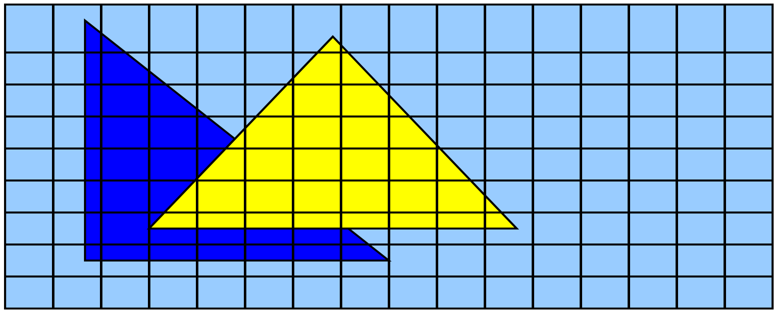
Recall: OpenGL - Image Space Approach

- Paint pixel with color of **closest** object

```
for (each pixel in image) {
    determine the object closest to the pixel
    draw the pixel using the object's color
}
```



Recall: Z (depth) Buffer Algorithm

Depth of polygon being rasterized at pixel (x, y)

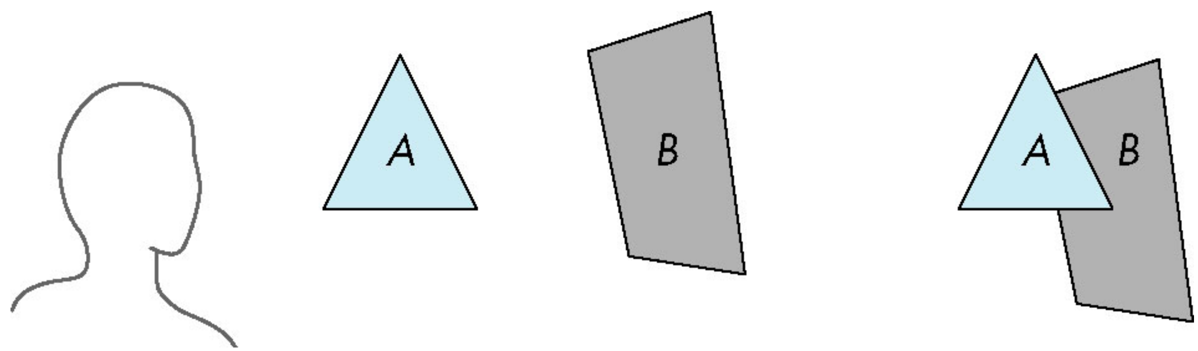
Largest depth seen so far Through pixel (x, y)

```
For  each polygon  {

   for each pixel (x,y) in polygon area  {

          if  (z_polygon_pixel(x,y) < depth_buffer(x,y) ) {

                 depth_buffer(x,y) = z_polygon_pixel(x,y);

                 color_buffer(x,y) = polygon color at (x,y)
          }
     }
}
```

Note: know depths at vertices. Interpolate for interior z_polygon_pixel(x, y) depths

Painter's HSR Algorithm

- Render polygons farthest to nearest
- Similar to painter layers oil paint



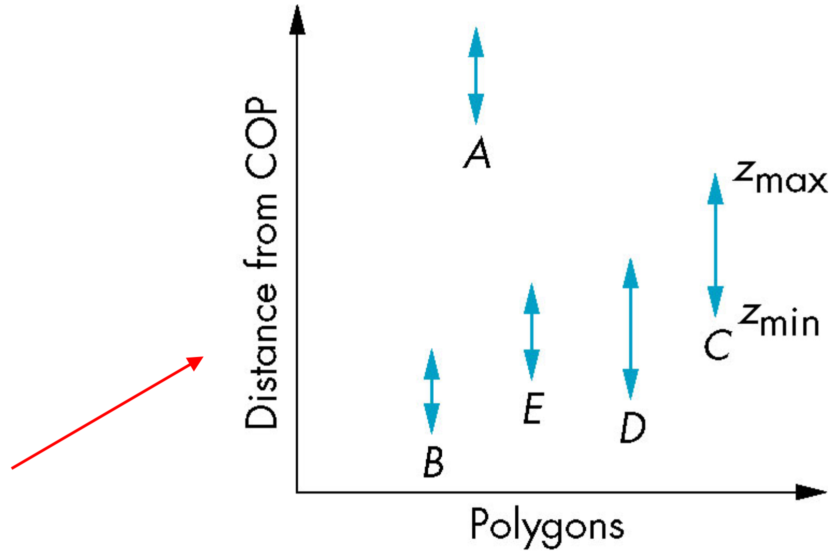
Viewer sees B behind A

Render B then A

Depth Sort

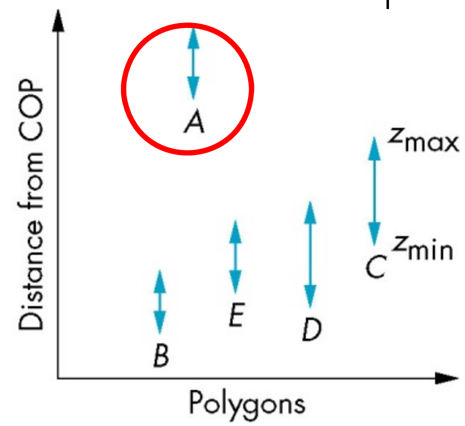
- Requires sorting polygons (based on depth)
  - O(n log n) complexity to sort n polygon depths
  - Not every polygon is clearly in front or behind other polygons

