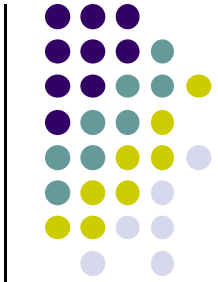
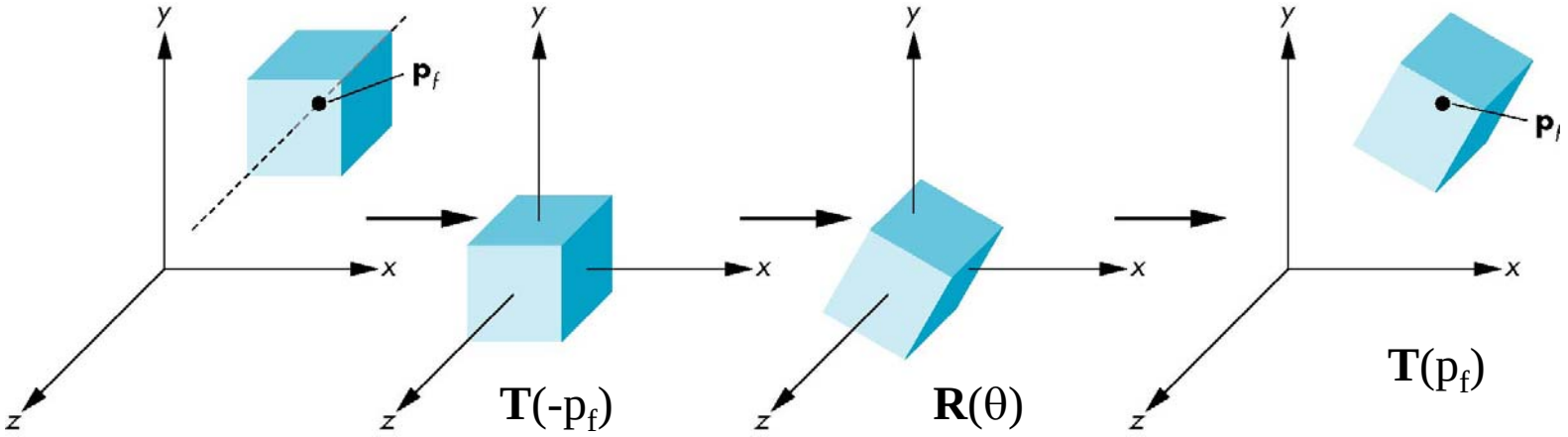
Rotation About Arbitrary Point other than the Origin

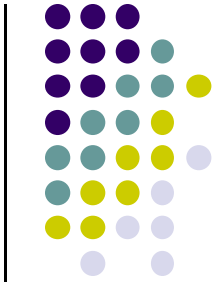


- Default rotation matrix is about origin
- How to rotate about any arbitrary point p_f (Not origin)?
  - Move fixed point to origin $\mathbf{T}(-p_f)$
  - Rotate $\mathbf{R}(\theta)$
  - Move fixed point back $\mathbf{T}(p_f)$

So, M = $\mathbf{T}(p_f)\ \mathbf{R}(\theta)\ \mathbf{T}(-p_f)$



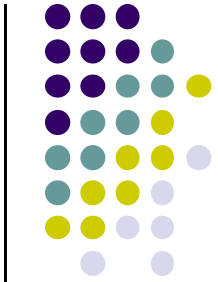
Scale about Arbitrary Center



- Similary, default scaling is about origin
- To scale about arbitrary point P = (Px, Py, Pz) by (Sx, Sy, Sz)
  1. **Translate** object by T(-Px, -Py, -Pz) so P coincides with origin
  2. **Scale** object by (Sx, Sy, Sz)
  3. **Translate** object back: T(Px, Py, Py)
- In matrix form: T(Px,Py,Pz) (Sx, Sy, Sz) T(-Px,-Py,-Pz) * P

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & Px \\ 0 & 1 & 0 & Py \\ 0 & 0 & 1 & Pz \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -Px \\ 0 & 1 & 0 & -Py \\ 0 & 0 & 1 & -Pz \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Example



