Computer Graphics (CS 543)
Lecture 10 (Part 1): 3D Clipping

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Liang-Barsky 3D Clipping

- **Goal:** Clip object edge-by-edge against Canonical View volume (CVV)

- **Problem:**
  - 2 end-points of edge: \( A = (Ax, Ay, Az, Aw) \) and \( C = (Cx, Cy, Cz, Cw) \)
  - If edge intersects with CVV, compute intersection point \( I = (lx, ly, lz, lw) \)
Determining if point is inside CVV

Problem: Determine if point \((x,y,z)\) is inside or outside CVV?

- CVV == 6 infinite planes \((x=-1,1;\ y=-1,1;\ z=-1,1)\)

Point \((x,y,z)\) is inside CVV if
- \((-1 \leq x \leq 1)\)
- \((-1 \leq y \leq 1)\)
- \((-1 \leq z \leq 1)\)
else point is outside CVV
Determining if point is inside CVV

If point specified as (x,y,z,w)
- Test \((x/w, \ y/w, \ z/w)!\)

Point \((x/w, \ y/w, \ z/w)\) is inside CVV

\[
\begin{align*}
\text{if} & \quad (-1 <= x/w <= 1) \\
\text{and} & \quad (-1 <= y/w <= 1) \\
\text{and} & \quad (-1 <= z/w <= 1)
\end{align*}
\]

else point is outside CVV
Modify Inside/Outside Tests Slightly

Our test: \((-1 < \frac{x}{w} < 1)\)

Point \((x,y,z,w)\) inside plane \(x = 1\) if

\[
\frac{x}{w} < 1 \\
\Rightarrow w - x > 0
\]

Point \((x,y,z,w)\) inside plane \(x = -1\) if

\[
-1 < \frac{x}{w} \\
\Rightarrow w + x > 0
\]
Numerical Example: Inside/Outside CVV Test

- Point \((x,y,z,w)\) is
  - inside plane \(x=-1\) if \(w+x > 0\)
  - inside plane \(x=1\) if \(w - x > 0\)

![Graph showing the range of \(x/w\) from -1 to 1]

- Example Point \((0.5, 0.2, 0.7)\) inside planes \((x = -1, 1)\) because \(-1 \leq 0.5 \leq 1\)

- If \(w = 10\), \((0.5, 0.2, 0.7) = (5, 2, 7, 10)\)
- Can either divide by \(w\) then test: \(-1 \leq 5/10 \leq 1\) OR
  - To test if inside \(x = -1\), \(w + x = 10 + 5 = 15 > 0\)
  - To test if inside \(x = 1\), \(w - x = 10 - 5 = 5 > 0\)
3D Clipping

- Do same for y, z to form boundary coordinates for 6 planes as:

<table>
<thead>
<tr>
<th>Boundary coordinate (BC)</th>
<th>Homogenous coordinate</th>
<th>Clip plane</th>
<th>Example (5,2,7,10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC0</td>
<td>w+x</td>
<td>x=-1</td>
<td>15</td>
</tr>
<tr>
<td>BC1</td>
<td>w-x</td>
<td>x=1</td>
<td>5</td>
</tr>
<tr>
<td>BC2</td>
<td>w+y</td>
<td>y=-1</td>
<td>12</td>
</tr>
<tr>
<td>BC3</td>
<td>w-y</td>
<td>y=1</td>
<td>8</td>
</tr>
<tr>
<td>BC4</td>
<td>w+z</td>
<td>z=-1</td>
<td>17</td>
</tr>
<tr>
<td>BC5</td>
<td>w-z</td>
<td>z=1</td>
<td>3</td>
</tr>
</tbody>
</table>

- Consider line that goes from point A to C
  - **Trivial accept**: 12 BCs (6 for pt. A, 6 for pt. C) > 0
  - **Trivial reject**: Both endpoints outside (-ve) for same plane
Edges as Parametric Equations

- Implicit form \( F(x, y) = 0 \)

- Parametric forms:
  - points specified based on single parameter value
  - Typical parameter: time \( t \)
    \[
P(t) = P_0 + (P_1 - P_0) \times t \quad 0 \leq t \leq 1
    \]

- Some algorithms work in parametric form
  - Clipping: exclude line segment ranges
  - Animation: Interpolate between endpoints by varying \( t \)

- Represent each edge parametrically as \( A + (C - A)t \)
  - at time \( t=0 \), point at \( A \)
  - at time \( t=1 \), point at \( C \)
Inside/outside?

