Recall: Indexing into Cube Map

• Compute $R = 2(N \cdot V)N - V$
• Object at origin
• Use **largest magnitude component** of $R$ to determine face of cube
• Other 2 components give texture coordinates
Cube Map Layout
Cube Map Texture Lookup: Given an (s,t,p) direction vector, what (r,g,b) does that correspond to?

- Let \( L \) be the texture coordinate of \((s, t, p)\) with the largest magnitude.
- \( L \) determines which of the 6 2D texture “walls” is being hit by the vector\(-X\) in this case.
- The texture coordinates in that texture are the remaining two texture coordinates divided by \( L \): \((a/L, b/L)\).

Built-in GLSL functions

```glsl
vec3 ReflectVector = reflect( vec3 eyeDir, vec3 normal );

vec3 RefractVector = refract( vec3 eyeDir, vec3 normal, float Eta );
```
Example

- \( \mathbf{R} = (-4, 3, -1) \)
- Same as \( \mathbf{R} = (-1, 0.75, -0.25) \)
- Use face \( x = -1 \) and \( y = 0.75, z = -0.25 \)
- Not quite right since cube defined by \( x, y, z = \pm 1 \) rather than \([0, 1]\) range needed for texture coordinates
- Remap by from \([-1,1]\) to \([0,1]\) range
- \( s = \frac{1}{2} + \frac{1}{2} y, \ t = \frac{1}{2} + \frac{1}{2} z \)
- Hence, \( s = 0.875, \ t = 0.375 \)
Sphere Environment Map

- Cube can be replaced by a sphere (sphere map)
Sphere Mapping

- Original environmental mapping technique
- Proposed by Blinn and Newell
- Uses lines of longitude and latitude to map parametric variables to texture coordinates
- OpenGL supports sphere mapping
- Requires a circular texture map equivalent to an image taken with a fisheye lens
Sphere Map

- A sphere map is basically a photograph of a reflective sphere in an environment

Paul DeBevec, www.debevec.org
Sphere map

- example

Sphere map (texture)
Sphere map applied on torus
Capturing a Sphere Map

Matt Loper, MERL
Normal Mapping

- Store normals in texture
- Very useful for making low-resolution geometry look like it’s much more detailed

original mesh
4M triangles

simplified mesh
500 triangles

simplified mesh
and normal mapping
500 triangles
Introduction to Shadows

- Shadows give information on relative positions of objects.

Use ambient + diffuse + specular components

Use just ambient component
Introduction to Shadows

- Two popular shadow rendering methods:
  1. Shadows as texture (projection)
  2. Shadow buffer

- Third method used in ray-tracing (covered in grad class)
Projective Shadows

- Oldest method: Used in early flight simulators
- Projection of polygon is polygon called **shadow polygon**
Projective Shadows

- Works for flat surfaces illuminated by point light
- For each face, project vertices $V$ to find $V'$ of shadow polygon
- Object shadow $= \text{union of projections of faces}$
Projective Shadow Algorithm

- Project light-object edges onto plane

Algorithm:
- First, draw ground plane/scene using specular+diffuse+ambient components
- Then, draw shadow projections (face by face) using only ambient component
Projective Shadows for Polygon

1. If light is at \((x_l, y_l, z_l)\)
2. Vertex at \((x, y, z)\)
3. Would like to calculate shadow polygon vertex \(V\) projected onto ground at \((x_p, 0, z_p)\)

Ground plane: \(y = 0\)
Projective Shadows for Polygon

- If we move original polygon so that light source is at origin
- Matrix $M$ projects a vertex $V$ to give its projection $V'$ in shadow polygon

$$m = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 \\ -y_l & 0 & 0 \end{bmatrix}$$
Building Shadow Projection Matrix

1. Translate source to origin with $T(-x_l, -y_l, -z_l)$
2. Perspective projection
3. Translate back by $T(x_l, y_l, z_l)$

$$M = \begin{bmatrix}
1 & 0 & 0 & x_l \\
0 & 1 & 0 & y_l \\
0 & 0 & 1 & z_l \\
0 & 0 & 0 & 1 \\
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & -1 & 0 & 0 \\
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 & -x_l \\
0 & 1 & 0 & -y_l \\
0 & 0 & 1 & -z_l \\
0 & 0 & 0 & 1 \\
\end{bmatrix}$$

Final matrix that projects Vertex $V$ onto $V'$ in shadow polygon
Code snippets?

- Set up projection matrix in OpenGL application

```c
float light[3];  // location of light
mat4 m;        // shadow projection matrix initially identity

M[3][1] = -1.0/light[1];
```

\[
M = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & y_l & 0 \\
0 & -y_l & 0 & 0 \\
\end{bmatrix}
\]
Projective Shadow Code

- Set up object (e.g. a square) to be drawn

```cpp
point4 square[4] = {vec4(-0.5, 0.5, -0.5, 1.0)
    {vec4(-0.5, 0.5, -0.5, 1.0)
    {vec4(-0.5, 0.5, -0.5, 1.0)
    {vec4(-0.5, 0.5, -0.5, 1.0)
```

- Copy square to VBO
- Pass modelview, projection matrices to vertex shader
What next?

- Next, we load `model_view` as usual then draw original polygon.
- Then load shadow projection matrix, change color to black, re-render polygon.

