Computer Graphics (CS 543)
Lecture 4a: Linear Algebra for Graphics (Points, Scalars, Vectors)

Prof Emmanuel Agu

Computer Science Dept.

Worcester Polytechnic Institute (WPI)

Announcements

- Sample exam 1 will be posted on class website
- Exam 1 next week in class, Sept 26
  - Exam review at the end of today's class
- **Date change:** Project 2 out next week, due Oct 10

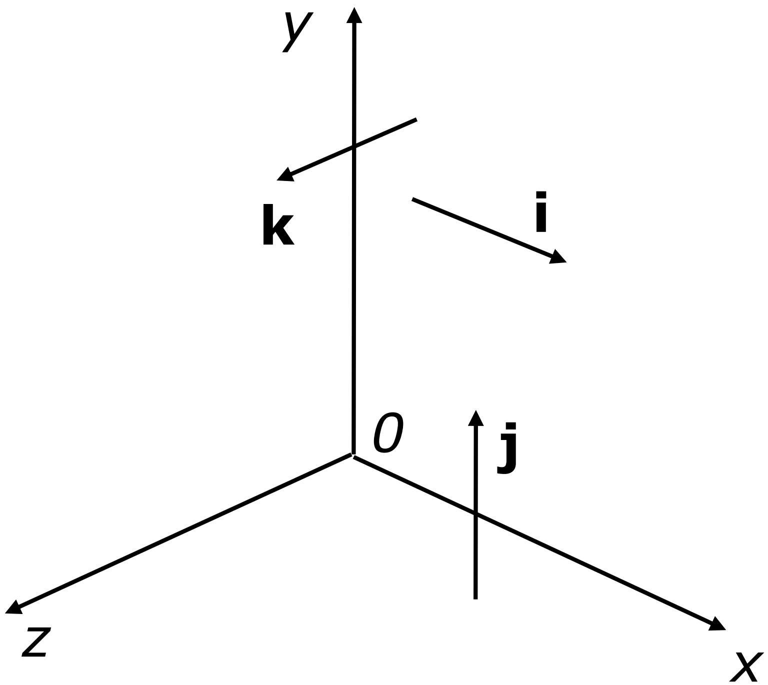
Standard Unit Vectors

Define

$$\mathbf{i} = (1,0,0)$$

$$\mathbf{j} = (0,1,0)$$

$$\mathbf{k} = (0,0,1)$$



So that any vector,

$$\mathbf{v} = (a, b, c) = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$$

Cross Product (Vector product)

If

$$\mathbf{a} = \left(a_x, a_y, a_z\right) \qquad \mathbf{b} = \left(b_x, b_y, b_z\right)$$

Then

$$\mathbf{a}\times\mathbf{b} = (a_y b_z - a_z b_y)\mathbf{i} - (a_x b_z - a_z b_x)\mathbf{j} + (a_x b_y - a_y b_x)\mathbf{k}$$

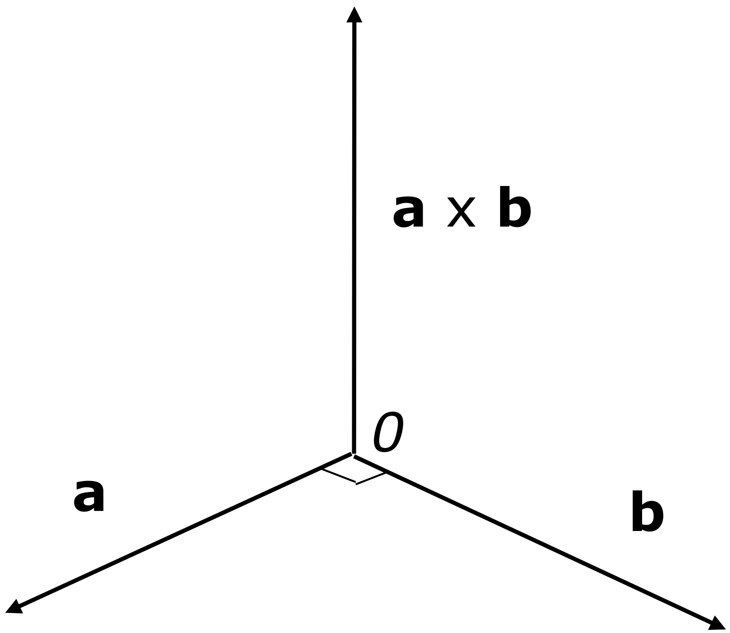
Remember using determinant

$$\begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Note: a x **b** is perpendicular to **a** and **b**

Cross Product

Note: a x **b** is perpendicular to both **a** and **b**



Cross Product (Vector product)

Calculate **a x b** if a = (3,0,2) and **b** = (4,1,8)

$$\mathbf{a} = (3,0,2) \qquad \mathbf{b} = (4,1,8)$$

Using determinant

$$\begin{vmatrix} i & j & k \\ 3 & 0 & 2 \\ 4 & 1 & 8 \end{vmatrix}$$

Then

$$\mathbf{a} \times \mathbf{b} = (0-2)\mathbf{i} - (24-8)\mathbf{j} + (3-0)\mathbf{k}$$

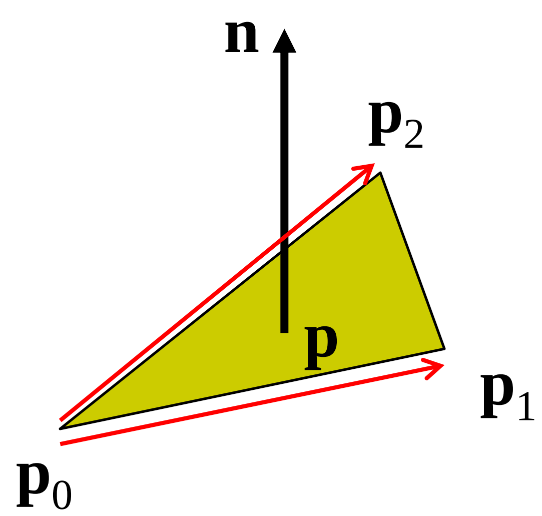
$$= -2\mathbf{i} - 16\mathbf{j} + 3\mathbf{k}$$

Normal for Triangle using Cross Product Method

plane $\mathbf{n} \cdot (\mathbf{p} - \mathbf{p}_0) = 0$

$\mathbf{n} = (\mathbf{p}_2 - \mathbf{p}_0) \times (\mathbf{p}_1 - \mathbf{p}_0)$

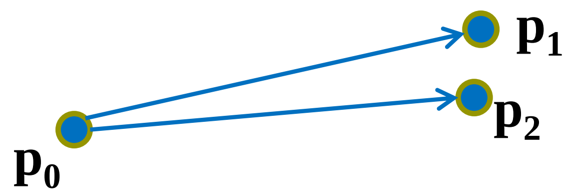
normalize $\mathbf{n} \leftarrow \mathbf{n}/|\mathbf{n}|$



Note that right-hand rule determines outward face

Newell Method for Normal Vectors

- Problems with cross product method:
  - calculation difficult by hand, tedious
  - If 2 vectors almost parallel, cross product is small
  - Numerical inaccuracy may result



- Proposed by Martin Newell at Utah (teapot guy)
  - Uses formulae, suitable for computer
  - Compute during mesh generation
  - Robust!

Newell Method Example

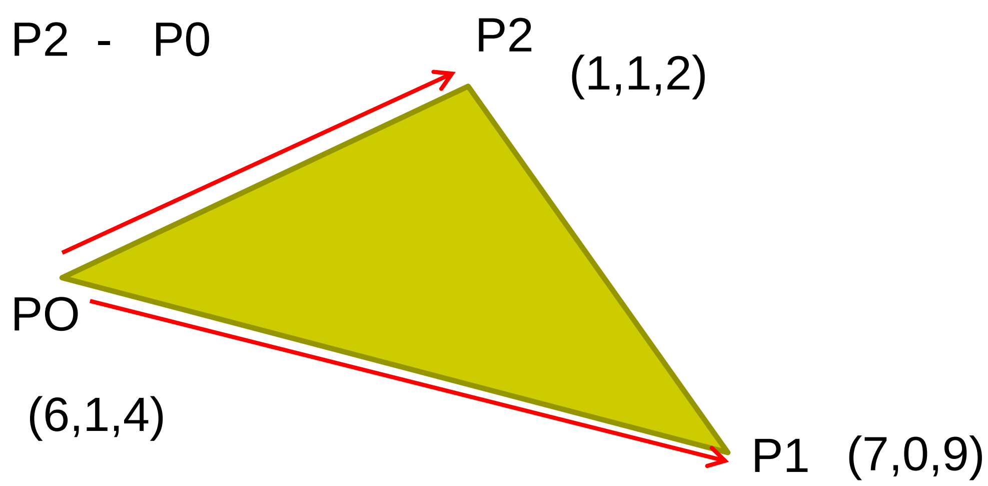
- Example: Find normal of polygon with vertices

  P0 = (6,1,4), P1=(7,0,9) and P2 = (1,1,2)

- Using simple cross product:

  ((7,0,9)-(6,1,4)) X ((1,1,2)-(6,1,4)) = (2,-23,-5)

P1 - P0 P2 - P0



Newell Method for Normal Vectors

- Formulae: Normal N = (mx, my, mz)

$$m_x = \sum_{i=0}^{N-1} \left(y_i - y_{next(i)}\right)\left(z_i + z_{next(i)}\right)$$

$$m_y = \sum_{i=0}^{N-1} \left(z_i - z_{next(i)}\right)\left(x_i + x_{next(i)}\right)$$

$$m_z = \sum_{i=0}^{N-1} \left(x_i - x_{next(i)}\right)\left(y_i + y_{next(i)}\right)$$