Polygons sorted by distance from COP

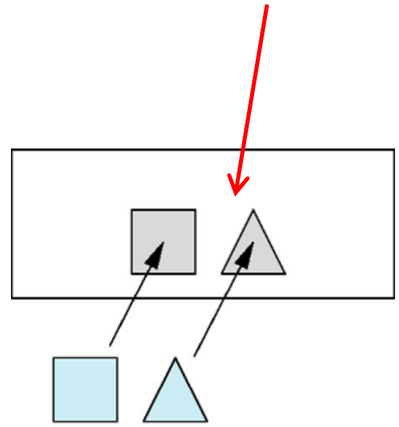


Easy Cases

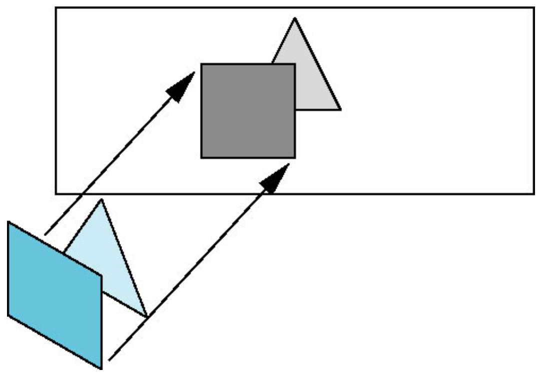
- Case a: A lies behind all polygons



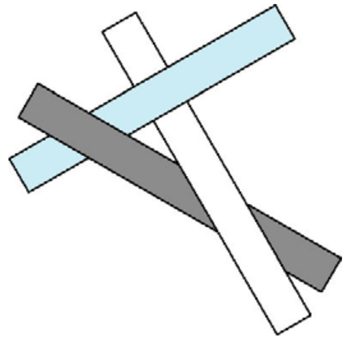
- Case b: Polygons overlap in z but **not** in x or y



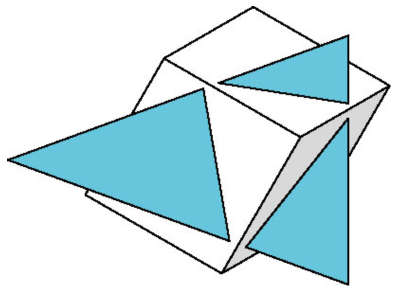
Hard Cases



Overlap in (x,y) and z ranges



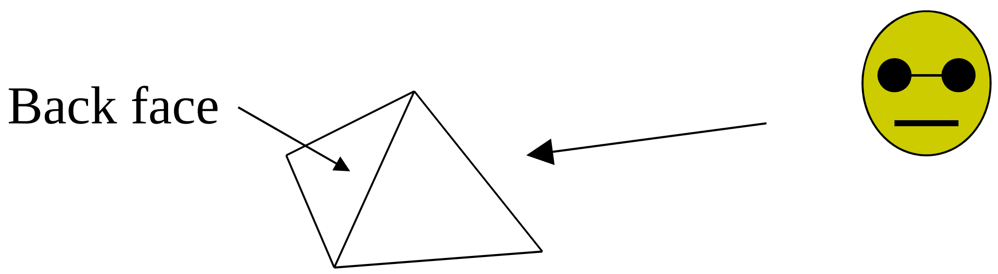
cyclic overlap



penetration

Back Face Culling

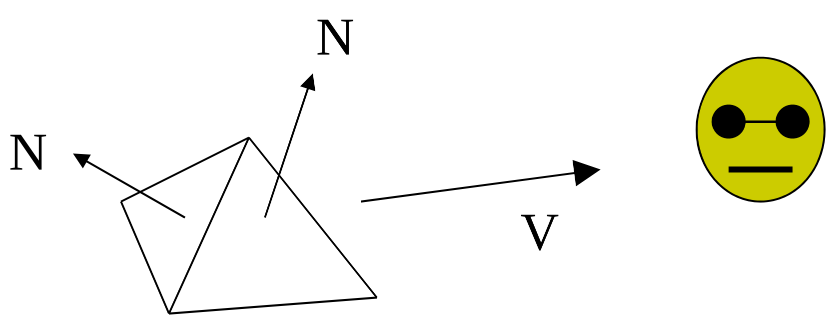
- **Back faces:** faces of opaque object that are "pointing away" from viewer
- **Back face culling:** do not draw back faces (saves resources)



- How to detect back faces?

Back Face Culling

- Goal: Test if a face F is is backface
- How? Form vectors
  - View vector, V
  - Normal N to face F



Backface test: F is backface if N.V < 0 why??

