Computer Graphics (CS 543)  
Lecture 5b: Implementing Transformations  

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Objectives

- Learn how to implement transformations in OpenGL
  - Rotation
  - Translation
  - Scaling
- Introduce mat.h and vec.h header files for transformations
  - Model-view
  - Projection
Affine Transformations

- Translate, Scale, Rotate, Shearing, are affine transforms
- **Rigid body transformations**: rotation, translation, scaling, shear
- **Line preserving**: important in graphics since we can
  1. Transform endpoints of line segments
  2. Draw line segment between the transformed endpoints

Vertices $u$ $v$

Straight line

Affine Transform

Transformed vertices $u'$ $v'$

Straight line
Previously: Transformations in OpenGL

- Pre 3.0 OpenGL had a set of transformation functions:
  - `glTranslate`
  - `glRotate()`
  - `glScale()`

- Previously, OpenGL would:
  - Receive transform commands (`glTranslate`, `glRotate`, `glScale`)
  - Multiply transform matrices together and maintain transform matrix stack known as `modelview matrix`
Previously: **Modelview Matrix Formed?**

```c
glMatrixMode(GL_MODELVIEW)
glLoadIdentity();
glScale(1,2,3);
glTranslate(3,6,4);
```

Specify transforms in OpenGL Program (.cpp file)

OpenGL implementations (glScale, glTranslate, etc) in Hardware (Graphics card)

OpenGL multiplies transforms together to form modelview matrix

Applies final matrix to vertices of objects

---

**Identity Matrix**

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

**glScale Matrix**

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 \\
0 & 0 & 3 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

**glTranslate Matrix**

\[
\begin{pmatrix}
1 & 0 & 0 & 3 \\
0 & 1 & 0 & 6 \\
0 & 0 & 1 & 4 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

**Modelview Matrix**

\[
\begin{pmatrix}
1 & 0 & 0 & 3 \\
0 & 2 & 0 & 12 \\
0 & 0 & 3 & 12 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]
Previously: OpenGL Matrices

- OpenGL maintained 4 matrix stacks maintained as part of OpenGL state
  - Model-View (**GL_MODELVIEW**)
  - Projection (**GL_PROJECTION**)
  - Texture (**GL_TEXTURE**)
  - Color (**GL_COLOR**)

OpenGL Matrices
Now: Transformations in OpenGL

- **From OpenGL 3.0:** No transform commands (scale, rotate, etc), matrices maintained by OpenGL!!

- `glTranslate`, `glScale`, `glRotate`, OpenGL modelview matrix all deprecated!!

- If programmer needs transforms, matrices implement it!

- **Optional:** Programmer *may* now choose to maintain transform matrices or **NOT**!
Current Transformation Matrix (CTM)

- Conceptually, the user can implement a 4 x 4 homogeneous coordinate matrix, the *Current Transformation Matrix (CTM)*.
- The CTM is defined and updated in the user program.

![Diagram](https://via.placeholder.com/150)

Implement in `.h` Header file

Implement in `Main .cpp` file

- Implement transforms: Scale, rotate, etc.
- Build rotate, scale matrices, put results in CTM

Transform Matrix (CTM)

\[
p' = Cp
\]

User space

Graphics card
Homegrown CTM Matrices

- CTM = modelview + projection
  - Model-View (`GL_MODELVIEW`)  
  - Projection (`GL_PROJECTION`) 
  - Texture (`GL_TEXTURE`) 
  - Color(`GL_COLOR`)
CTM Functionality

1. We need to implement our own transforms i.e. math functions to transform points

\[
\begin{pmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1 \\
\end{pmatrix} \times \begin{pmatrix}
  1 & 0 & 0 & 0 \\
  0 & 2 & 0 & 0 \\
  0 & 0 & 3 & 0 \\
  0 & 0 & 0 & 1 \\
\end{pmatrix} \times \begin{pmatrix}
  1 & 0 & 0 & 3 \\
  0 & 1 & 0 & 6 \\
  0 & 0 & 1 & 4 \\
  0 & 0 & 0 & 1 \\
\end{pmatrix} = \begin{pmatrix}
  1 & 0 & 0 & 3 \\
  0 & 2 & 0 & 12 \\
  0 & 0 & 3 & 12 \\
  0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

Identity Matrix  Scale Matrix  Translate Matrix  CTM Matrix

2. Multiply our transforms together to form CTM matrix
3. Apply final matrix to vertices of objects
Implementing Transforms and CTM

Where to implement transforms and CTM?