- Rotation about z axis by 30 degrees about a fixed point (1.0, 2.0, 3.0)

```
mat 4 m = Identity();
m = Translate(1.0, 2.0, 3.0)*
  Rotate(30.0, 0.0, 0.0, 1.0)*
  Translate(-1.0, -2.0, -3.0);
```

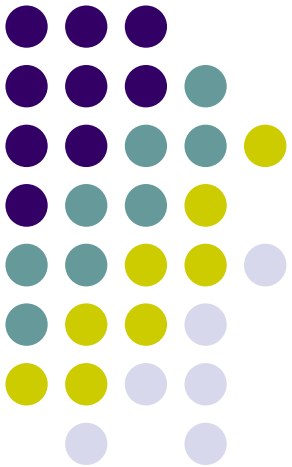
- Remember last matrix specified in program (i.e. translate matrix in example) is first applied

Computer Graphics (CS 4731)
Lecture 11: Hierarchical 3D Models

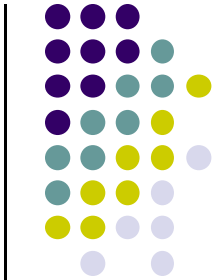
Prof Emmanuel Agu

Computer Science Dept.

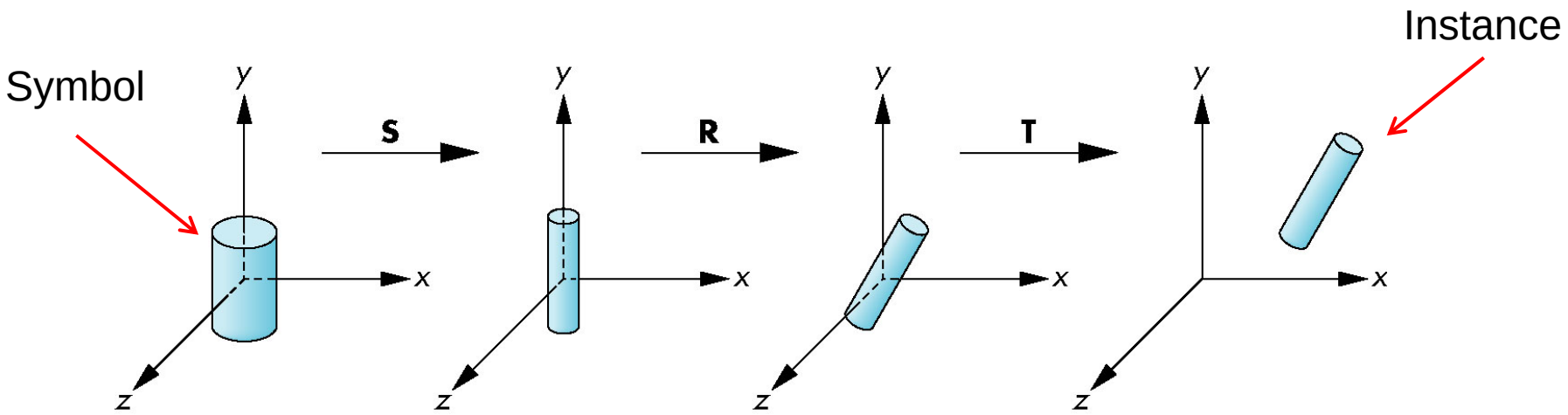
Worcester Polytechnic Institute (WPI)



Instance Transformation



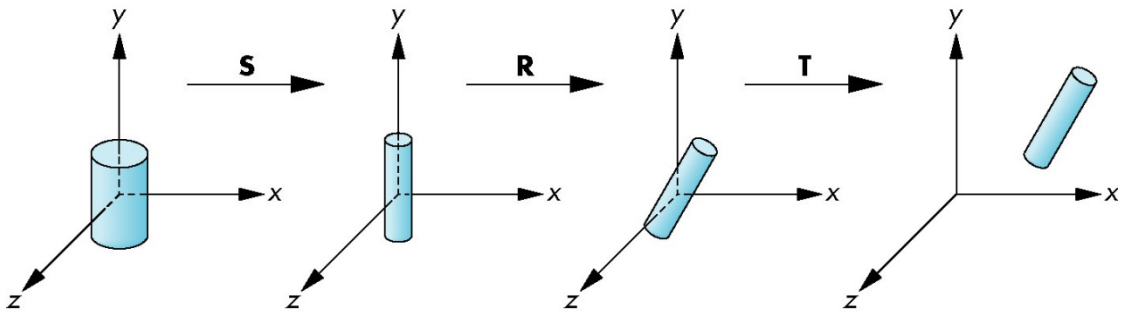
- Start with unique object (a *symbol*)
- Each appearance of object in model is an ***instance***
  - Must scale, orient, position
  - Defines instance transformation