Newell Method for Normal Vectors

- Calculate x component of normal

$$m_x = \sum_{i=0}^{N-1} \left(y_i - y_{next(i)}\right)\left(z_i + z_{next(i)}\right)$$

$$m_x = (1)(13) + (-1)(11) + (0)(6)$$

$$m_x = 13 - 11 + 0$$

$$m_x = 2$$

	x	y	z
P0	6	1	4
P1	7	0	9
P2	1	1	2
P0	6	1	4

Newell Method for Normal Vectors

- Calculate y component of normal

$$m_y = \sum_{i=0}^{N-1} \left(z_i - z_{next(i)}\right)\left(x_i + x_{next(i)}\right)$$

$$m_y = (-5)(13) + (7)(8) + (-2)(7)$$

$$m_y = -65 + 56 - 14$$

$$m_y = -23$$

$$\begin{array}{c|ccc|} & x & y & z \\ \mathbf{P0} & 6 & 1 & 4 \\ \mathbf{P1} & 7 & 0 & 9 \\ \mathbf{P2} & 1 & 1 & 2 \\ \mathbf{P0} & 6 & 1 & 4 \end{array}$$

Newell Method for Normal Vectors

- Calculate z component of normal

$$m_z = \sum_{i=0}^{N-1} \left(x_i - x_{next(i)}\right)\left(y_i + y_{next(i)}\right)$$

	x	y	z
P0	6	1	4
P1	7	0	9
P2	1	1	2
P0	6	1	4

$$m_z = (-1)(1) + (6)(1) + (-5)(2)$$

$$m_z = -1 + 6 - 10$$

$$m_z = -5$$

Note: Using Newell method yields same result as Cross product method (2,-23,-5)

Finding Vector Reflected From a Surface

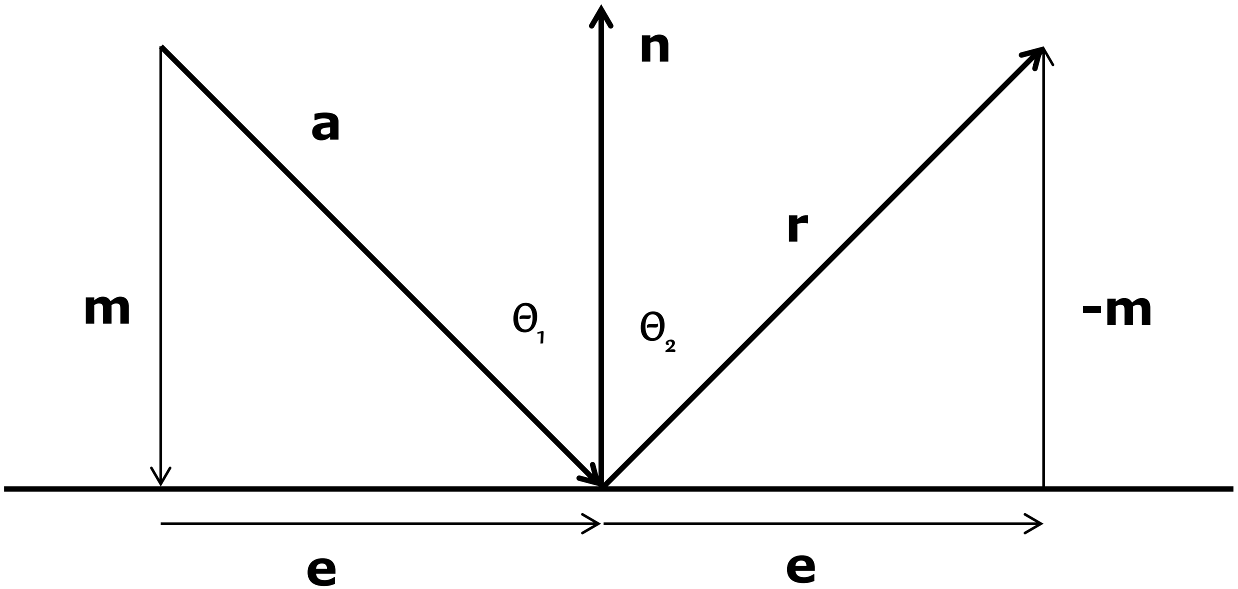
- **a** = original vector
- **n** = normal vector
- **r** = reflected vector
- **m** = projection of **a** along **n**
- **e** = projection of **a** orthogonal to **n**

Note: $\Theta_1 = \Theta_2$

$$\mathbf{e} = \mathbf{a} - \mathbf{m}$$

$$\mathbf{r} = \mathbf{e} - \mathbf{m}$$

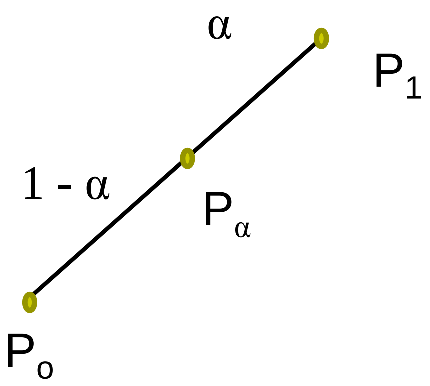
$$=> \mathbf{r} = \mathbf{a} - 2\mathbf{m}$$



Forms of Equation of a Line

- Two-dimensional forms of a line
  - **Explicit:** $y = mx + h$
  - **Implicit:** $ax + by + c = 0$
  - **Parametric:**

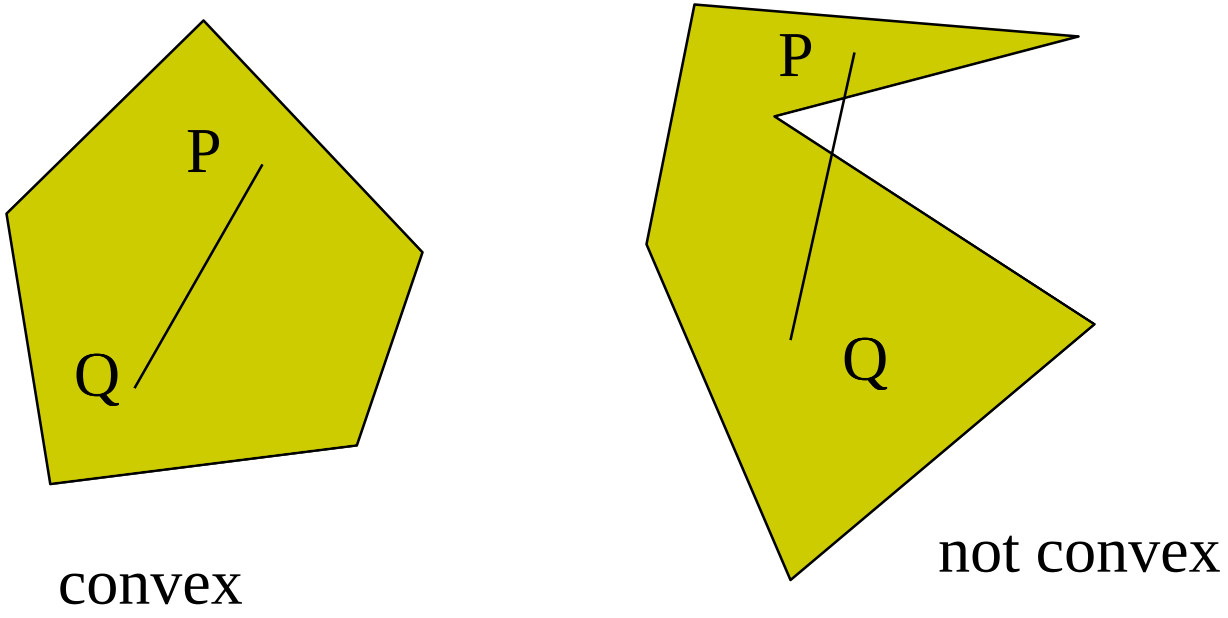
$$x(\alpha) = \alpha x_0 + (1-\alpha)x_1$$
$$y(\alpha) = \alpha y_0 + (1-\alpha)y_1$$



- Parametric form of line
  - More robust and general than other forms
  - Extends to curves and surfaces

Convexity

- An object is *convex* iff for any two points in the object all points on the straight line between these points are also in the object



References

- Angel and Shreiner, Interactive Computer Graphics, 6th edition, Chapter 3
- Hill and Kelley, Computer Graphics using OpenGL, 3rd edition, Sections 4.2 - 4.4

Computer Graphics (CS 543)
Lecture 4a: Building 3D Models

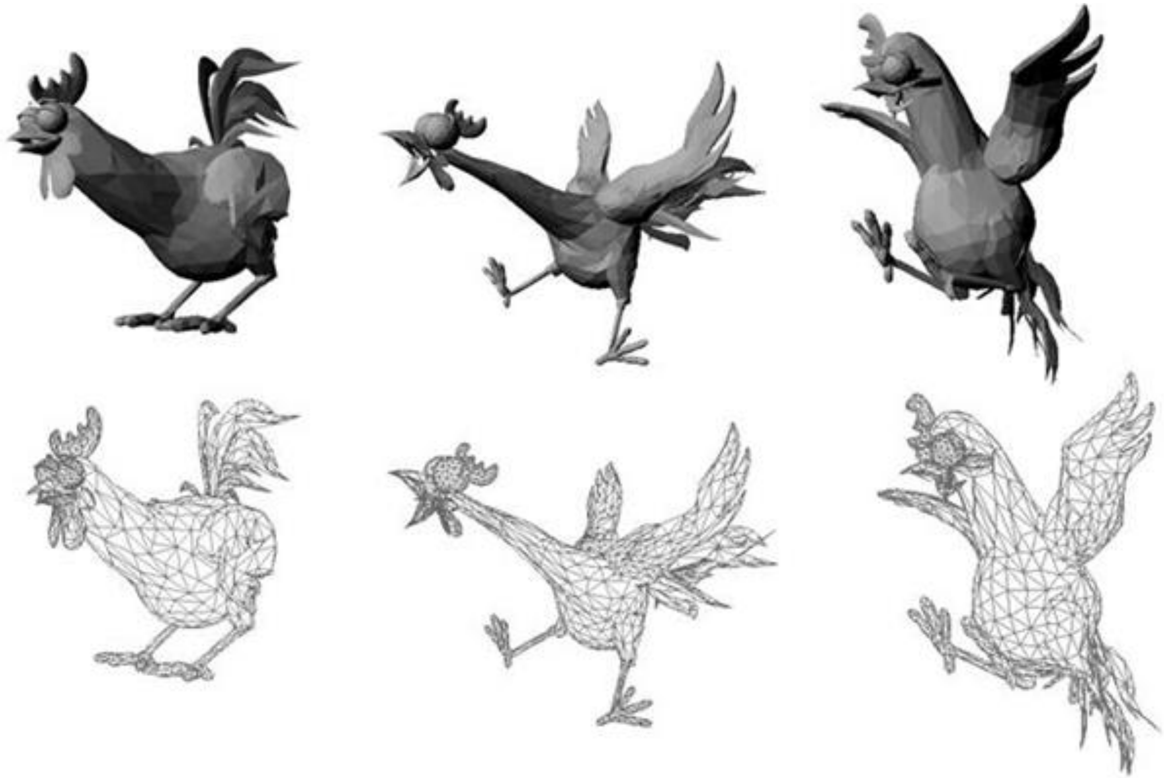
Prof Emmanuel Agu

Computer Science Dept.

