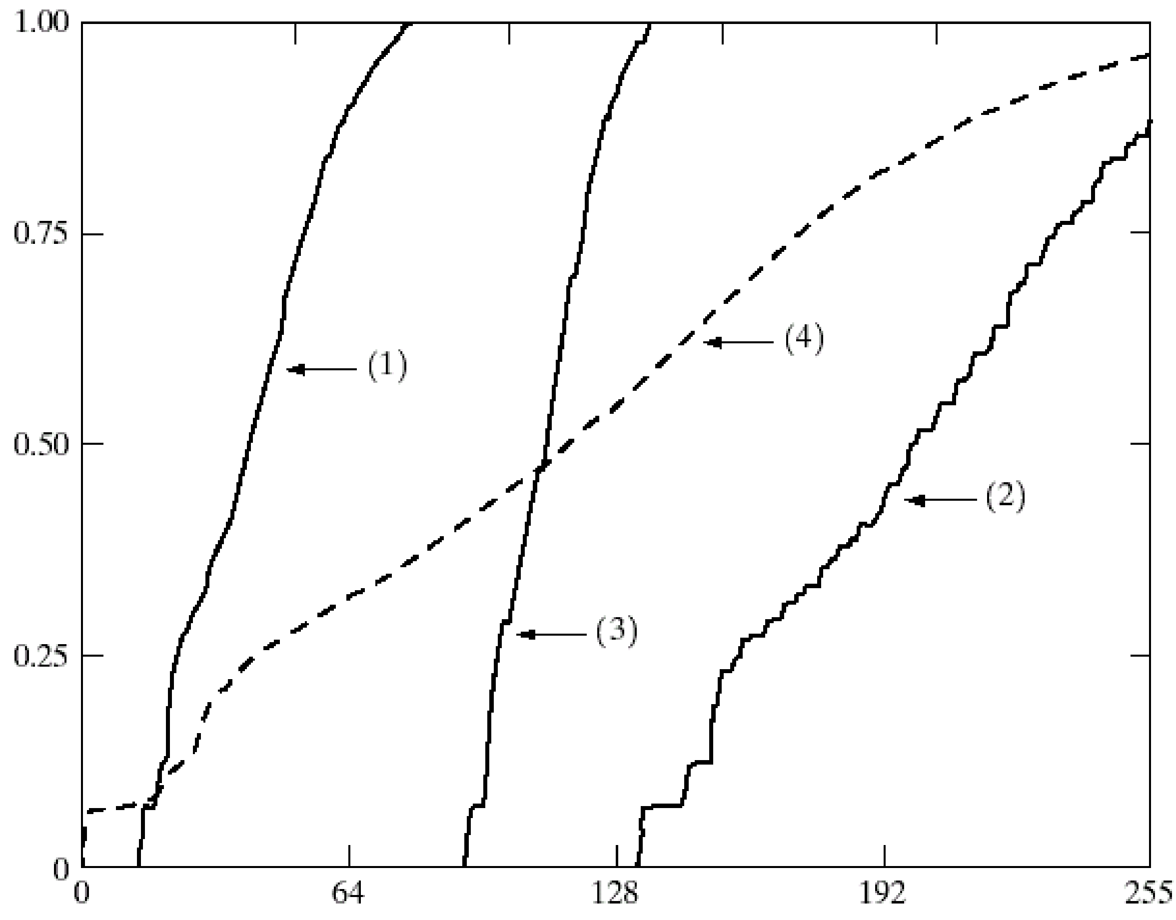




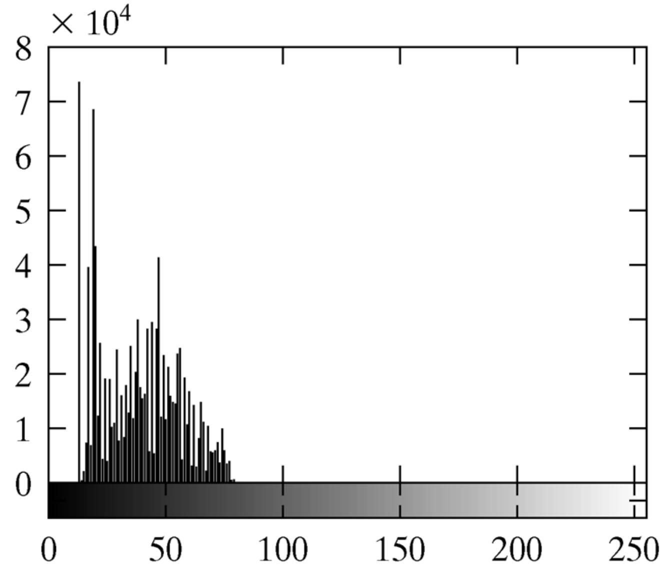
Equalization Transformation Functions

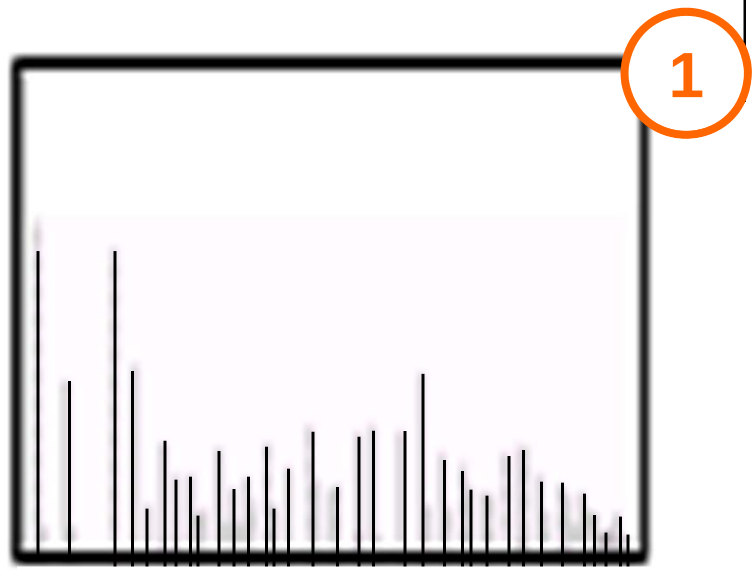
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

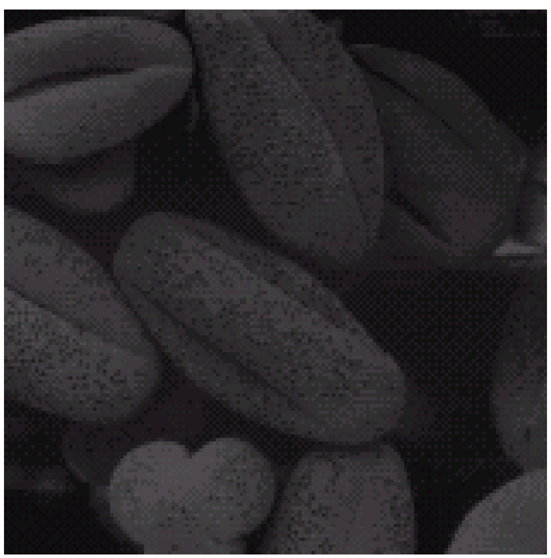
Different equalization function (1-4) may be used