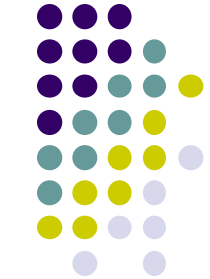
Symbol-Instance Table

Can store **intances** + **instance transformations**

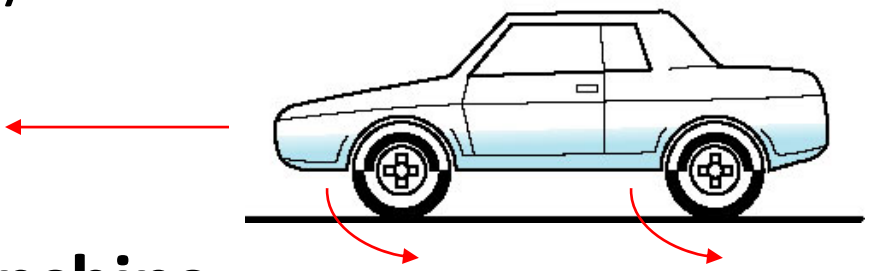
Symbol	Scale	Rotate	Translate
1	s_x, s_y, s_z	$\theta_x, \theta_y, \theta_z$	d_x, d_y, d_z
2			
3			
1			
1			
.			
.			



Problems with Symbol-Instance Table

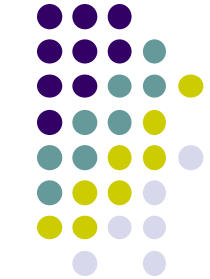


- Symbol-instance table does not show relationships between parts of model
- Consider model of car
  - Chassis (body) + 4 identical wheels
  - Two symbols

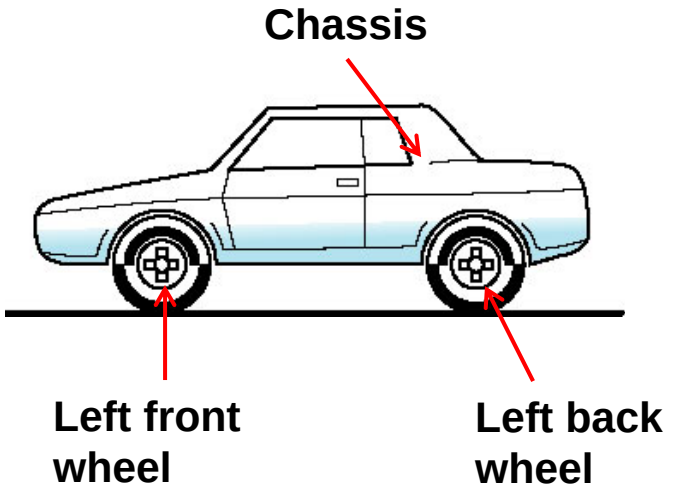


- **Relationships:**
  - Wheels connected to chassis
  - Chassis motion determined by rotational speed of wheels

Structure Program Using Function Calls?

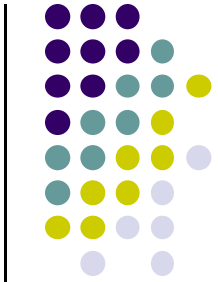


```
car(speed)
{
    chassis()
    wheel(right_front);
    wheel(left_front);
    wheel(right_rear);
    wheel(left_rear);
}
```