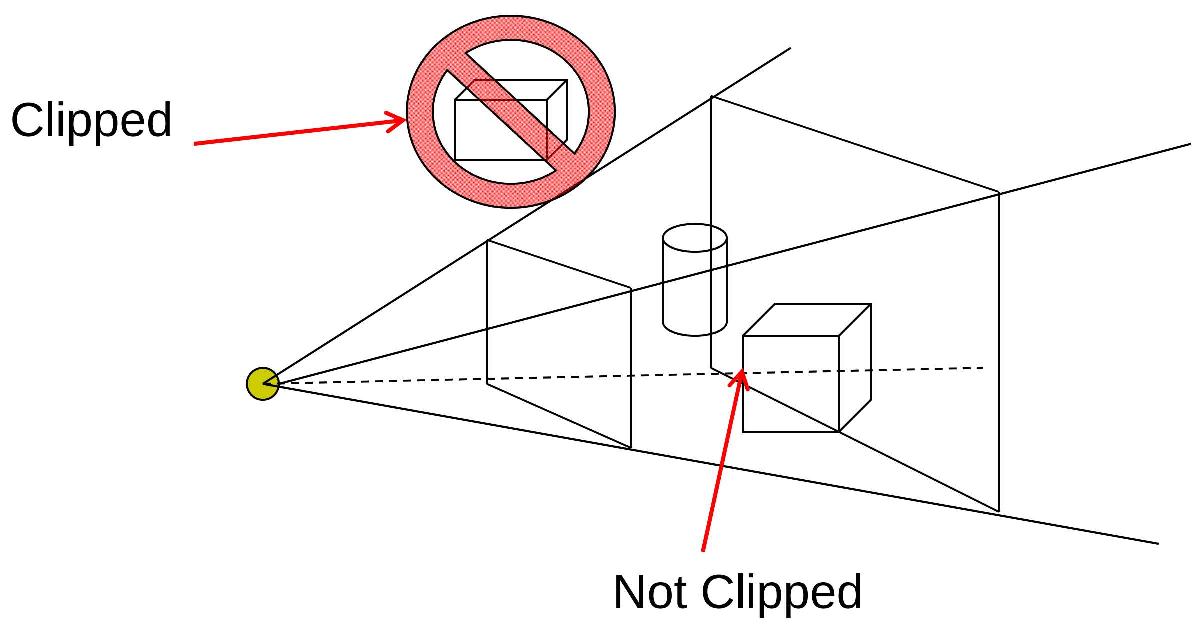
Back Face Culling: Draw mesh front faces

```
void drawFrontFaces( )
{
    for(int f = 0;f < numFaces; f++)
    {
        if(isBackFace(f, ….) continue;
        glDrawArrays(GL_POLYGON, 0, N);
    }
```

if N.V < 0

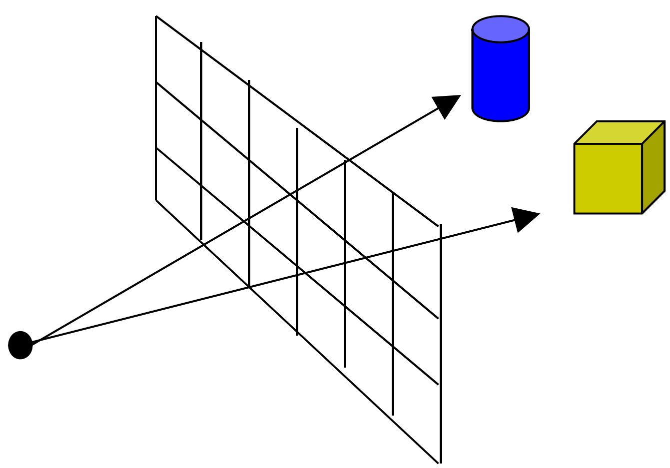
View-Frustum Culling

- **Goal:** Remove objects outside view frustum
- Done by 3D clipping algorithm (e.g. Liang-Barsky)



Ray Tracing

- Ray tracing is another image space method
- Ray tracing: Cast a ray from eye through each pixel into world.
- Ray tracing algorithm figures out: what object seen in direction through given pixel?