Worcester Polytechnic Institute (WPI)

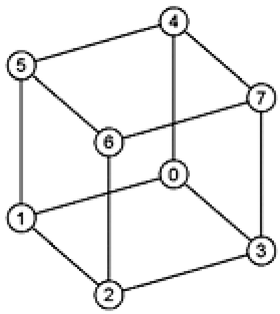
3D Applications

- **2D points:** (x,y) coordinates
- **3D points:** have (x,y,z) coordinates



Setting up 3D Applications: Main Steps

- Programming 3D similar to 2D
  1. Load representation of 3D object into data structure

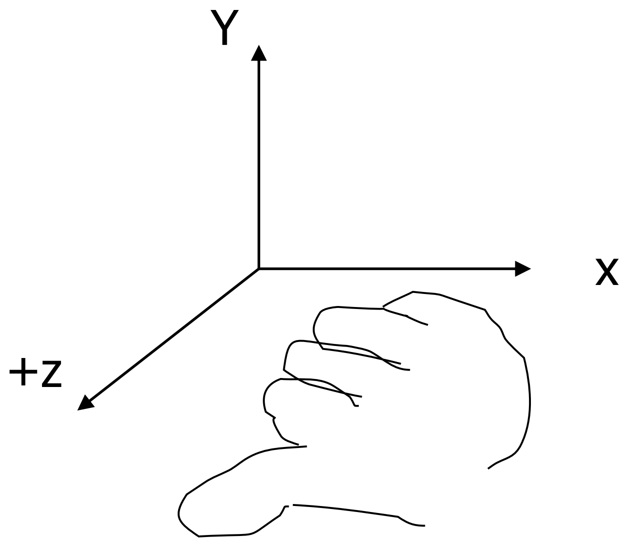


Each vertex has (x,y,z) coordinates.
Store as **vec3 NOT vec2**

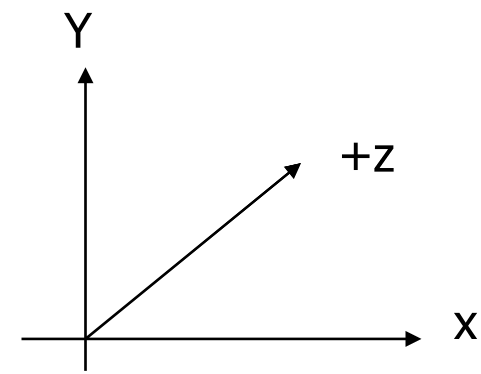
  2. Draw 3D object
  3. **Set up Hidden surface removal:** Correctly determine order in which primitives (triangles, faces) are rendered (e.g Blocked faces **NOT** drawn)

3D Coordinate Systems

- Vertex (x,y,z) positions specified on coordinate system
- OpenGL uses **right hand coordinate system**



Right hand coordinate system
Tip: sweep fingers x-y: thumb is z



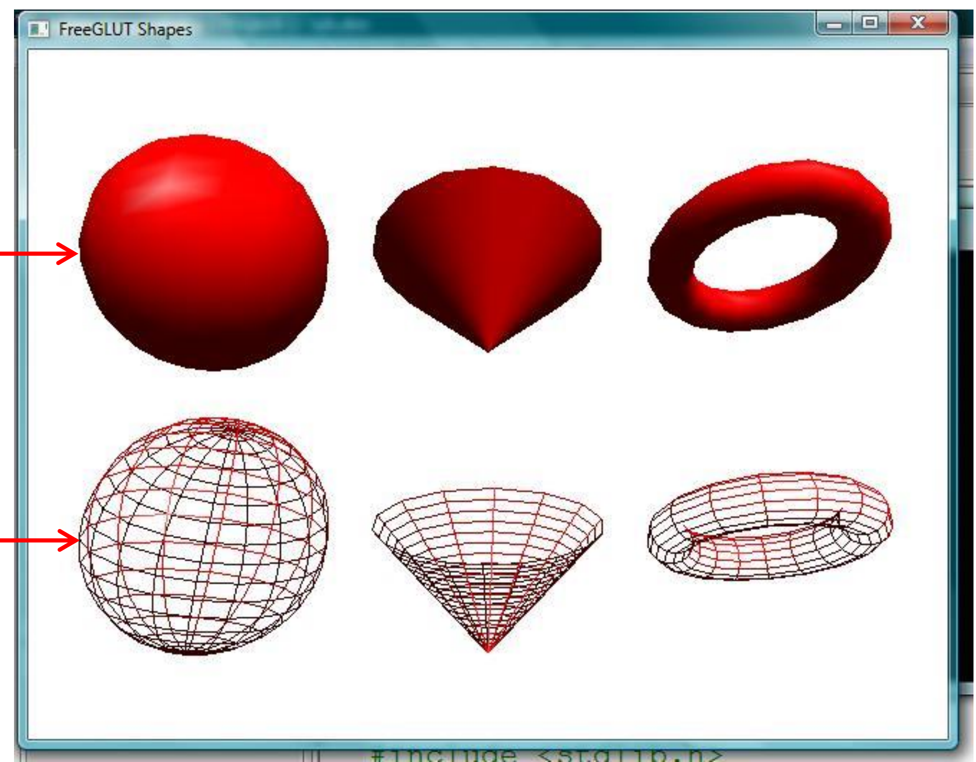
Left hand coordinate system
•Not used in OpenGL

Generating 3D Models: GLUT Models

- Make GLUT 3D calls in **OpenGL program** to generate vertices describing different shapes (Restrictive?)
- Two types of GLUT models:
  - Wireframe Models
  - Solid Models