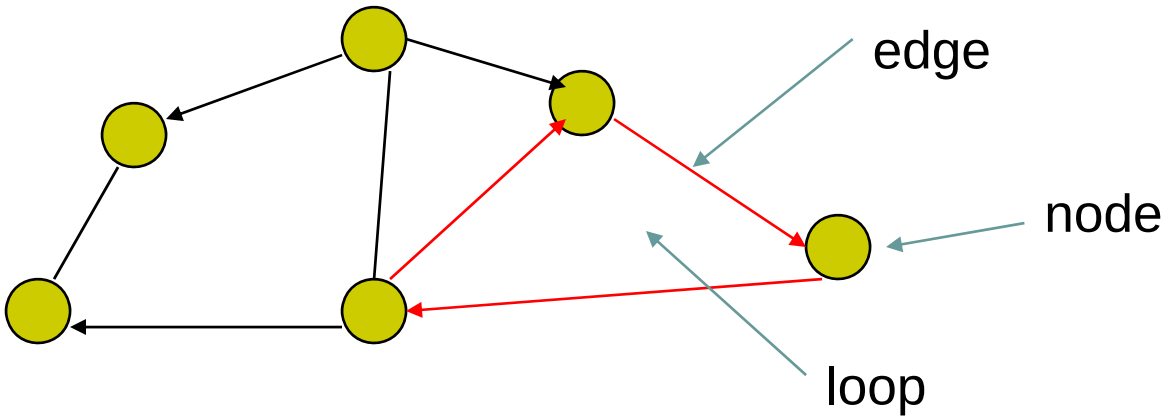


- Fails to show relationships between parts
- Look into graph representation

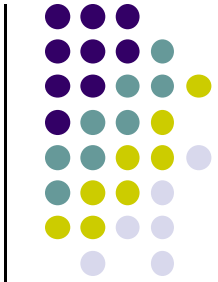
Graphs



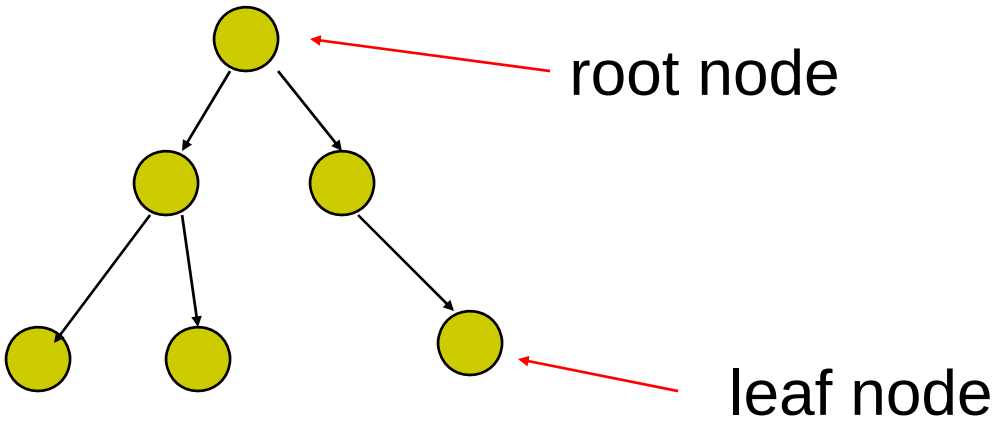
- Set of ***nodes*** + ***edges (links)***
- **Edge** connects a pair of nodes
  - Directed or undirected
- ***Cycle*****:** directed path that is a loop