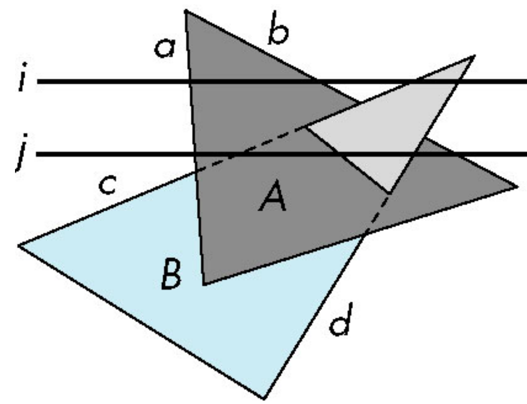


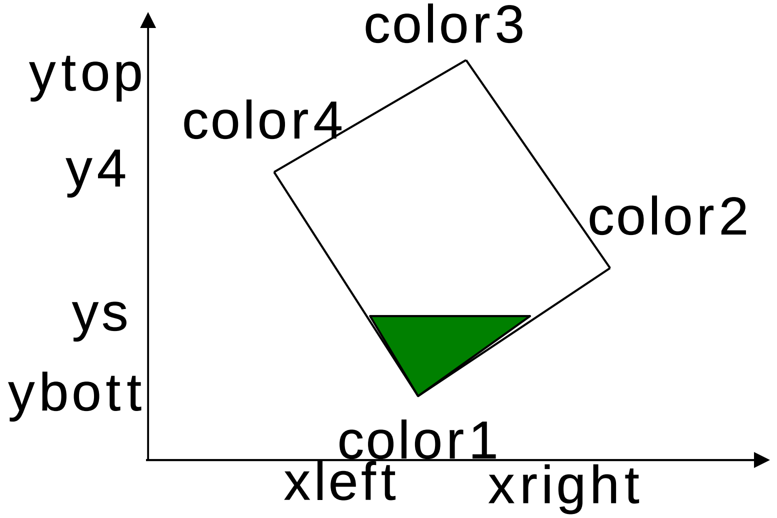
Topic of grad class

Combined z-buffer and Gouraud Shading (Hill)

- Can combine shading and hsr through scan line algorithm

```
for(int y = ybott; y <= ytop; y++)  // for each scan line
{
      for(each polygon){
      find xleft and xright
      find dleft,  dright, and dinc
      find colorleft and colorright, and colorinc
      for(int x = xleft, c = colorleft, d = dleft; x <= xright;
                                    x++, c+= colorinc, d+= dinc)
      if(d < d[x][y])
      {
         put c into the pixel at (x, y)
         d[x][y] = d; // update closest depth
      }
}
```





Computer Graphics (CS 4731)
Lecture 24: Rasterization: Line Drawing

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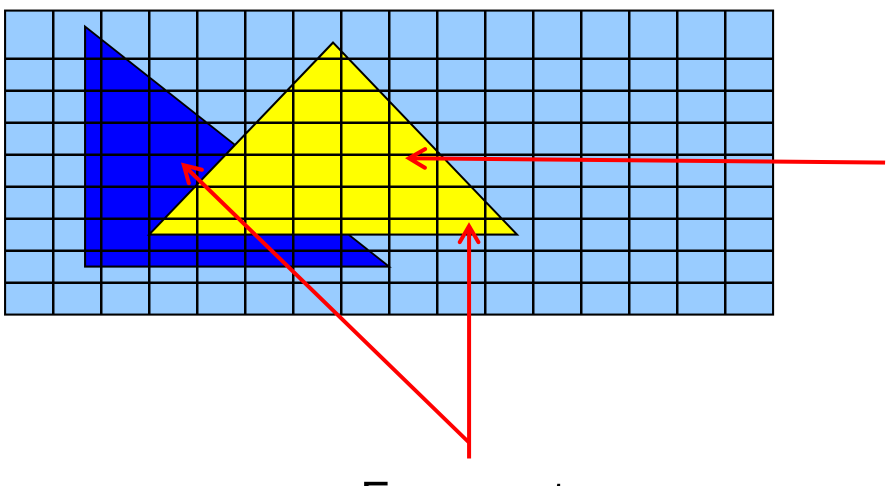
Rasterization

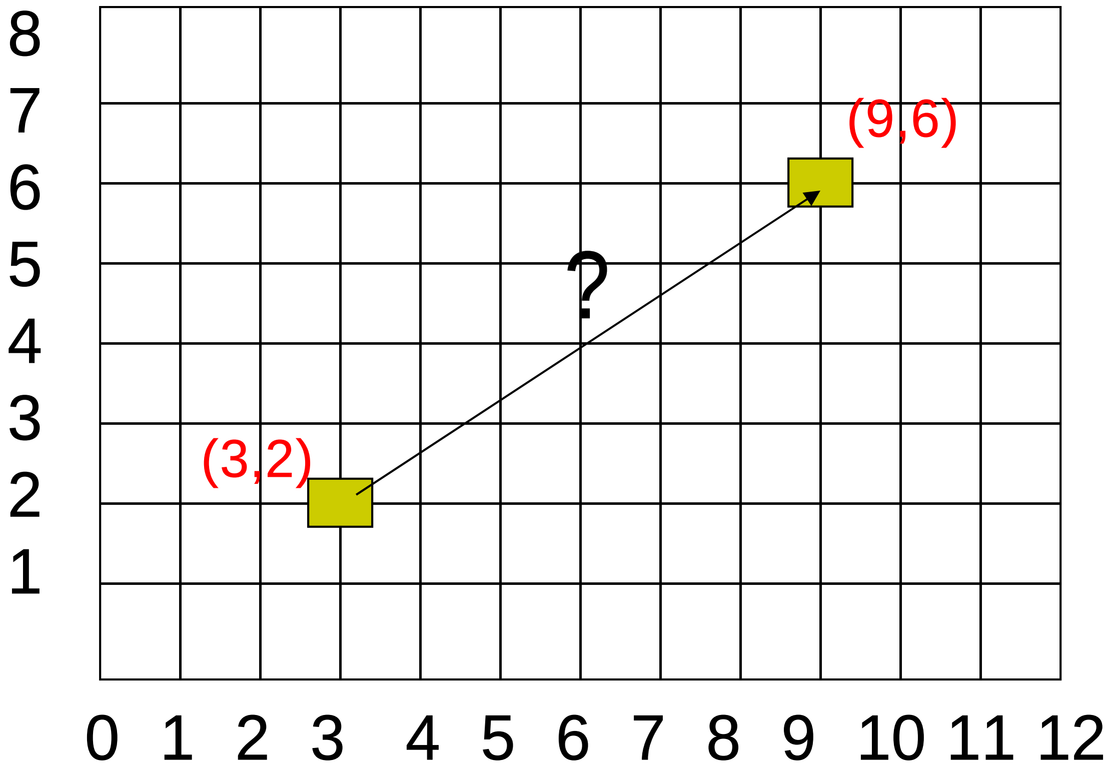
- Rasterization produces set of **fragments**
- Implemented by graphics hardware
- Rasterization algorithms for primitives (e.g lines, circles, triangles, polygons)