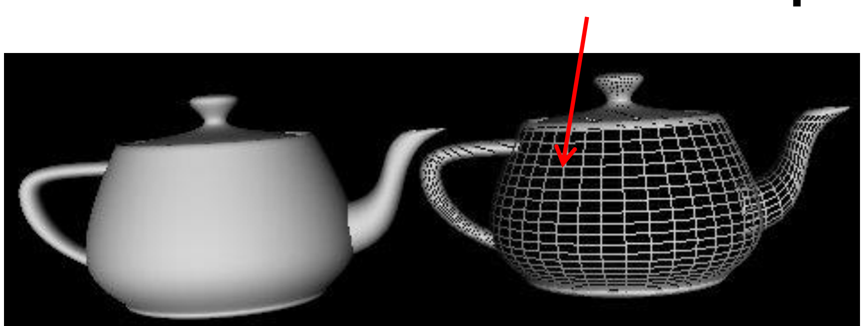
Solid models

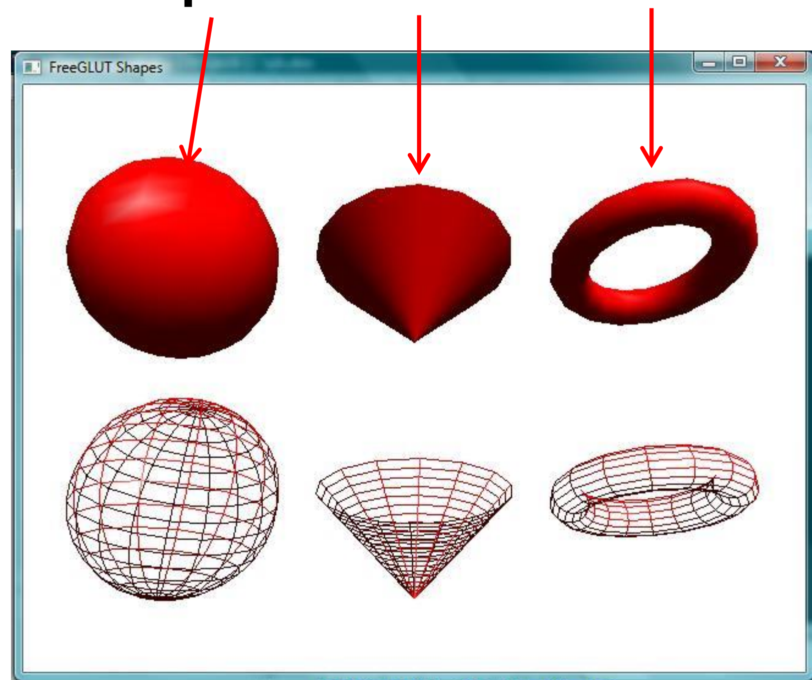
Wireframe models



3D Modeling: GLUT Models

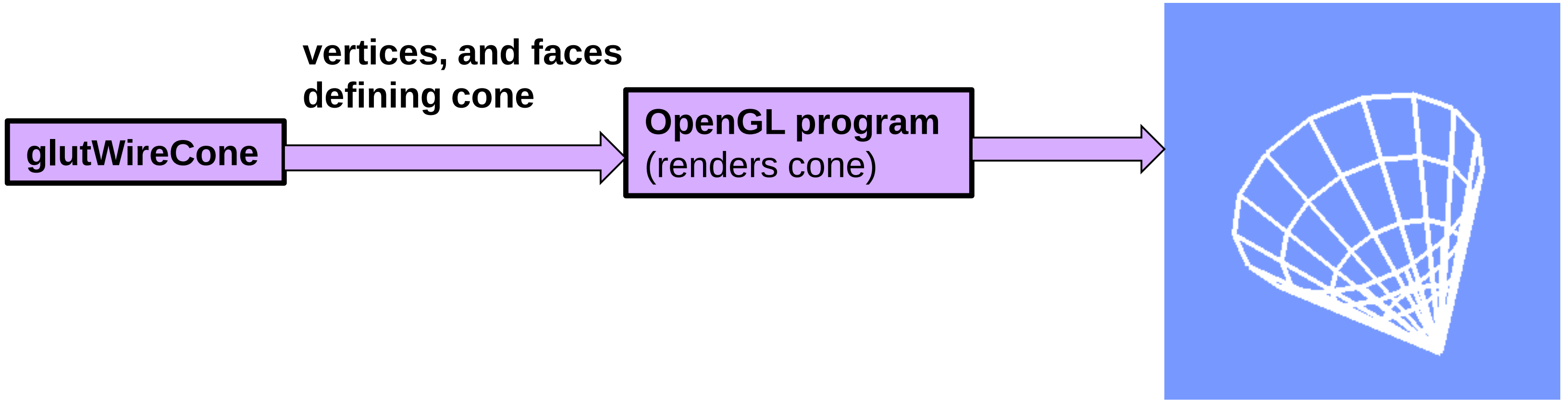
- Basic Shapes
  - **Cone:** glutWireCone( ), glutSolidCone( )
  - **Sphere:** glutWireSphere( ), glutSolidSphere( )
  - **Cube:** glutWireCube( ), glutSolidCube( )
- More advanced shapes:
  - Newell Teapot: (symbolic)
  - Dodecahedron, Torus





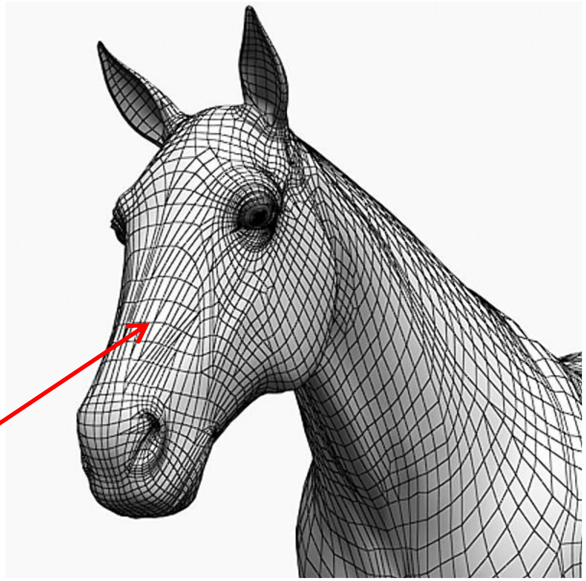
3D Modeling: GLUT Models

- Glut functions under the hood
  - generate sequence of points that define a shape
  - Generated vertices and faces passed to OpenGL for rendering
- **Example:** glutWireCone generates sequence of vertices, and faces defining cone and connectivity



Polygonal Meshes

- Modeling with GLUT shapes (cube, sphere, etc) too restrictive
- Difficult to approach realism. E.g. model a horse
- Preferred way is using polygonal meshes:
  - Collection of polygons, or faces, that form “skin” of object
  - More flexible, represents complex surfaces better
  - Examples:
    - Human face
    - Animal structures
    - Furniture, etc

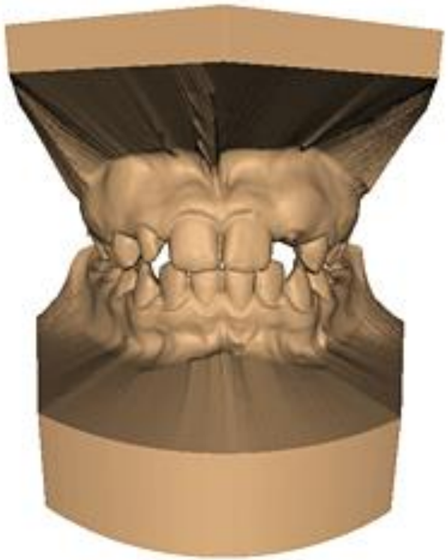


Each face of mesh is a polygon

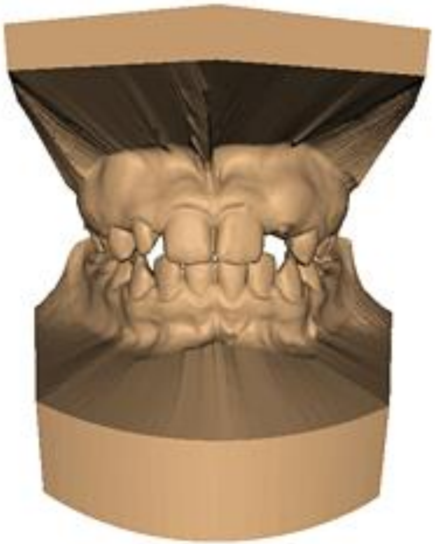
Polygonal Meshes

- Mesh = sequence of polygons forming thin skin around object
- OpenGL good at drawing polygons, triangles
- Meshes now standard in graphics
- Simple meshes exact. (e.g barn)
- Complex meshes approximate (e.g. human face)

Same Mesh at Different Resolutions



Original: 424,000 triangles



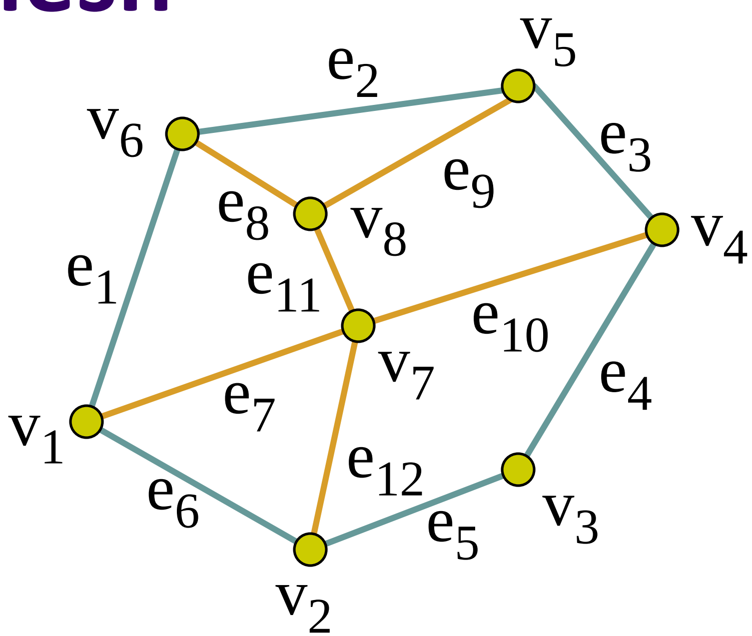
60,000 triangles (14%).

1000 triangles (0.2%)

(courtesy of Michael Garland and Data courtesy of Iris Development.)

Representing a Mesh

- Consider a mesh



- There are 8 vertices and 12 edges
  - 5 interior polygons
  - 6 interior (shared) edges (shown in orange)
- Each vertex has a location $v_i = (x_i\ y_i\ z_i)$

Simple Representation

- Define each polygon by (x,y,z) locations of its vertices
- OpenGL code

```
vertex[i]   = vec3(x1, y1, z1);
vertex[i+1] = vec3(x6, y6, z6);
vertex[i+2] = vec3(x7, y7, z7);
i+=3;
```

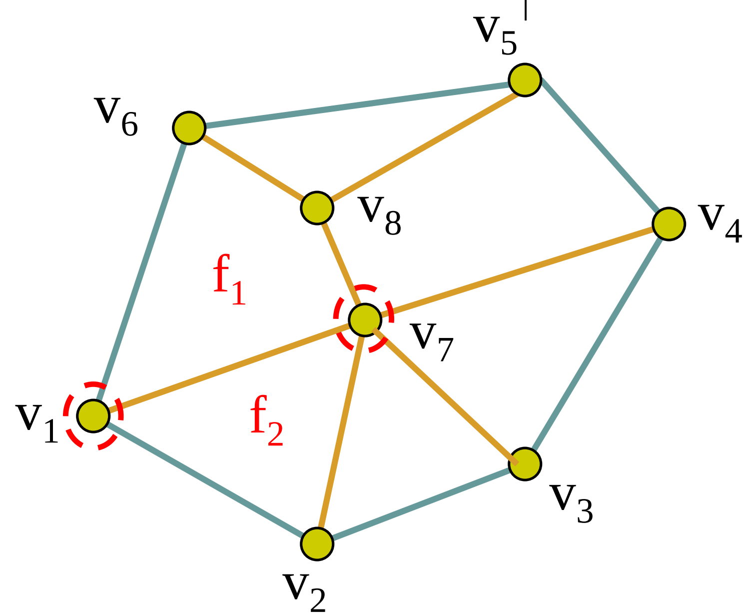
Issues with Simple Representation

- Declaring face f1

```
vertex[i]   = vec3(x1, y1, z1);
vertex[i+1] = vec3(x7, y7, z7);
vertex[i+2] = vec3(x8, y8, z8);
vertex[i+3] = vec3(x6, y6, z6);
```

- Declaring face f2

```
vertex[i]   = vec3(x1, y1, z1);
vertex[i+1] = vec3(x2, y2, z2);
vertex[i+2] = vec3(x7, y7, z7);
```



