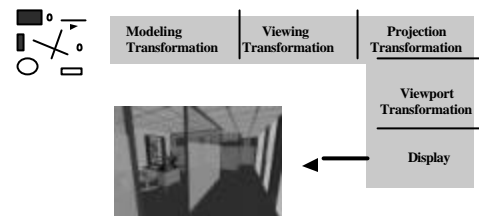


3D viewing under the hood



3D viewing under the hood

- Topics of Interest:
 - Viewing transformation
 - Projection transformation

Viewing Transformation

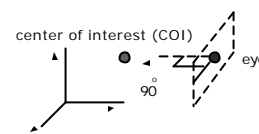
- Transform the object from world to eye space
 - Construct eye coordinate frame
 - Construct matrix to perform coordinate transformation
 - Flexible Camera Control

Viewing Transformation

- Recall OpenGL way to set camera:
 - `gluLookAt (Ex, Ey, Ez, cx, cy, cz, Up_x, Up_y, Up_z)`
 - The view up vector is usually (0,1,0)
 - Remember to set the OpenGL matrix mode to `GL_MODELVIEW` first
- Modelview matrix:
 - combination of modeling matrix M and Camera transforms V
- `gluLookAt` fills V part of modelview matrix
- What does `gluLookAt` do with parameters (*eye*, *COI*, *up vector*) you provide?

Eye Coordinate Frame

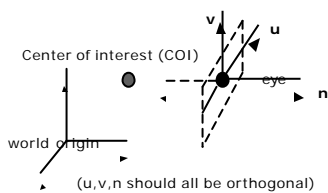
- Known: eye position, center of interest, view-up vector
- To find out: new origin and three basis vectors



Assumption: direction of view is orthogonal to view plane (plane that objects will be projected onto)

Eye Coordinate Frame

- Origin: eye position (that was easy)
- Three basis vectors:
 - one is the normal vector (\mathbf{n}) of the viewing plane,
 - other two (\mathbf{u} and \mathbf{v}) span the viewing plane



\mathbf{n} is pointing away from the world because we use left hand coordinate system

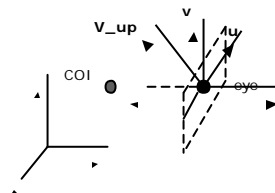
$$\mathbf{N} = \text{eye} - \text{COI}$$

$$\mathbf{n} = \mathbf{N} / |\mathbf{N}|$$

Remember $\mathbf{u}, \mathbf{v}, \mathbf{n}$ should be all unit vectors

Eye Coordinate Frame

- How about \mathbf{u} and \mathbf{v} ?



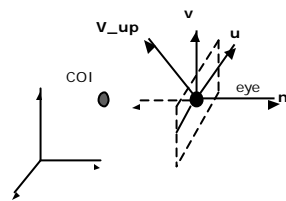
- We can get \mathbf{u} first -
- \mathbf{u} is a vector that is perp to the plane spanned by \mathbf{N} and view up vector ($\mathbf{V_up}$)

$$\mathbf{U} = \mathbf{V_up} \times \mathbf{n}$$

$$\mathbf{u} = \mathbf{U} / |\mathbf{U}|$$

Eye Coordinate Frame

- How about v ?



Knowing n and u , getting v is easy

$$v = n \times u$$

v is already normalized

Eye Coordinate Frame

- Put it all together

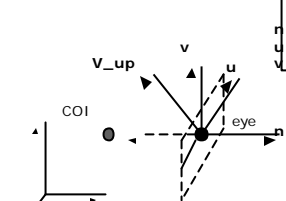
Eye space origin:
(Eye.x, Eye.y, Eye.z)

Basis vectors:

$$n = (eye - COI) / |eye - COI|$$

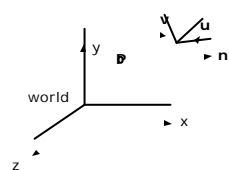
$$u = (V_up \times n) / |V_up \times n|$$

$$v = n \times u$$



World to Eye Transformation

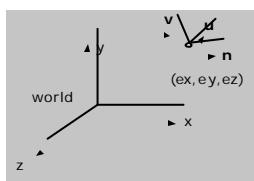
- Transformation matrix (M_{w2e}) ?
 $P' = M_{w2e} \times P$



- Come up with the transformation sequence to move eye coordinate frame to the world
- And then apply this sequence to the point P in a reverse order

World to Eye Transformation

- Rotate the eye frame to "align" it with the world frame
- Translate $(-ex, -ey, -ez)$



Rotation:

ux	uy	uz	0
vx	vy	vz	0
nx	ny	nz	0
0	0	0	1

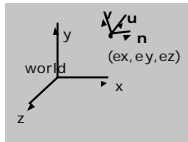
Translation:

1	0	0	$-ex$
0	1	0	$-ey$
0	0	1	$-ez$
0	0	0	1

World to Eye Transformation

- Transformation order: apply the transformation to the object in a reverse order - translation first, and then rotate

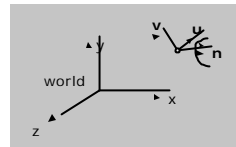
$$M_{w2e} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



World to Eye Transformation

- Head tilt: Rotate your head by δ
- Just rotate the object about the eye space z

$$M_{w2e} = \begin{bmatrix} \cos(-\delta) & -\sin(-\delta) & 0 & 0 \\ \sin(-\delta) & \cos(-\delta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Why $-\delta$?

When you rotate your head by δ , it is like rotate the object by $-\delta$