Rasterization: Determine Pixels (fragments) each primitive covers

Line drawing algorithm

- Programmer specifies (x,y) of end pixels
- Need algorithm to determine pixels on line path

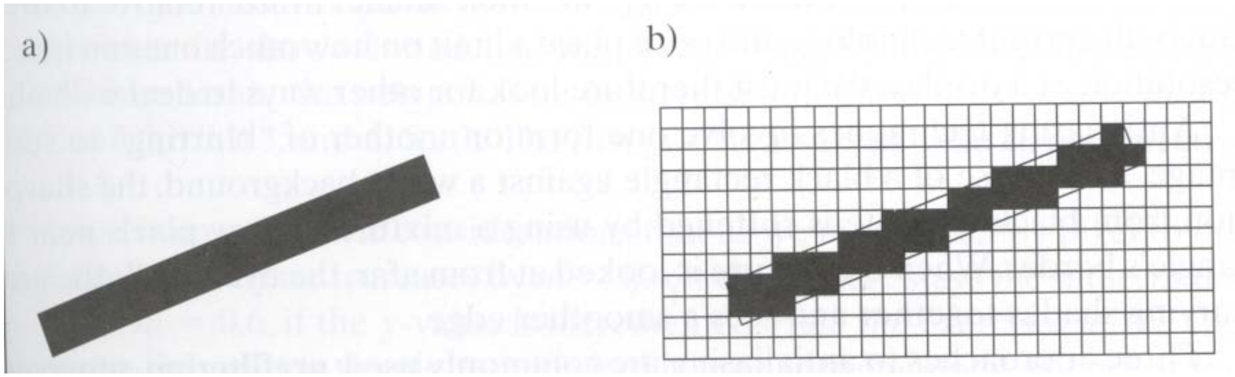


Line: (3,2) -> (9,6)

Which intermediate pixels to turn on?

Line drawing algorithm

- Pixel (x,y) values constrained to integer values
- Computed intermediate values may be floats
- Rounding may be required. E.g. (10.48, 20.51) rounded to (10, 21)
- Rounded pixel value is off actual line path (jaggy!!)
- Sloped lines end up having jaggies
- Vertical, horizontal lines, no jaggies



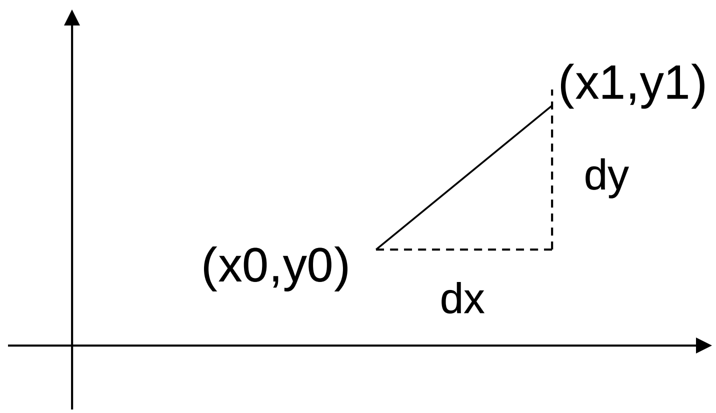
Line Drawing Algorithm

- Slope-intercept line equation
  - y = mx + b
  - Given 2 end points (x0,y0), (x1, y1), how to compute m and b?