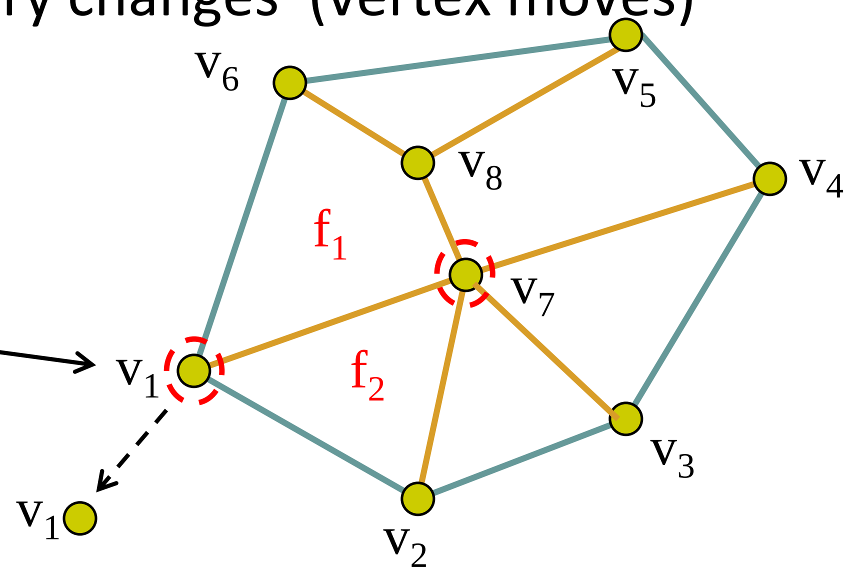
- Inefficient and unstructured
  - **Repeats:** vertices **v1 and v7 repeated** while declaring f1 and f2
  - Shared vertices shared declared multiple times
  - Delete vertex? Move vertex? Search for all occurences of vertex

Geometry vs Topology

- **Geometry:** (x,y,z) locations of the vertices
- **Topology:** How vertices and edges are connected

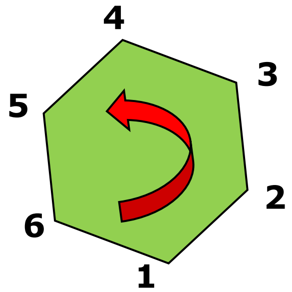
- Good data structures separate **geometry** from **topology**
  - **Example:**
    - A polygon is **ordered list** of vertices
    - An edge connects successive pairs of vertices
  - Topology holds even if geometry changes (vertex moves)

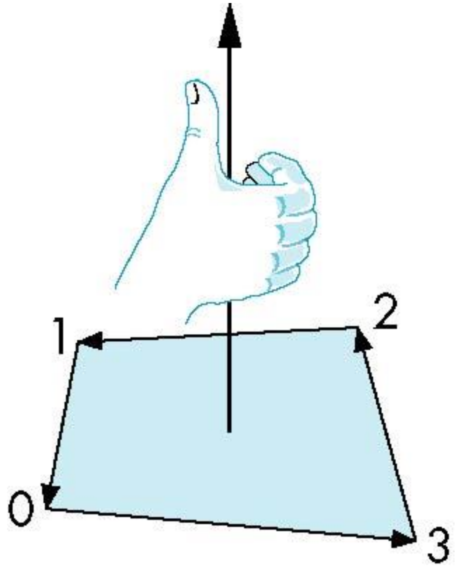
Example: even if we move (x,y,z) location of v1, v1 still connected to v6, v7 and v2

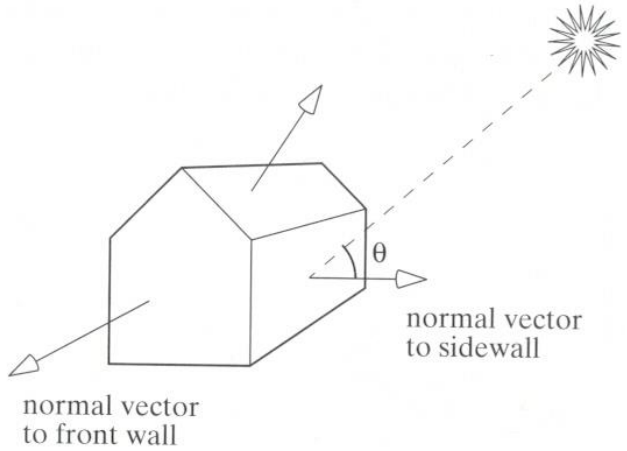


Polygon Traversal Convention

- **Convention:** traverse vertices counter-clockwise around normal
- Focus on direction of traversal
  - Orders $\{v_1, v_0, v_3\}$ and $\{v_3, v_2, v_1\}$ are same *(ccw)*
  - Order $\{v_1, v_2, v_3\}$ is different *(clockwise)*
- **Normal vector:** Direction each polygon is facing



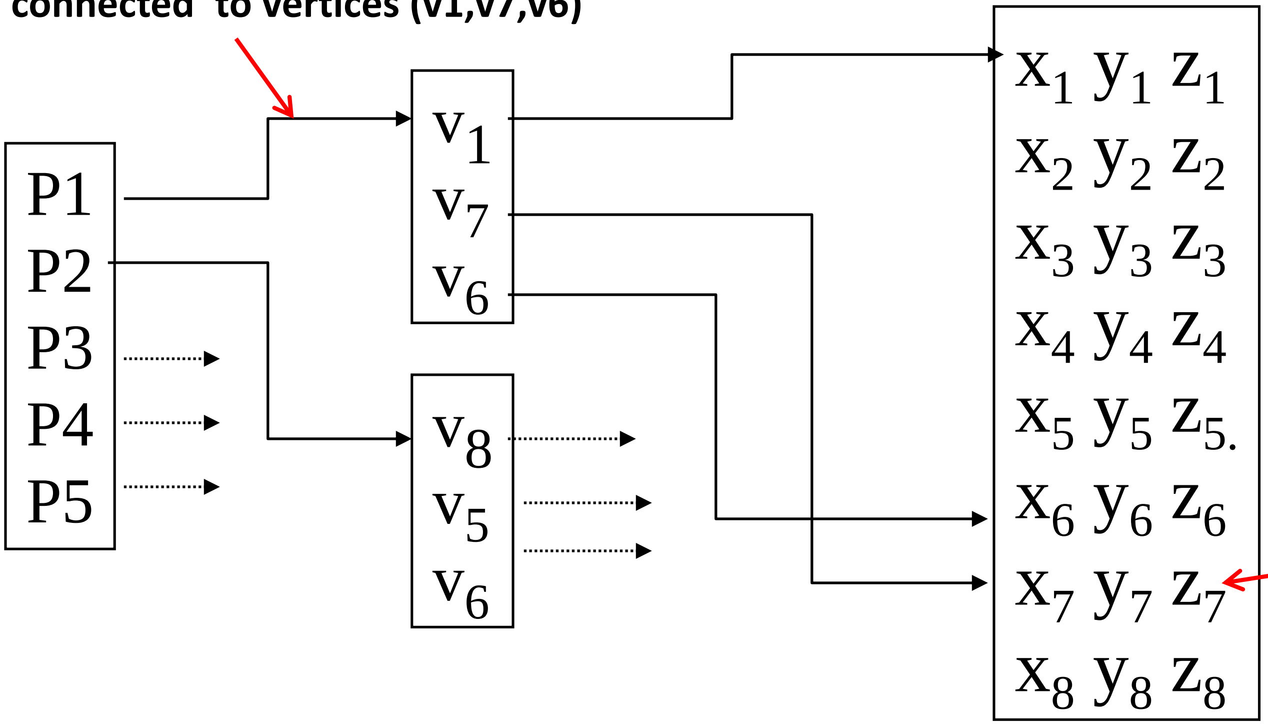




Vertex Lists

- **Vertex list:** (x,y,z) of vertices (its geometry) are put in array
- Use pointers from vertices into vertex list
- **Polygon list:** vertices connected to each polygon (face)

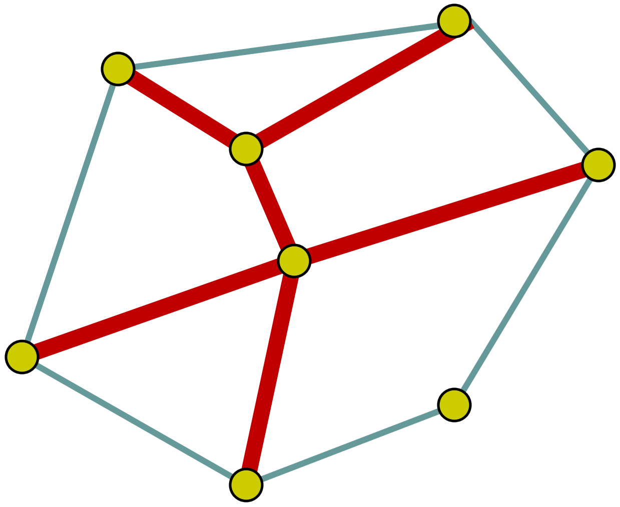
Topology example: Polygon P1 of mesh is connected to vertices (v1,v7,v6)



Geometry example: Vertex v7 coordinates are (x7,y7,z7). Note: If v7 moves, changed once in vertex list