- Test A, C against 6 walls \((x=-1,1;\ y=-1,1;\ z=-1,1)\)
- There is an intersection if BCs have opposite signs. i.e. if either
  - A is outside \((< 0)\), C is inside \((> 0)\) or
  - A inside \((> 0)\), C outside \((< 0)\)
- Edge intersects with plane at some \(t_{\text{hit}}\) between \([0,1]\)
Calculating hit time (t_hit)

How to calculate t_hit?

Represent an edge t as:

$$\text{Edge}(t) = ((Ax + (Cx - Ax)t, (Ay + (Cy - Ay)t, (Az + (Cz - Az)t, (Aw + (Cw - Aw)t})$$

- E.g. If x = 1, 
  $$\frac{Ax + (Cx - Ax)t}{Aw + (Cw - Aw)t} = 1$$

- Solving for t above,
  $$t = \frac{Aw - Ax}{(Aw - Ax) - (Cw - Cx)}$$
Inside/outside?

- t_hit can be “entering (t_in) ” or ”leaving (t_out)”
- Define: “entering” if A outside, C inside
  - Why? As t goes [0-1], edge goes from outside (at A) to inside (at C)
- Define “leaving” if A inside, C outside
  - Why? As t goes [0-1], edge goes from inside (at A) to inside (at C)
Chop step by Step against 6 planes

- Initially

\[ t_{in} = 0, \quad t_{out} = 1 \]
Candidate Interval (CI) = [0 to 1]

- Chop against each of 6 planes

\[ t_{in} = 0, \quad t_{out} = 0.74 \]
Candidate Interval (CI) = [0 to 0.74]
Chop step by Step against 6 planes

- Initially
  \[ t_{\text{out}} = 0.74 \]
  \[ t_{\text{in}} = 0, \quad t_{\text{out}} = 0.74 \]
  Candidate Interval (CI) = [0 to 0.74]

- Then
  Plane \( x = -1 \)
  \[ t_{\text{out}} = 0.74 \]
  \[ t_{\text{in}} = 0.36, \quad t_{\text{out}} = 0.74 \]
  Candidate Interval (CI) CI = [0.36 to 0.74]

**Why \( t_{\text{in}} \)?**
Candidate Interval

- Candidate Interval (CI): time interval during which edge might still be inside CVV. i.e. CI = t_in to t_out
- Initialize CI to [0,1]
- For each of 6 planes, calculate t_in or t_out, shrink CI
- Conversely: values of t outside CI = edge is outside CVV
Shortening Candidate Interval

- **Algorithm:**
  - Test for trivial accept/reject (stop if either occurs)
  - Set CI to [0,1]
  - For each of 6 planes:
    - Find hit time $t_{\text{hit}}$
    - If $t_{\text{in}}$, new $t_{\text{in}} = \max(t_{\text{in}}, t_{\text{hit}})$
    - If $t_{\text{out}}$, new $t_{\text{out}} = \min(t_{\text{out}}, t_{\text{hit}})$
    - If $t_{\text{in}} > t_{\text{out}} \Rightarrow$ exit (no valid intersections)

![Diagram showing CI, t, t_in, t_out](image)