$$m = \frac{dy}{dx} = \frac{y1 - y0}{x1 - x0}$$

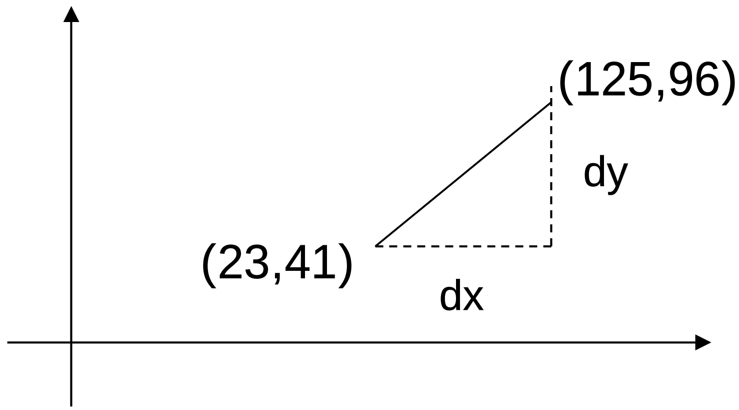
$$y0 = m * x0 + b$$

$$\Rightarrow b = y0 - m * x0$$



Line Drawing Algorithm

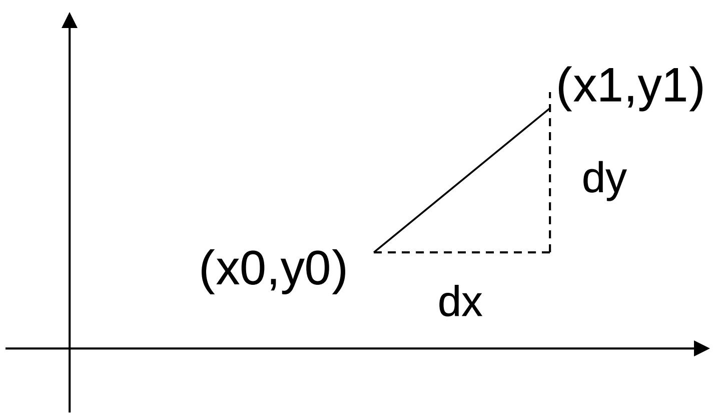
- Numerical example of finding slope m:
  - (Ax, Ay) = (23, 41), (Bx, By) = (125, 96)

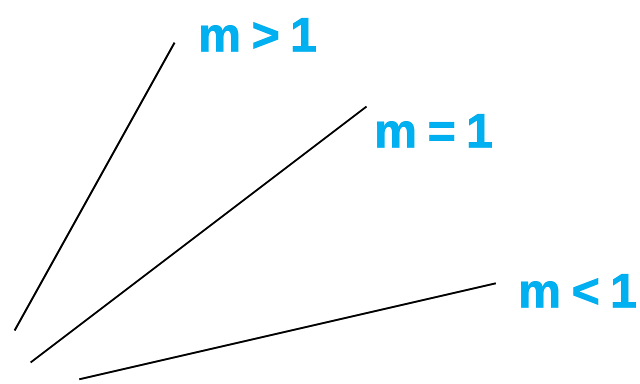


$$m = \frac{By - Ay}{Bx - Ax} = \frac{96 - 41}{125 - 23} = \frac{55}{102} = 0.5392$$

Digital Differential Analyzer (DDA): Line Drawing Algorithm

Consider slope of line, m:



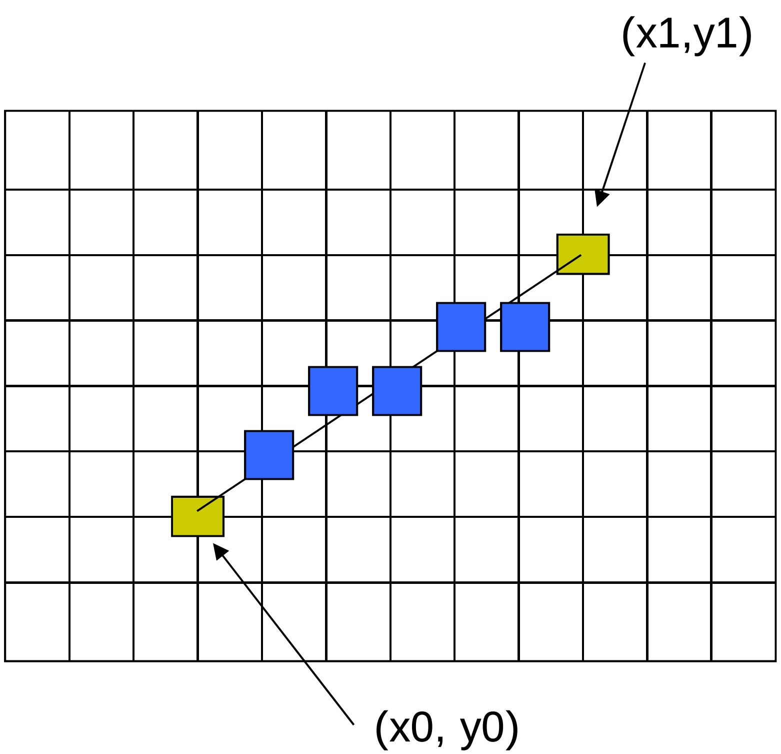


- Step through line, starting at (x0,y0)
- **Case a: (m < 1)** x incrementing faster
  - Step in x=1 increments, compute y (a fraction) and round
- **Case b: (m > 1)** y incrementing faster
  - Step in y=1 increments, compute x (a fraction) and round

DDA Line Drawing Algorithm (Case a: m < 1)

$$m = \frac{\Delta y}{\Delta x} = \frac{y_{k+1} - y_k}{x_{k+1} - x_k} = \frac{y_{k+1} - y_k}{1}$$

$$\Rightarrow y_{k+1} = y_k + m$$



x = x0 y = y0

Illuminate pixel (x, round(y))

x = x + 1 y = y + m

Illuminate pixel (x, round(y))

x = x + 1 y = y + m

Illuminate pixel (x, round(y))

...