Vertex List Issue: Shared Edges

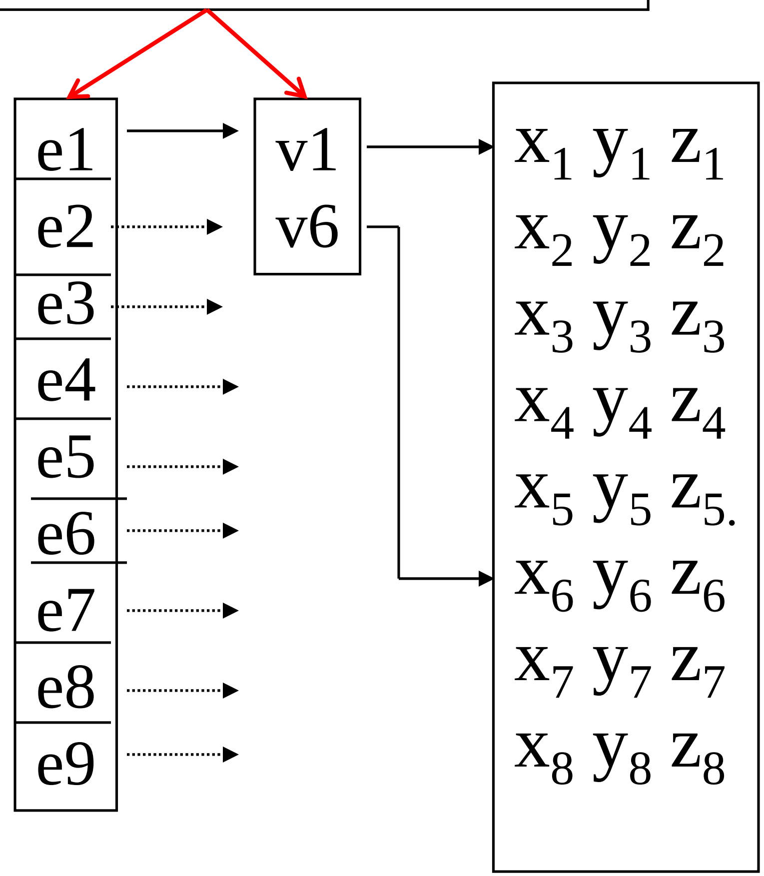
- Vertex lists draw filled polygons correctly
- If each polygon is drawn by its edges, shared edges are drawn twice

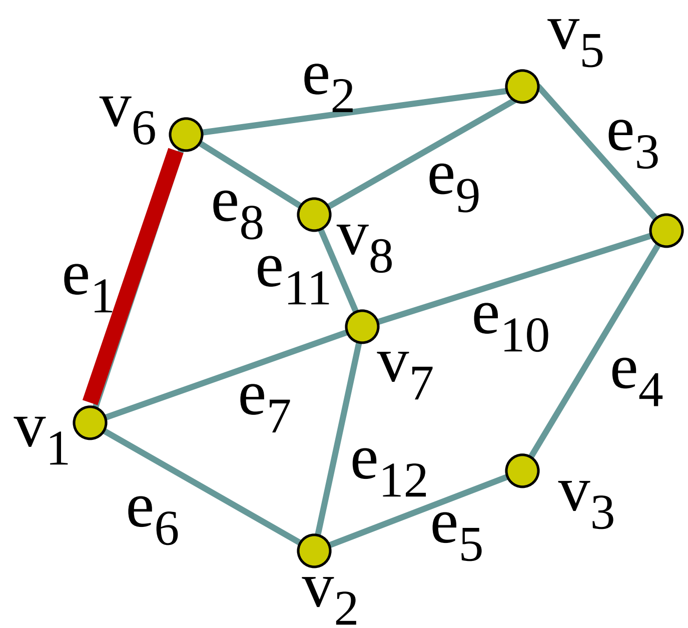


- **Alternatively:** Can store mesh by *edge list*

Edge List

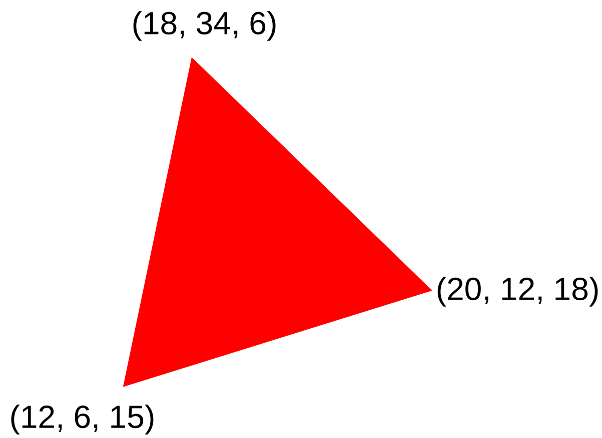
Simply draw each edges once
E.g e1 connects v1 and v6





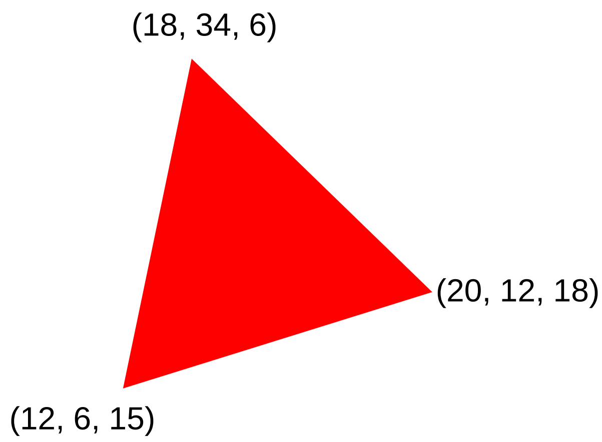
Note polygons are not represented

Vertex Attributes

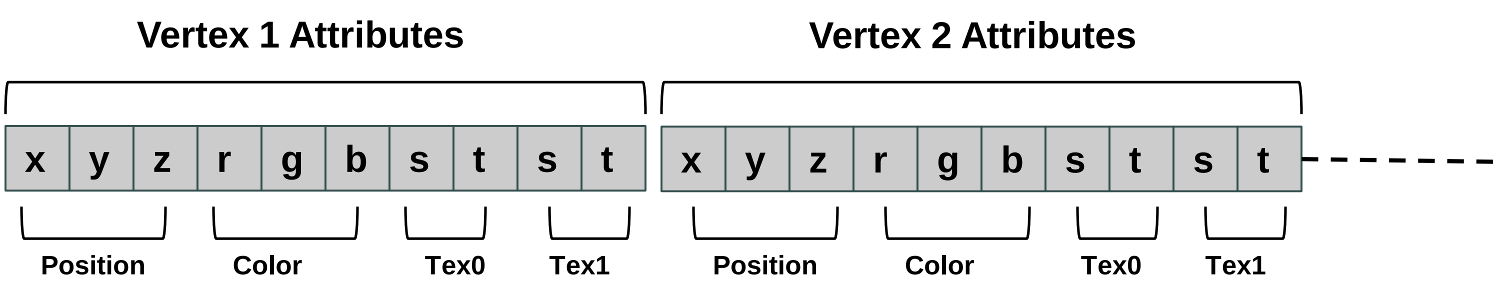


- Vertices can have attributes
  - Position (x, y, z) E.g (20, 12, 18)
  - Color (R,G,B) E.g. (1,0,0) or (red)
  - Normal (x,y,z)
  - Texture coordinates

Vertex Attributes

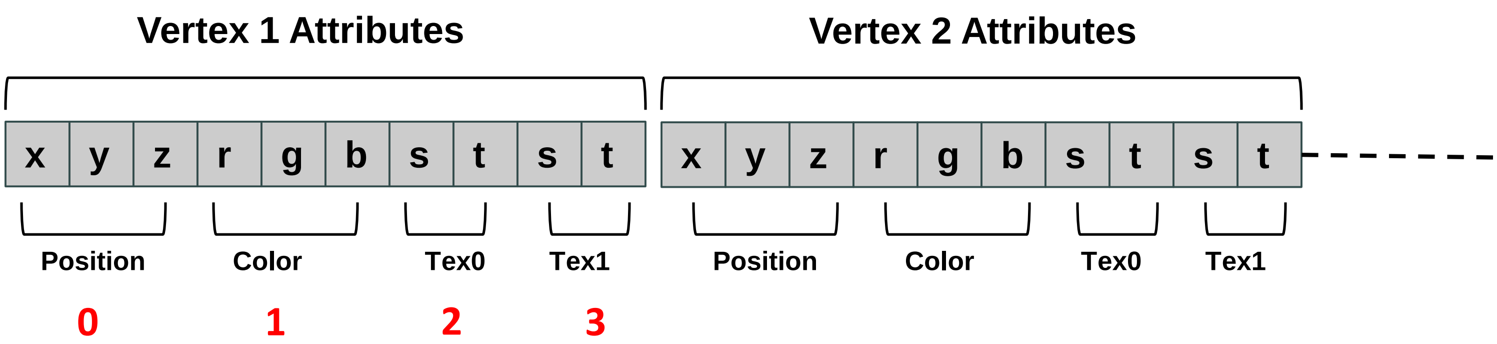


- Store vertex attributes in single Array (array of structures)
- **Later:** pass array to OpenGL, specify attributes, order, position using `glVertexAttribPointer`



Declaring Array of Vertex Attributes

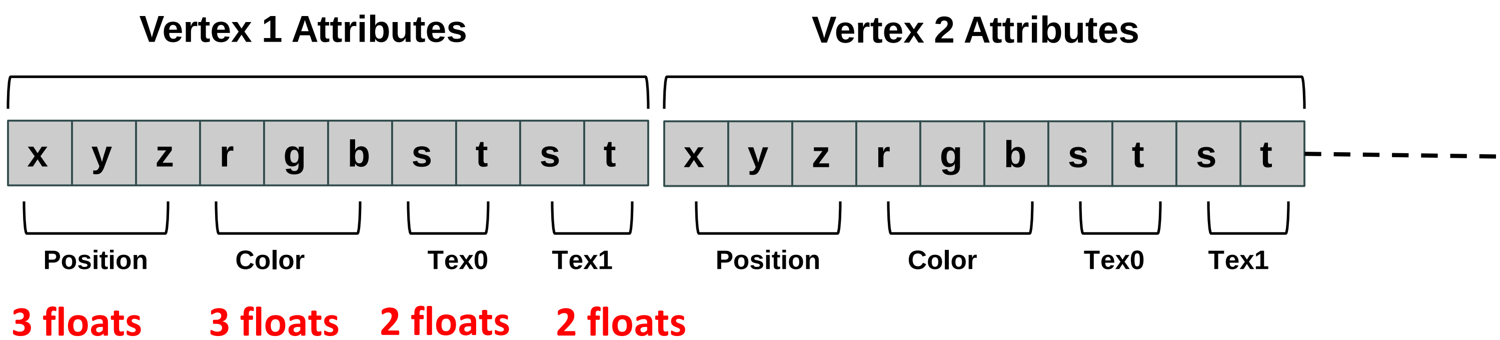
- Consider the following array of vertex attributes