We implement CTM in 3 parts

1. mat.h (Header file)
   - Implementations of translate( ), scale( ), etc

2. Application code (.cpp file)
   - Multiply together translate( ), scale( ) = final CTM matrix

3. GLSL functions (vertex and fragment shader)
   - Apply final CTM matrix to vertices
Implementing Transforms and CTM

- We just have to include mat.h (#include "mat.h"), use it
- **Uniformity:** mat.h syntax resembles GLSL language in shaders
- **Matrix Types:** mat4 (4x4 matrix), mat3 (3x3 matrix).
  ```
  class mat4 {
    vec4 _m[4];
    .......
  }
  ```
- Can declare CTM as mat4 type
  ```
  mat4 ctm = Translate(3,6,4);
  ```
- **mat.h also has transform functions:** Translate, Scale, Rotate, etc.

```
mat4 Translate(const GLfloat x, const GLfloat y, const GLfloat z )
mat4 Scale( const GLfloat x, const GLfloat y, const GLfloat z )
```
CTM operations

The CTM can be altered either by loading a new CTM or by postmultiplication

Load identity matrix: $C \leftarrow I$
Load arbitrary matrix: $C \leftarrow M$

Load a translation matrix: $C \leftarrow T$
Load a rotation matrix: $C \leftarrow R$
Load a scaling matrix: $C \leftarrow S$

Postmultiply by an arbitrary matrix: $C \leftarrow CM$
Postmultiply by a translation matrix: $C \leftarrow CT$
Postmultiply by a rotation matrix: $C \leftarrow CR$
Postmultiply by a scaling matrix: $C \leftarrow CS$
Example: Creating Identity Matrix

- All transforms (translate, scale, rotate) converted to 4x4 matrix
- We put 4x4 transform matrix into **CTM**
- Example: Create an identity matrix

```cpp
mat4 m = Identity();
```

**CTM Matrix**

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

**mat4** type stores 4x4 matrix
Defined in mat.h
Transformation matrices Formed?

```plaintext
mat4 m = Identity();
mat4 t = Translate(3,6,4);
m = m*t;
```

<table>
<thead>
<tr>
<th>Identity Matrix</th>
<th>Translation Matrix</th>
<th>CTM Matrix</th>
</tr>
</thead>
</table>
| \[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\] | \[
\begin{pmatrix}
1 & 0 & 0 & 3 \\
0 & 1 & 0 & 6 \\
0 & 0 & 1 & 4 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\] | \[
\begin{pmatrix}
1 & 0 & 0 & 3 \\
0 & 1 & 0 & 6 \\
0 & 0 & 1 & 4 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\] |
Transformation matrices Formed?

- Consider following code snippet

```cpp
mat4 m = Identity();
mat4 s = Scale(1, 2, 3);
m = m * s;
```

<table>
<thead>
<tr>
<th>Identity Matrix</th>
<th>Scaling Matrix</th>
<th>CTM Matrix</th>
</tr>
</thead>
</table>
| \[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
| \[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 \\
0 & 0 & 3 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
| \[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 \\
0 & 0 & 3 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
```
Transformation matrices Formed?

- What of translate, then scale, then ....
- Just multiply them together. Evaluated in reverse order!! E.g:

```cpp
mat4 m = Identity();
mat4 s = Scale(1,2,3);
mat4 t = Translate(3,6,4);
m = m*s*t;
```

<table>
<thead>
<tr>
<th>Identity Matrix</th>
<th>Scale Matrix</th>
<th>Translate Matrix</th>
<th>Final CTM Matrix</th>
</tr>
</thead>
</table>
| \[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\] | \[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 \\
0 & 0 & 3 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\] | \[
\begin{pmatrix}
1 & 0 & 0 & 3 \\
0 & 1 & 0 & 6 \\
0 & 0 & 1 & 4 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\] | \[
\begin{pmatrix}
1 & 0 & 0 & 3 \\
0 & 2 & 0 & 12 \\
0 & 0 & 3 & 12 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\] |
How are Transform matrices Applied?

```cpp
mat4 m = Identity();
mat4 s = Scale(1,2,3);
mat4 t = Translate(3,6,4);
m = m*s*t;

colorcube();
```

1. In application:
Load object vertices into points[ ] array -> VBO
Call glDrawArrays

2. CTM built in application, passed to vertex shader

3. In vertex shader: Each vertex of object (cube) is multiplied by CTM
to get transformed vertex position

CTM Matrix

```
| 1 0 0 3 |
| 0 2 0 12 |
| 0 0 3 12 |
| 0 0 0 1 |
```

```
| 4 |
| 14 |
| 15 |
| 1 |
```

gl_Position = model_view*vPosition;
Passing CTM to Vertex Shader

- Build CTM (modelview) matrix in application program
- Pass matrix to shader

```cpp
void display() {
    // find location of matrix variable “model_view” in shader
    // then pass matrix to shader
    mat4 m = Identity();
    mat4 s = Scale(1, 2, 3);
    mat4 t = Translate(3, 6, 4);
    m = m * s * t;

    int matrix_loc = glGetUniformLocation(program, "model_view");
    glUniformMatrix4fv(matrix_loc, 1, GL_TRUE, m);
    ....
}
```

CTM matrix `m` in application is same as `model_view` in shader.
Implementation: Vertex Shader

- On `glDrawArrays()`, vertex shader invoked with different `vPosition` per shader
- E.g. If `colorcube()` generates 8 vertices, each vertex shader receives a vertex stored in `vPosition`
- Shader calculates modified vertex position, stored in `gl_Position`

```
in vec4 vPosition;
uniform mat4 model_view;

void main( )
{
    gl_Position = model_view*vPosition;
}
```
What Really Happens to Vertex Position Attributes?

Image credit: Arcsynthesis tutorials
What About Multiple Vertex Attributes?

Positions

Colors

Image credit: Arcsynthesis tutorials
Transformation matrices Formed?

- Example: Vertex (1, 1, 1) is one of 8 vertices of cube

In application

```cpp
mat4 m = Identity();
mat4 s = Scale(1,2,3);
m = m*s;
colorcube();
```

In vertex shader

```
CTM (m)          p         p'
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 \\
0 & 0 & 3 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
1 \\
1 \\
1 \\
1 \\
\end{pmatrix}
= \begin{pmatrix}
1 \\
2 \\
3 \\
1 \\
\end{pmatrix}
```

Each vertex of cube is multiplied by modelview matrix to get scaled vertex position.
Transformation matrices Formed?

- Another example: Vertex (1, 1, 1) is one of 8 vertices of cube

In application

```cpp
mat4 m = Identity();
mat4 s = Scale(1,2,3);
mat4 t = Translate(3,6,4);
m = m*s*t;
colorcube();
```

In vertex shader