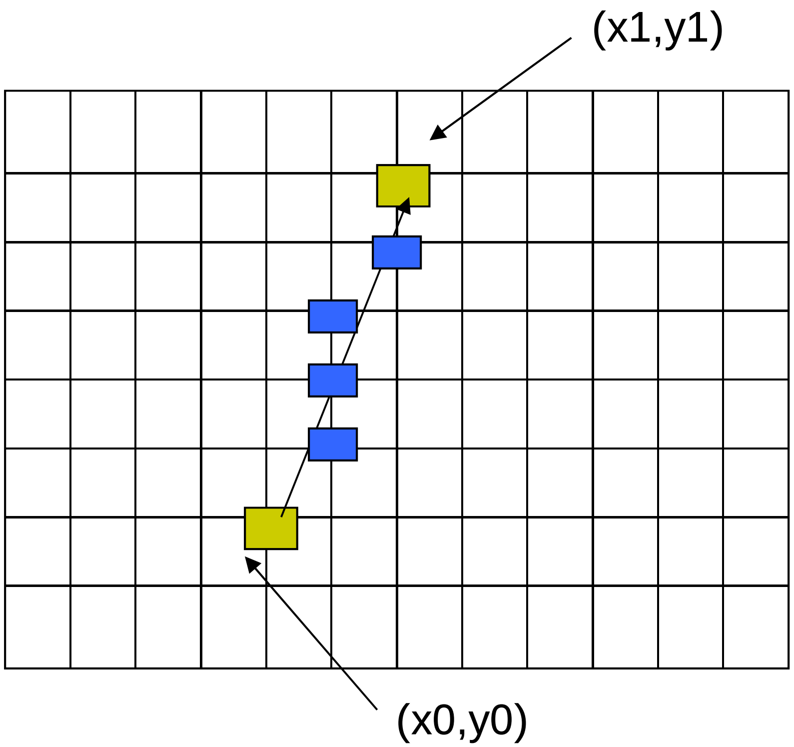
Until x == x1

Example, if first end point is (0,0)
Example, if m = 0.2
Step 1: x = 1, y = 0.2 => shade (1,0)
Step 2: x = 2, y = 0.4 => shade (2, 0)
Step 3: x= 3, y = 0.6 => shade (3, 1)
... etc

DDA Line Drawing Algorithm (Case b: m > 1)

$$m = \frac{\Delta y}{\Delta x} = \frac{y_{k+1} - y_k}{x_{k+1} - x_k} = \frac{1}{x_{k+1} - x_k}$$

$$\Rightarrow x_{k+1} = x_k + \frac{1}{m}$$



x = x0 y = y0

Illuminate pixel (round(x), y)

y = y + 1 x = x + 1/m

Illuminate pixel (round(x), y)

y = y + 1 x = x + 1 /m

Illuminate pixel (round(x), y)

...

Until y == y1

Example, if first end point is (0,0)
if 1/m = 0.2
Step 1: y = 1, x = 0.2 => shade (0,1)
Step 2: y = 2, x = 0.4 => shade (0, 2)
Step 3: y= 3, x = 0.6 => shade (1, 3)
... etc

DDA Line Drawing Algorithm Pseudocode

```
compute m;
if m < 1:
{
   float y = y0;        // initial value
   for(int x = x0;  x <= x1;  x++, y += m)
                  setPixel(x, round(y));
}
else   // m > 1
{
   float x = x0;        // initial value
   for(int y = y0;  y <= y1;  y++, x += 1/m)
                 setPixel(round(x), y);
}
```

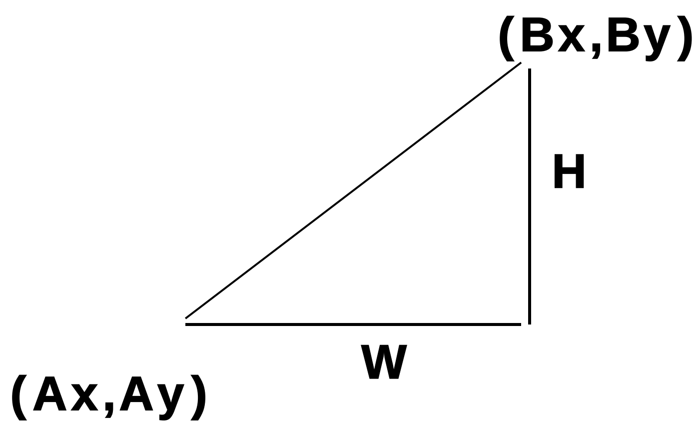
- **Note:** `setPixel(x, y)` writes current color into pixel in column x and row y in frame buffer