- So we can define attribute positions (per vertex)

```
#define VERTEX_POS_INDEX            0
#define VERTEX_COLOR_INDEX          1
#define VERTEX_TEXCOORD0_INDX       2
#define VERTEX_TEXCOORD1_INDX       3
```

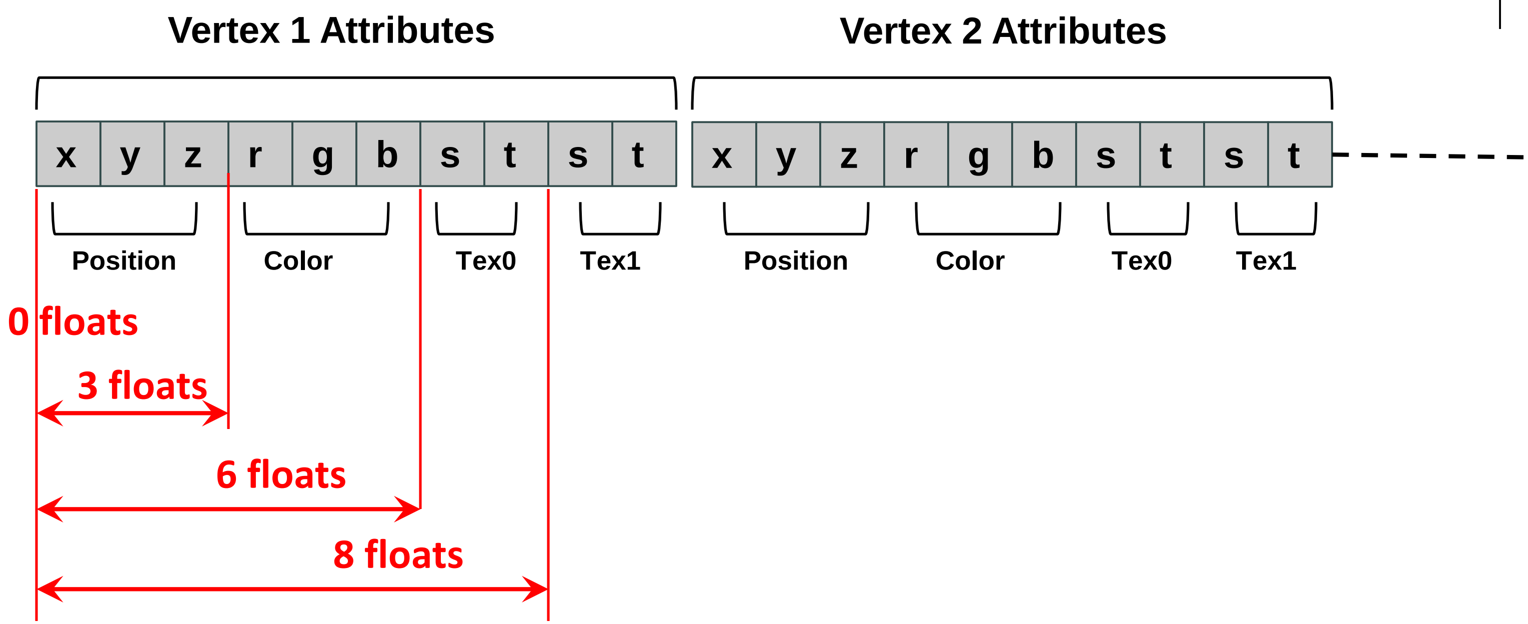
Declaring Array of Vertex Attributes



- Also define number of floats (storage) for each vertex attribute

```
#define VERTEX_POS_SIZE             3    // x, y and z
#define VERTEX_COLOR_SIZE           3    // r, g and b
#define VERTEX_TEXCOORD0_SIZE       2    // s and t
#define VERTEX_TEXCOORD1_SIZE       2    // s and t

#define VERTEX_ATTRIB_SIZE    VERTEX_POS_SIZE + VERTEX_COLOR_SIZE + \
                              VERTEX_TEXCOORD0_SIZE + \
                              VERTEX_TEXCOORD1_SIZE
```

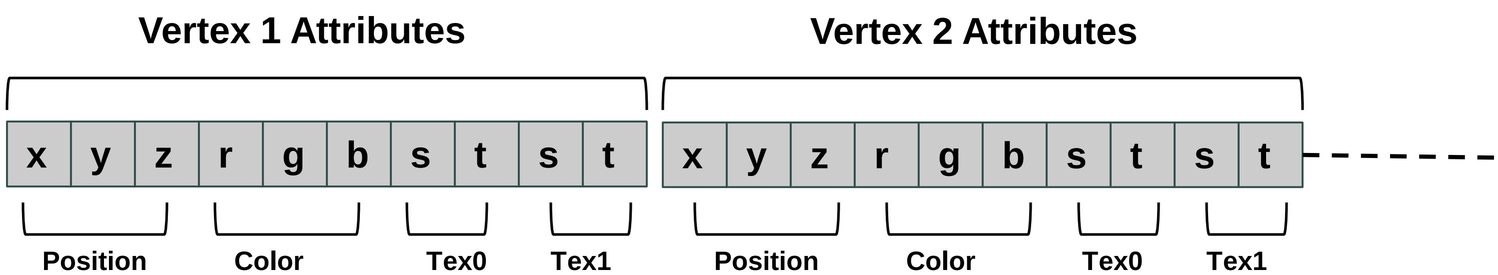
Declaring Array of Vertex Attributes



- Define offsets (# of floats) of each vertex attribute from beginning

```
#define VERTEX_POS_OFFSET          0
#define VERTEX_COLOR_OFFSET        3
#define VERTEX_TEXCOORD0_OFFSET    6
#define VERTEX_TEXCOORD1_OFFSET    8
```

Allocating Array of Vertex Attributes



- Allocate memory for entire array of vertex attributes

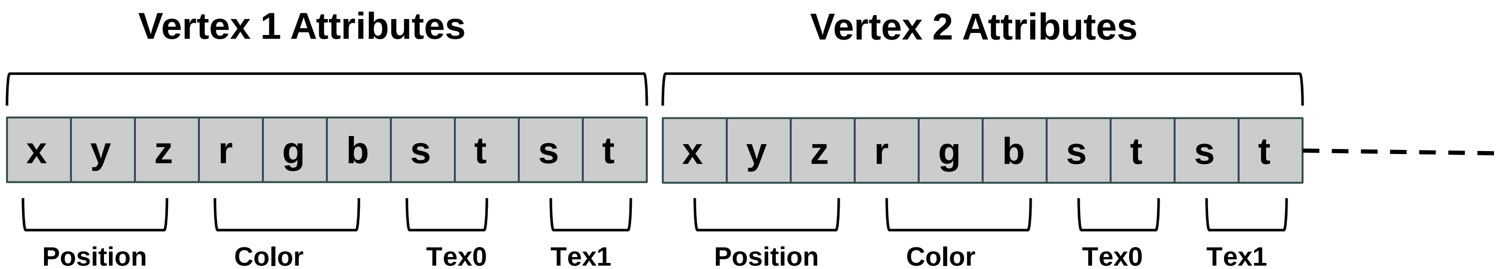
Recall

```
#define VERTEX_ATTRIB_SIZE    VERTEX_POS_SIZE + VERTEX_COLOR_SIZE + \
                              VERTEX_TEXCOORD0_SIZE + \
                              VERTEX_TEXCOORD1_SIZE
```

```
float *p = malloc(numVertices * VERTEX_ATTRIB_SIZE * sizeof(float));
```

Allocate memory for all vertices

Specifying Array of Vertex Attributes



- `glVertexAttribPointer` used to specify vertex attributes
- Example: to specify vertex position attribute

Position 0

3 values (x, y, z)

```
glVertexAttribPointer(VERTEX_POS_INDX, VERTEX_POS_SIZE,
                      GL_FLOAT, GL_FALSE,
                      VERTEX_ATTRIB_SIZE * sizeof(float), p);
```

Data should not Be normalized

Data is floats

Stride: distance between consecutive vertices

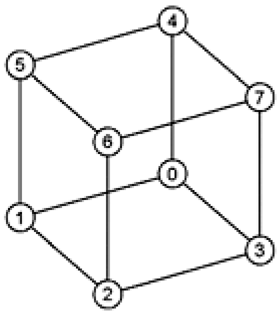
Pointer to data

```
glEnableVertexAttribArray(0);
```

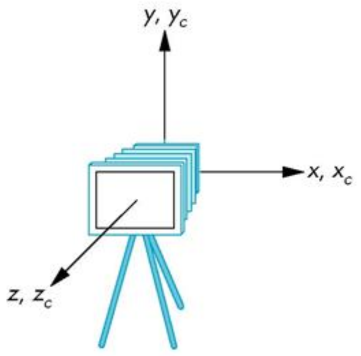
- `do same for normal, tex0 and tex1`

Full Example: Rotating Cube in 3D

- **Desired Program behaviour:**
  - Draw colored cube
  - Continuous rotation about X,Y or Z axis
    - Idle function called repeatedly when nothing to do
    - Increment angle of rotation in idle function
  - Use 3-button mouse to change direction of rotation
    - Click left button -> rotate cube around X axis
    - Click middle button -> rotate cube around Y axis
    - Click right button -> rotate cube around Z axis



- Use default camera
  - If we don't set camera, we get a default camera
  - Located at origin and points in the negative z direction

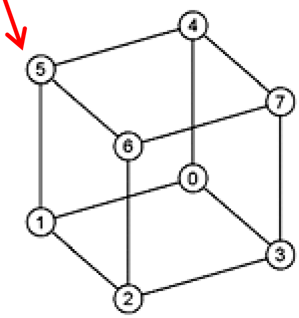


(a)