**Note:** seeking smallest valid CI without $t_{\text{in}}$ crossing $t_{\text{out}}$
Calculate chopped A and C

- If valid t_in, t_out, calculate adjusted edge endpoints A, C as

  - \( A_{\text{chop}} = A + t_{\text{in}} \ (C - A) \) (calculate for \(Ax, Ay, Az\))
  - \( C_{\text{chop}} = A + t_{\text{out}} \ (C - A) \) (calculate for \(Cx, Cy, Cz\))
3D Clipping Implementation

- Function clipEdge()
- Input: two points A and C (in homogenous coordinates)
- Output:
  - 0, if AC lies complete outside CVV
  - 1, complete inside CVV
  - Returns clipped A and C otherwise
- Calculate 6 BCs for A, 6 for C
Store BCs as Outcodes

- Use outcodes to track in/out
  - Number walls $x = +1, -1; y = +1, -1, \text{ and } z = +1, -1$ as 0 to 5
  - Bit $i$ of A’s outcode $= 1$ if A is outside $i$th wall
  - 1 otherwise

- Example: outcode for point outside walls 1, 2, 5

<table>
<thead>
<tr>
<th>Wall no.</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>OutCode</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Trivial Accept/Reject using Outcodes

- **Trivial accept**: inside (not outside) any walls

<table>
<thead>
<tr>
<th>Wall no.</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Outcode</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C OutCode</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Logical bitwise test: \( A \mid C == 0 \)

- **Trivial reject**: point outside *same* wall. Example Both A and C outside wall 1

<table>
<thead>
<tr>
<th>Wall no.</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Outcode</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>C OutCode</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Logical bitwise test: \( A \& C != 0 \)
3D Clipping Implementation

- Compute BCs for A, C store as outcodes
- Test A, C outcodes for trivial accept, trivial reject
- If not trivial accept/reject, for each wall:
  - Compute tHit
  - Update t_in, t_out
  - If t_in > t_out, early exit
3D Clipping Pseudocode

```c
int clipEdge(Point4& A, Point4& C) {
    double tIn = 0.0, tOut = 1.0, tHit;
    double aBC[6], cBC[6];
    int aOutcode = 0, cOutcode = 0;

    .....find BCs for A and C
    .....form outcodes for A and C

    if((aOutCode & cOutcode) != 0) // trivial reject
        return 0;
    if((aOutCode | cOutcode) == 0) // trivial accept
        return 1;
}
```
3D Clipping Pseudocode

for(i=0;i<6;i++) // clip against each plane
{
    if(cBC[i] < 0) // C is outside wall i (exit so tOut)
    {
        tHit = aBC[i]/(aBC[i] – cBC[i]); // calculate tHit
        tOut = MIN(tOut, tHit);
    }
    else if(aBC[i] < 0) // A is outside wall I (enters so tIn)
    {
        tHit = aBC[i]/(aBC[i] – cBC[i]); // calculate tHit
        tIn = MAX(tIn, tHit);
    }
    if(tIn > tOut) return 0; // CI is empty: early out
}
3D Clipping Pseudocode

Point4 tmp; // stores homogeneous coordinates
If(aOutcode != 0) // A is outside: tIn has changed. Calculate A_chop
{
    tmp.x = A.x + tIn * (C.x – A.x);
    // do same for y, z, and w components
}
If(cOutcode != 0) // C is outside: tOut has changed. Calculate C_chop
{
    C.x = A.x + tOut * (C.x – A.x);
    // do same for y, z and w components
}
A = tmp;
Return 1; // some of the edges lie inside CVV
Polygon Clipping

- Not as simple as line segment clipping
  - Clipping a line segment yields at most one line segment
  - Clipping a polygon can yield multiple polygons
- Clipping a convex polygon can yield at most one other polygon
Clipping Polygons

- Need more sophisticated algorithms to handle polygons:
  - *Sutherland-Hodgman*: any a given polygon against a convex clip polygon (or window)
  - *Weiler-Atherton*: Both subject polygon and clip polygon can be concave
Tessellation and Convexity

- One strategy is to replace nonconvex (*concave*) polygons with a set of triangular polygons (a *tessellation*)
- Also makes fill easier
References