1. Load modelview
draw polygon as usual

2. Modify modelview with Shadow projection matrix
Re-render as black (or ambient)
Shadow projection Display( ) Function

```c
void display( )
{
    mat4 mm;
    // clear the window
    glClear(GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT);

    // render red square (original square) using modelview matrix as usual (previously set up)
    glUniform4fv(color_loc, 1, red);
    glDrawArrays(GL_TRIANGLE_STRIP, 0, 4);
}
```
Shadow projection Display() Function

// modify modelview matrix to project square
// and send modified model_view matrix to shader
mm = model_view

    * Translate(light[0], light[1], light[2])
    *m

    * Translate(-light[0], -light[1], -light[2]);
glUniformMatrix4fv(matrix_loc, 1, GL_TRUE, mm);

// and re-render square as
// black square (or using only ambient component)
glUniform4fv(color_loc, 1, black);
glDrawArrays(GL_TRIANGLE_STRIP, 0, 4);
glutSwapBuffers();

\[
M = \begin{pmatrix}
1 & 0 & 0 & x_l \\
0 & 1 & 0 & y_l \\
0 & 0 & 1 & z_l \\
0 & 0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
1 & 0 & 0 & -x_l \\
0 & 1 & 0 & -y_l \\
0 & 0 & 1 & -z_l \\
0 & 0 & 0 & 1
\end{pmatrix}
\]
Shadow Buffer Approach

- Uses second depth buffer called **shadow buffer**
- Pros: not limited to plane surfaces
- Cons: needs lots of memory
- Depth buffer?
OpenGL Depth Buffer (Z Buffer)

- **Depth:** While drawing objects, depth buffer stores distance of each polygon from viewer.
- **Why?** If multiple polygons overlap a pixel, only closest one polygon is drawn.

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Setting up OpenGL Depth Buffer

- **Note:** You did this in order to draw solid cube, meshes

1. `glutInitDisplayMode(GLUT_DEPTH | GLUT_RGB)` instructs OpenGL to create depth buffer

2. `glEnable(GL_DEPTH_TEST)` enables depth testing

3. `glClear(GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT)`
   
   Initializes depth buffer every time we draw a new picture
Shadow Buffer Theory

- Along each path from light
  - Only closest object is lit
  - Other objects on that path in shadow
- Shadow buffer stores closest object on each path
Shadow Buffer Approach

- Rendering in two stages:
  - Loading shadow buffer
  - Render the scene
Loading Shadow Buffer

- Initialize each element to 1.0
- Position a camera at light source
- Rasterize each face in scene updating closest object
- Shadow buffer tracks smallest depth on each path
Shadow Buffer (Rendering Scene)

- Render scene using camera as usual
- While rendering a pixel find:
  - pseudo-depth $D$ from light source to $P$
  - Index location $[i][j]$ in shadow buffer, to be tested
  - Value $d[i][j]$ stored in shadow buffer
- If $d[i][j] < D$ (other object on this path closer to light)
  - point $P$ is in shadow
  - lighting = ambient
- Otherwise, not in shadow
  - Lighting = $amb + diffuse + specular$
Loading Shadow Buffer

- Shadow buffer calculation is independent of eye position
- In animations, shadow buffer loaded once
- If eye moves, no need for recalculation
- If objects move, recalculation required
Soft Shadows

- Point light sources => simple hard shadows, unrealistic
- Extended light sources => more realistic
- Shadow has two parts:
  - Umbra (Inner part) => no light
  - Penumbra (outer part) => some light
Fog example

- Fog is atmospheric effect
  - Better realism, helps determine distances
Fog

- Fog was part of OpenGL fixed function pipeline
- Programming fixed function fog
  - **Parameters:** Choose fog color, fog model
  - **Enable:** Turn it on
- Fixed function fog **deprecated!!**
- Shaders can implement even better fog
- **Shaders implementation:** fog applied in fragment shader just before display
Rendering Fog

- Mix some color of fog: $\mathbf{c}_f + \text{color of surface: } \mathbf{c}_s$

\[ \mathbf{c}_p = f\mathbf{c}_f + (1 - f)\mathbf{c}_s \quad f \in [0,1] \]

- If $f = 0.25$, output color = 25% fog + 75% surface color
  - $f$ computed as function of distance $z$
  - 3 ways: linear, exponential, exponential-squared

- Linear:
  \[ f = \frac{Z_{end} - Z_p}{Z_{end} - Z_{start}} \]

$Z_{start}$, $Z_{end}$, $Z_p$
Fog Shader Fragment Shader Example

float dist = abs(Position.z);
Float fogFactor = (Fog.maxDist - dist)/
    Fog.maxDist - Fog.minDist);
fogFactor = clamp(fogFactor, 0.0, 1.0);

vec3 shadeColor = ambient + diffuse + specular
vec3 color = mix(Fog.color, shadeColor,fogFactor);
FragColor = vec4(color, 1.0);

\[ f = \frac{Z_{end} - Z_p}{Z_{end} - Z_{start}} \]

\[ \mathbf{c}_p = f \mathbf{c}_f + (1 - f) \mathbf{c}_s \]
**Fog**

- **Exponential** \( f = e^{-d_f z_p} \)
- **Squared exponential** \( f = e^{-(d_f z_p)^2} \)
- **Exponential derived from Beer’s law**
  - **Beer’s law**: intensity of outgoing light diminishes exponentially with distance

![Graph](image-url)

**fog factor equations**

- linear
- exp2 0.33
- exp2 0.66
- exp 0.33
- exp 0.66

**Axes:**
- Surface color amount
- Relative distance
Fog Optimizations

- $f$ values for different depths ($z_p$) can be pre-computed and stored in a table on GPU.
- Distances used in $f$ calculations are planar.
- Can also use Euclidean distance from viewer or radial distance to create \textit{radial fog}.
References

- Real Time Rendering by Akenine-Moller, Haines and Hoffman