Cube Vertices

Declare array of (x,y,z,w) vertex positions
for a unit cube centered at origin
(Sides aligned with axes)

```
point4 vertices[8] = {
0   point4( -0.5, -0.5,  0.5, 1.0 ),
1   point4( -0.5,  0.5,  0.5, 1.0 ),
2   point4(  0.5,  0.5,  0.5, 1.0 ),
3   point4(  0.5, -0.5,  0.5, 1.0 ),
4   point4( -0.5, -0.5, -0.5, 1.0 ),
5   point4( -0.5,  0.5, -0.5, 1.0 ),
6   point4(  0.5,  0.5, -0.5, 1.0 ),
7   point4(  0.5, -0.5, -0.5, 1.0 )
};
              x      y      z     1
```



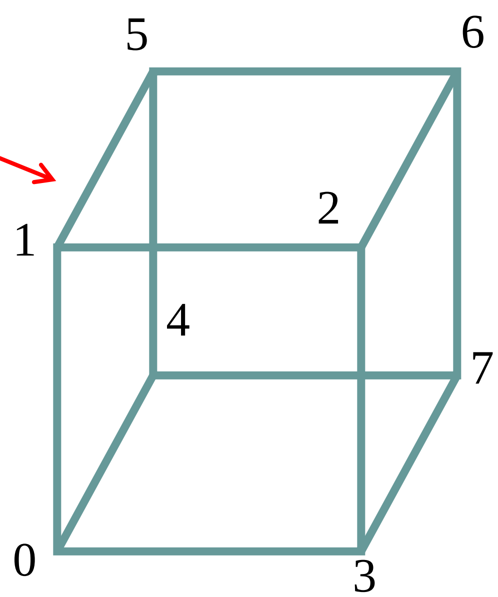

```
color4 vertex_colors[8] = {
    color4( 0.0, 0.0, 0.0, 1.0 ),  // black
    color4( 1.0, 0.0, 0.0, 1.0 ),  // red
    color4( 1.0, 1.0, 0.0, 1.0 ),  // yellow
    color4( 0.0, 1.0, 0.0, 1.0 ),  // green
    color4( 0.0, 0.0, 1.0, 1.0 ),  // blue
    color4( 1.0, 0.0, 1.0, 1.0 ),  // magenta
    color4( 1.0, 1.0, 1.0, 1.0 ),  // white
    color4( 0.0, 1.0, 1.0, 1.0 )   // cyan
};
            r    g    b    1
```

Declare array of vertex colors
(set of RGBA colors vertex can have)

Color Cube

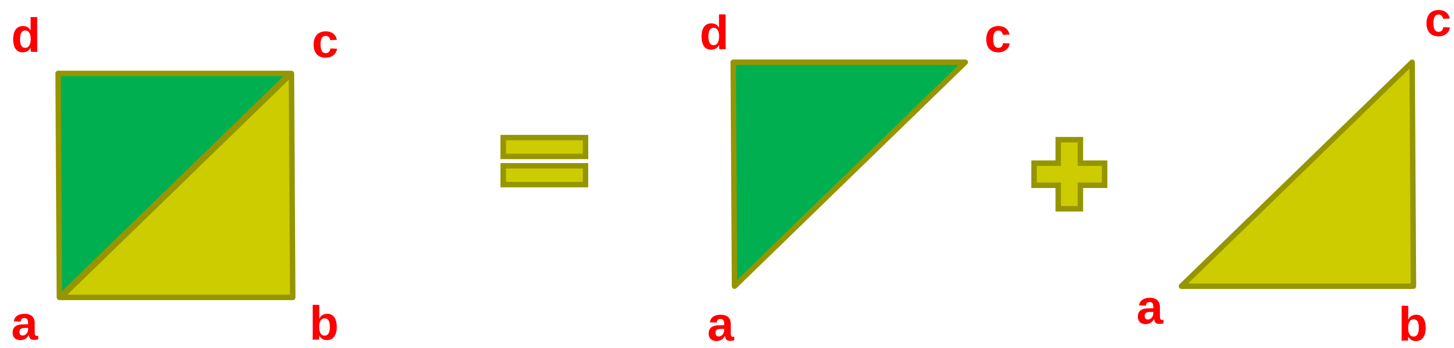
```
// generate 6 quads,
// sides of cube

void colorcube()
{
    quad( 1, 0, 3, 2 );
    quad( 2, 3, 7, 6 );
    quad( 3, 0, 4, 7 );
    quad( 6, 5, 1, 2 );
    quad( 4, 5, 6, 7 );
    quad( 5, 4, 0, 1 );
}
```

```
point4 vertices[8] = {
  0 point4( -0.5, -0.5,  0.5, 1.0 ),
  1 point4( -0.5,  0.5,  0.5, 1.0 ),
    point4(  0.5,  0.5,  0.5, 1.0 ),
    point4(  0.5, -0.5,  0.5, 1.0 ),
  4 point4( -0.5, -0.5, -0.5, 1.0 ),
  5 point4( -0.5,  0.5, -0.5, 1.0 ),
    point4(  0.5,  0.5, -0.5, 1.0 ),
    point4(  0.5, -0.5, -0.5, 1.0 )
};
```



Function **quad** is
Passed vertex indices

Quad Function




```
// quad generates two triangles (a,b,c) and (a,c,d) for each face
// and assigns colors to the vertices

int Index = 0;  // Index goes 0 to 5, one per vertex of face

void quad( int a, int b, int c, int d )
{
 0  colors[Index] = vertex_colors[a]; points[Index] = vertices[a]; Index++;
 1  colors[Index] = vertex_colors[b]; points[Index] = vertices[b]; Index++;
 2  colors[Index] = vertex_colors[c]; points[Index] = vertices[c]; Index++;
 3  colors[Index] = vertex_colors[a]; points[Index] = vertices[a]; Index++;
 4  colors[Index] = vertex_colors[c]; points[Index] = vertices[c]; Index++;
 5  colors[Index] = vertex_colors[d]; points[Index] = vertices[d]; Index++;
}
```

0	1	2	3	4	5
a	b	c	a	c	d

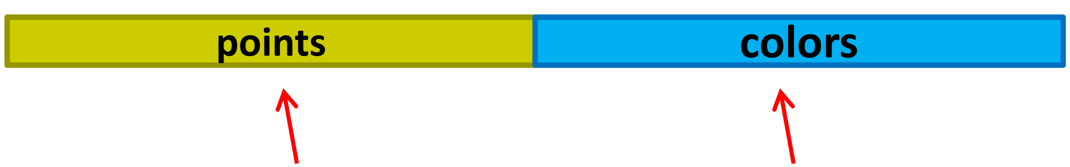
Points[ ] array to be
Sent to GPU

Read from appropriate index
of unique positions declared

quad 0 = points[0 - 5]
quad 1 = points[6 – 11]
quad 2 = points [12 – 17] …etc

Initialization I

```
void init()
{
    colorcube(); // Generates cube data in application using quads

    // Create a vertex array object
    GLuint vao;
    glGenVertexArrays ( 1, &vao );
    glBindVertexArray ( vao );

    // Create a buffer object and move data to GPU
    GLuint buffer;
    glGenBuffers( 1, &buffer );
    glBindBuffer( GL_ARRAY_BUFFER, buffer );
    glBufferData( GL_ARRAY_BUFFER, sizeof(points) +
                                  sizeof(colors), NULL, GL_STATIC_DRAW );
```



Points[ ] array of vertex positions sent to GPU

colors[ ] array of vertex colors sent to GPU

Initialization II

Send points[ ] and colors[ ] data to GPU separately using **`glBufferSubData`**

```
glBufferSubData( GL_ARRAY_BUFFER, 0, sizeof(points), points );
glBufferSubData( GL_ARRAY_BUFFER, sizeof(points), sizeof(colors), colors );
```


```
// Load vertex and fragment shaders and use the resulting shader program
GLuint program = InitShader( "vshader36.glsl", "fshader36.glsl" );
glUseProgram( program );
```

Initialization III

```
// set up vertex arrays

GLuint vPosition = glGetAttribLocation( program, "vPosition" );
glEnableVertexAttribArray( vPosition );
glVertexAttribPointer( vPosition, 4, GL_FLOAT, GL_FALSE, 0,
                                                 BUFFER_OFFSET(0) );

GLuint vColor = glGetAttribLocation( program, "vColor" );
glEnableVertexAttribArray( vColor );
glVertexAttribPointer( vColor, 4, GL_FLOAT, GL_FALSE, 0,
                                       BUFFER_OFFSET(sizeof(points)) );
```

Specify vertex data

Specify color data

```
theta = glGetUniformLocation( program, "theta" );
```

Want to Connect rotation variable theta in program to variable in shader

Display Callback

```
void display( void )
{
    glClear( GL_COLOR_BUFFER_BIT|GL_DEPTH_BUFFER_BIT );

    glUniform3fv( theta, 1, theta );
    glDrawArrays( GL_TRIANGLES, 0, NumVertices );

    glutSwapBuffers();
}
```

Draw series of triangles forming cube

Mouse Callback

```
…
enum { Xaxis = 0, Yaxis = 1, Zaxis = 2, NumAxes = 3 };



void mouse( int button, int state, int x, int y )
{
    if ( state == GLUT_DOWN ) {
        switch( button ) {
            case GLUT_LEFT_BUTTON:    axis = Xaxis;  break;
            case GLUT_MIDDLE_BUTTON:  axis = Yaxis;  break;
            case GLUT_RIGHT_BUTTON:   axis = Zaxis;  break;
        }
    }
}
```

Select axis (x,y,z) to rotate around
Using mouse click

Idle Callback

```
void idle( void )
{
    theta[axis] += 0.01;

    if ( theta[axis] > 360.0 ) {
        theta[axis] -= 360.0;
    }

    glutPostRedisplay();
}
```

The idle( ) function is called whenever nothing to do

Use it to increment rotation angle in steps of theta = 0.01 around currently selected axis

```
void main( void ){
  .........

  glutIdleFunc( idle );
  .........
}
```

Note: still need to:

- Apply rotation by (theta) in shader

References

- Angel and Shreiner, Interactive Computer Graphics, 6th edition, Chapter 3
- Hill and Kelley, Computer Graphics using OpenGL, 3rd edition