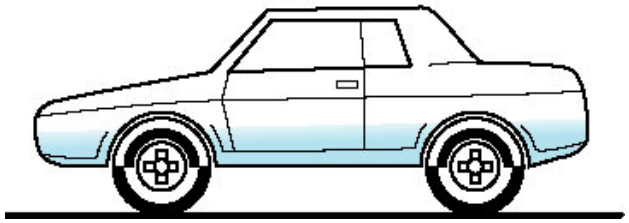
Tree

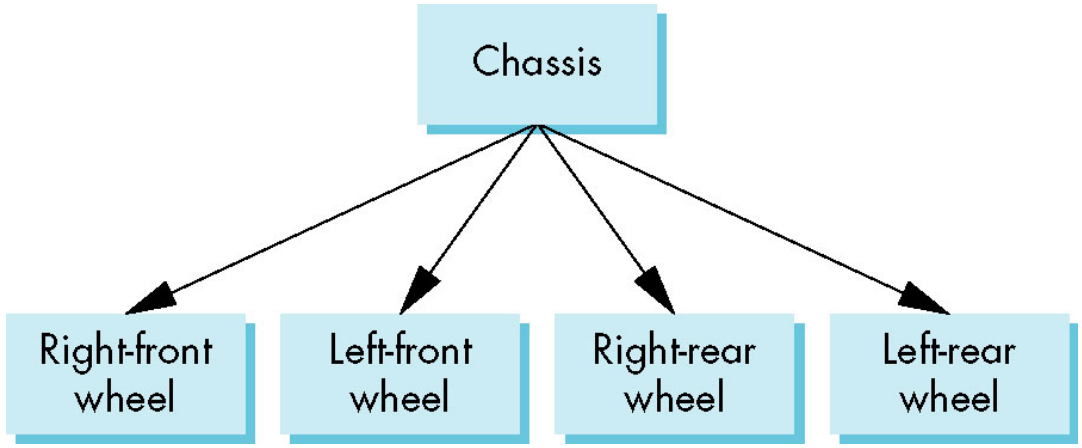


- Graph in which each node (except root) has exactly one parent node
  - A parent may have multiple children
  - Leaf node: no children