```
\[
\begin{pmatrix}
1 & 0 & 0 & 3 \\
0 & 2 & 0 & 12 \\
0 & 0 & 3 & 12 \\
0 & 0 & 0 & 1
\end{pmatrix}
\times
\begin{pmatrix}
1 \\
1 \\
1
\end{pmatrix}
= \begin{pmatrix}
4 \\
14 \\
15
\end{pmatrix}
\]
```

CTM Matrix

Original vertex

Transformed vertex

Each vertex of cube is multiplied by modelview matrix to get scaled vertex position
Arbitrary Matrices

- Can multiply by matrices from transformation commands (Translate, Rotate, Scale) into CTM
- Can also load arbitrary 4x4 matrices into CTM

Load into CTM Matrix:

\[
\begin{pmatrix}
1 & 0 & 15 & 3 \\
0 & 2 & 0 & 12 \\
34 & 0 & 3 & 12 \\
0 & 24 & 0 & 1 \\
\end{pmatrix}
\]
Example: Rotation about a Fixed Point

- We want $C = TRT^{-1}$
- Be careful with order. Do operations in following order

\[
\begin{align*}
C & \leftarrow I \\
C & \leftarrow CT \\
C & \leftarrow CR \\
C & \leftarrow CT^{-1}
\end{align*}
\]

- Each operation corresponds to one function call in the program.
- **Note:** last operation specified is first executed
Matrix Stacks

- CTM is actually not just 1 matrix but a matrix STACK
  - Multiple matrices in stack, “current” matrix at top
  - Can save transformation matrices for use later (push, pop)
- E.g: Traversing hierarchical data structures (Ch. 8)
- Pre 3.1 OpenGL also maintained matrix stacks
- Right now just implement 1-level CTM
- Matrix stack later for hierarchical transforms
Reading Back State

- Can also access OpenGL variables (and other parts of the state) by *query* functions
  
  ```c
  glGetIntegerv
  glGetFloatv
  glGetDoublev
  glGetBooleanv
  glEnable
  ```

- Example: to find out max. number texture units on GPU
  
  ```c
  glGetIntegerv(GL_MAX_TEXTURE_UNITS, &MaxTextureUnits);
  ```
Using Transformations

- **Example:** use idle function to rotate a cube and mouse function to change direction of rotation
- Start with program that draws cube as before
  - Centered at origin
  - Sides aligned with axes
Recall: `main.c`

```c
void main(int argc, char **argv)
{
    glutInit(&argc, argv);
    glutInitDisplayMode(GLUT_DOUBLE | GLUT_RGB | GLUT_DEPTH);
    glutInitWindowSize(500, 500);
    glutCreateWindow("colorcube");
    glutReshapeFunc(myReshape);
    glutDisplayFunc(display);
    glutIdleFunc(spinCube);
    glutMouseFunc(mouse);
    glEnable(GL_DEPTH_TEST);
    glutMainLoop();
}
```

Calls `spinCube` continuously whenever OpenGL program is idle.
Recall: Idle and Mouse callbacks

```c
void spinCube()
{
    theta[axis] += 2.0;
    if( theta[axis] > 360.0 ) theta[axis] -= 360.0;
    glutPostRedisplay();
}

void mouse(int button, int state, int x, int y)
{
    if(button==GLUT_LEFT_BUTTON && state == GLUT_DOWN)
        axis = 0;
    if(button==GLUT_MIDDLE_BUTTON && state == GLUT_DOWN)
        axis = 1;
    if(button==GLUT_RIGHT_BUTTON && state == GLUT_DOWN)
        axis = 2;
}
```
void display()
{
    glClear(GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT);
    ctm = RotateX(theta[0])*RotateY(theta[1])*RotateZ(theta[2]);
    glUniformMatrix4fv(matrix_loc,1,GL_TRUE,ctm);
    glDrawArrays(GL_TRIANGLES, 0, N);
    glutSwapBuffers();
}

- Alternatively, we can
  - send rotation angle + axis to vertex shader,
  - Let shader form CTM then do rotation
- Inefficient: if mesh has 10,000 vertices each one forms CTM, redundant!!!!
Using the Model-view Matrix

- In OpenGL the model-view matrix used to
  - Transform 3D models (translate, scale, rotate)
  - Position camera (using LookAt function) (next)
- The projection matrix used to define view volume and select a camera lens (later)
- Although these matrices no longer part of OpenGL, good to create them in our applications (as CTM)
References