Line Drawing Algorithm Drawbacks

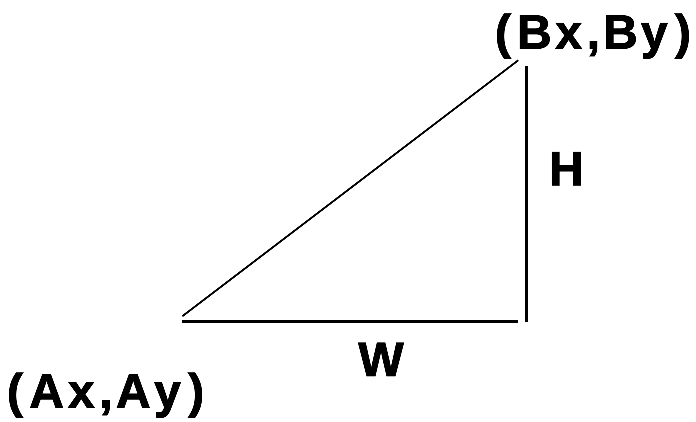
- DDA is the simplest line drawing algorithm
  - Not very efficient
  - Round operation is expensive
- Optimized algorithms typically used.
  - Integer DDA
  - E.g.Bresenham algorithm
- Bresenham algorithm
  - Incremental algorithm: current value uses previous value
  - Integers only: avoid floating point arithmetic
  - Several versions of algorithm: we'll describe midpoint version of algorithm

Bresenham's Line-Drawing Algorithm

- **Problem:** Given endpoints (Ax, Ay) and (Bx, By) of line, determine intervening pixels
- First make two simplifying assumptions (remove later):
  - (Ax < Bx) and
  - $(0 < m < 1)$
- Define
  - Width W = Bx – Ax
  - Height H = By - Ay



Bresenham's Line-Drawing Algorithm



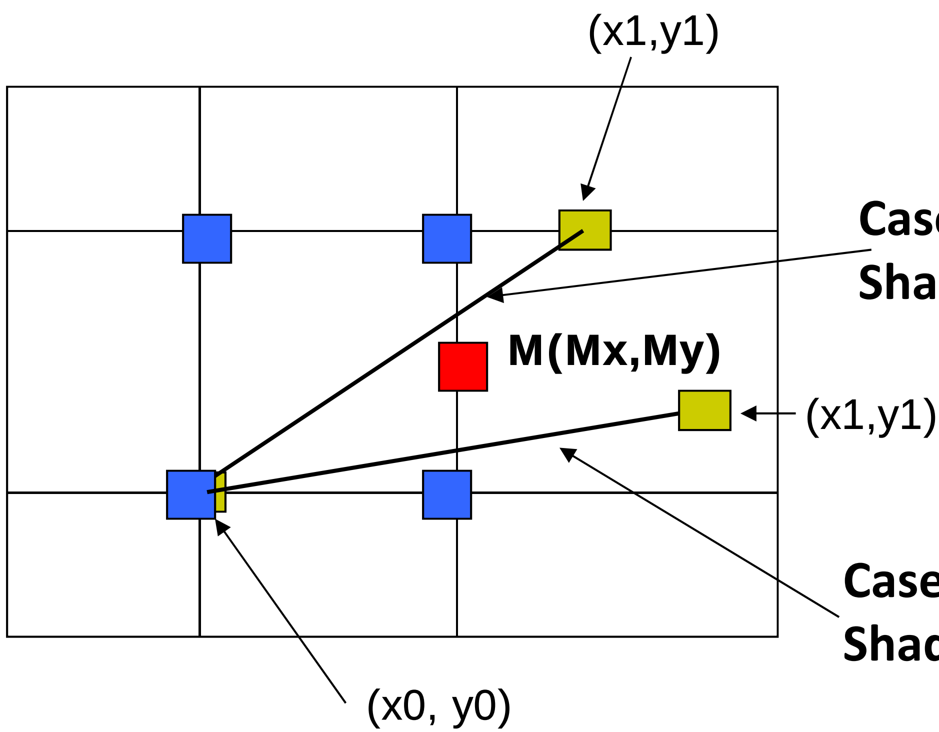
- Based on assumptions (Ax < Bx) and (0 < m < 1)
  - W, H are +ve
  - H < W
- Increment x by +1, y incr by +1 or stays same
- Midpoint algorithm determines which happens

Bresenham's Line-Drawing Algorithm

What Pixels to turn on or off?

Consider pixel midpoint M(Mx, My) = (x + 1, y + ½)

Build equation of actual line, compare to midpoint



Case a: If midpoint (red dot) is below line, Shade upper pixel, (x + 1, y + 1)

Case b: If midpoint (red dot) is above line, Shade lower pixel, (x + 1, y)

References

- Angel and Shreiner, Interactive Computer Graphics, 6th edition
- Hill and Kelley, Computer Graphics using OpenGL, 3rd edition, Chapter 9