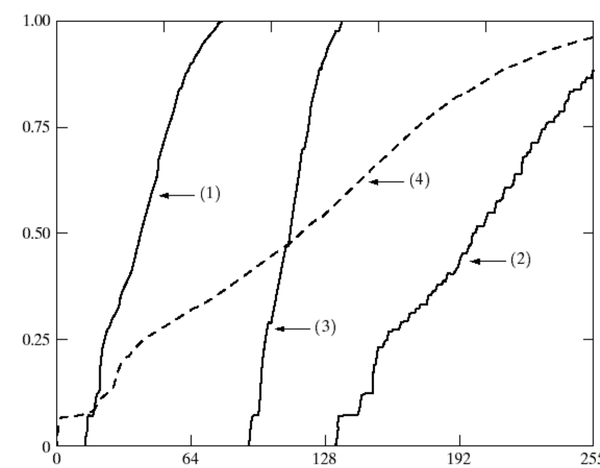


Equalization Examples



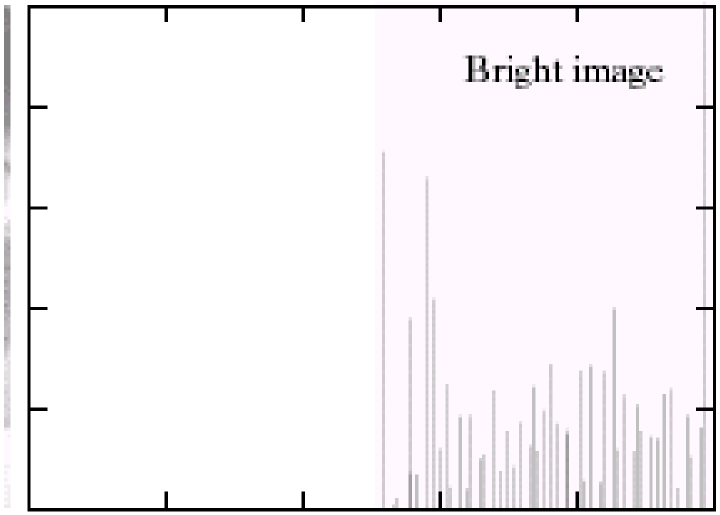


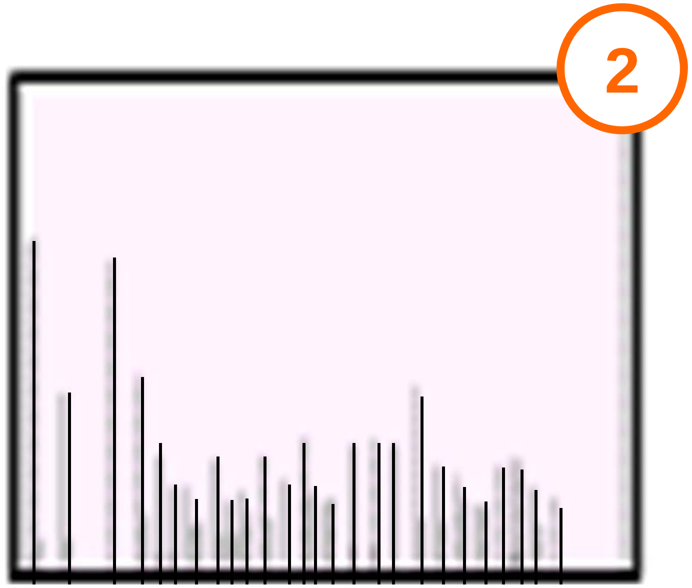


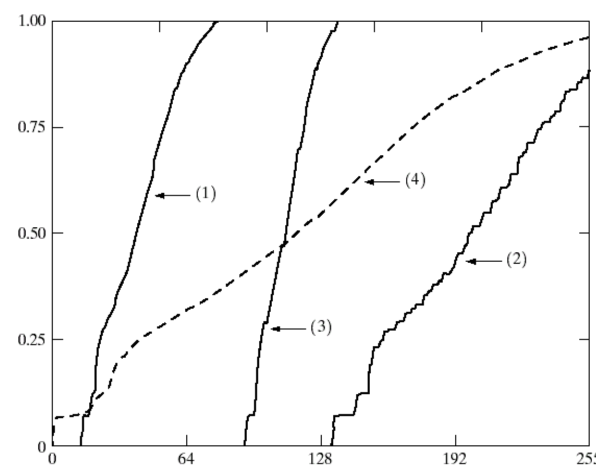


Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Equalization Examples

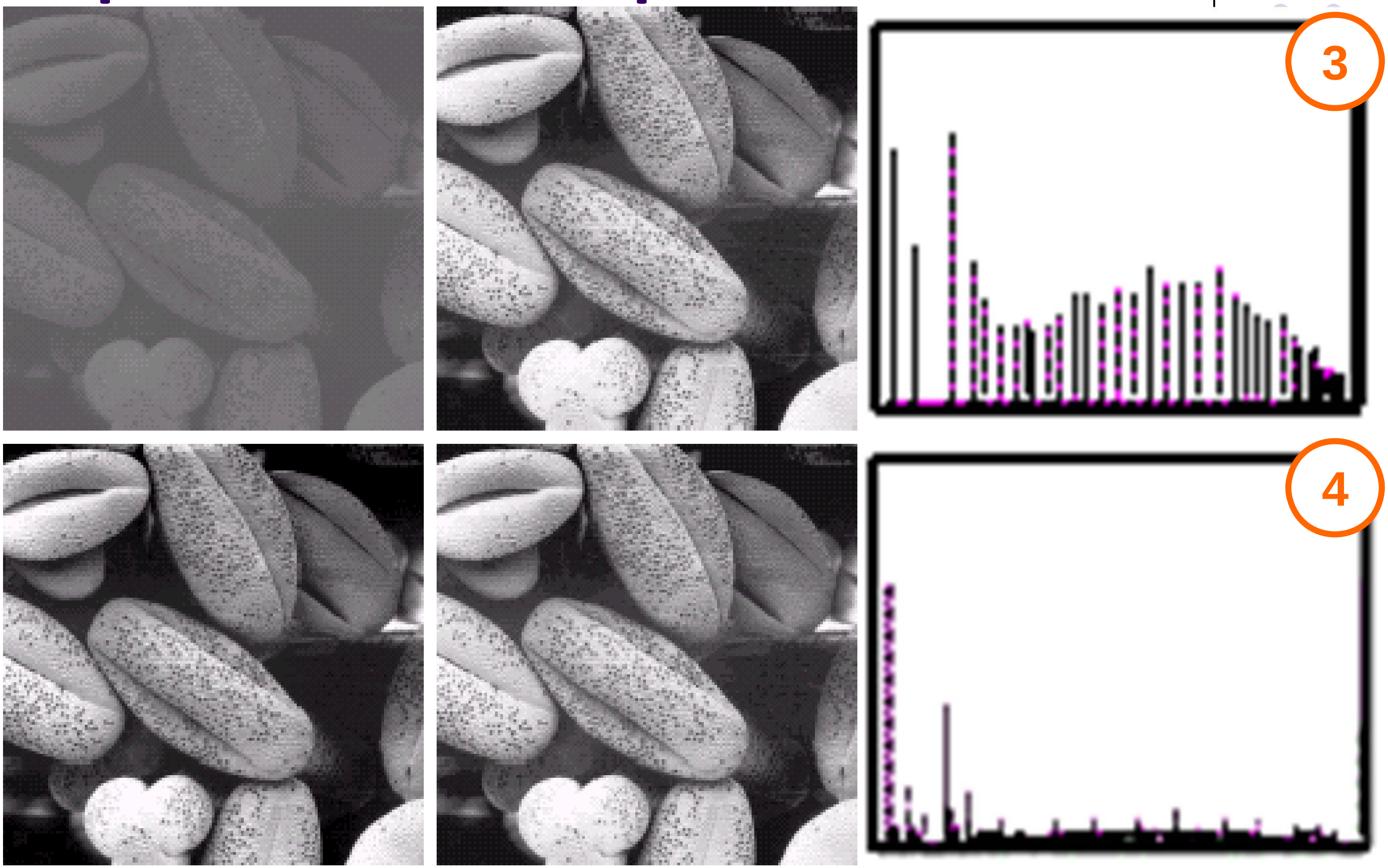






Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Equalization Examples



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

References

- Wilhelm Burger and Mark J. Burge, Digital Image Processing, Springer, 2008
  - Histograms (Ch 4)
  - Point operations (Ch 5)
- University of Utah, CS 4640: Image Processing Basics, Spring 2012
- Rutgers University, CS 334, Introduction to Imaging and Multimedia, Fall 2012
- Gonzales and Woods, Digital Image Processing (3rd edition), Prentice Hall