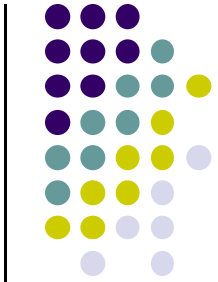


Tree Model of Car

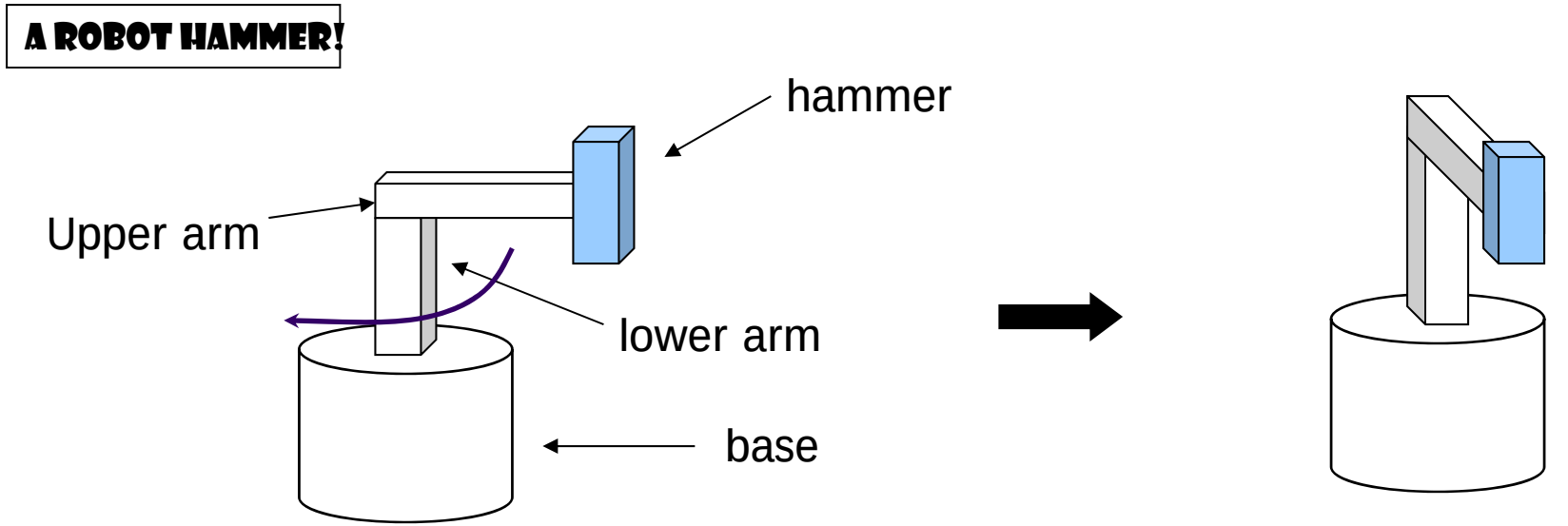




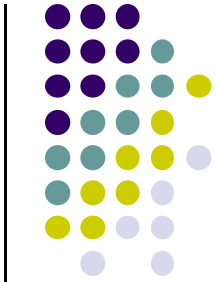
Hierarchical Transforms



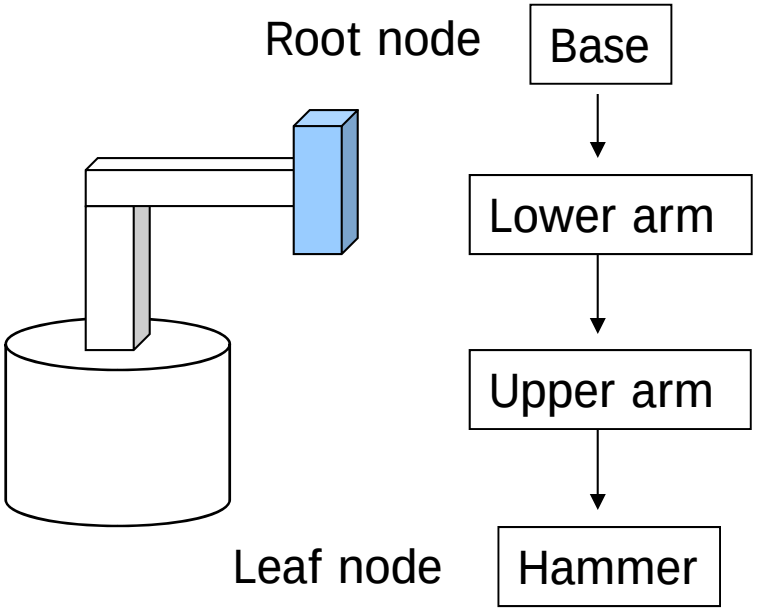
- **Robot arm:** Many small **connected** parts
- Attributes (position, orientation, etc) depend on each other



Hierarchical Transforms



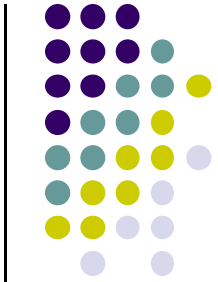
- Object dependency description using tree structure



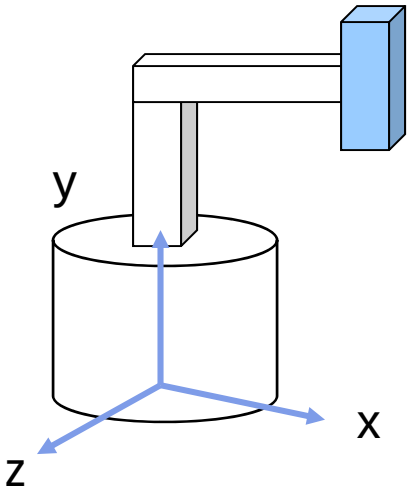
Object position and orientation can be affected by its parent, grand-parent, grand-grand-parent ... nodes

Hierarchical representation is known as a **Scene Graph**

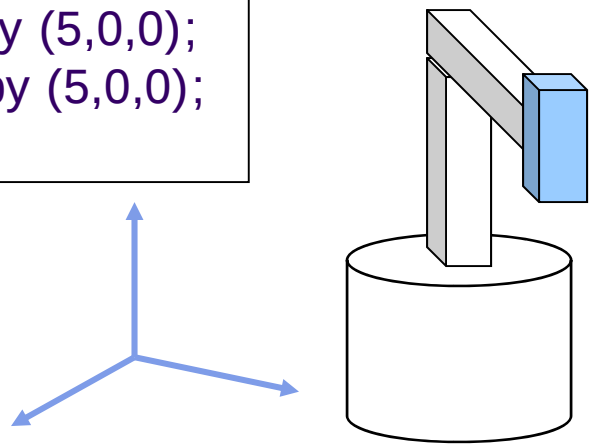
Transformations



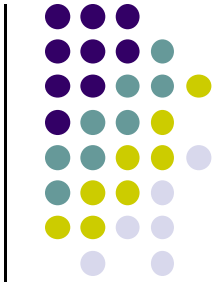
- Two ways to specify transformations:
  - **(1) Absolute transformation:** each part transformed independently (relative to origin)




```
Translate the base by (5,0,0);
Translate the lower arm by (5,0,0);
Translate the upper arm by (5,0,0);
...
```

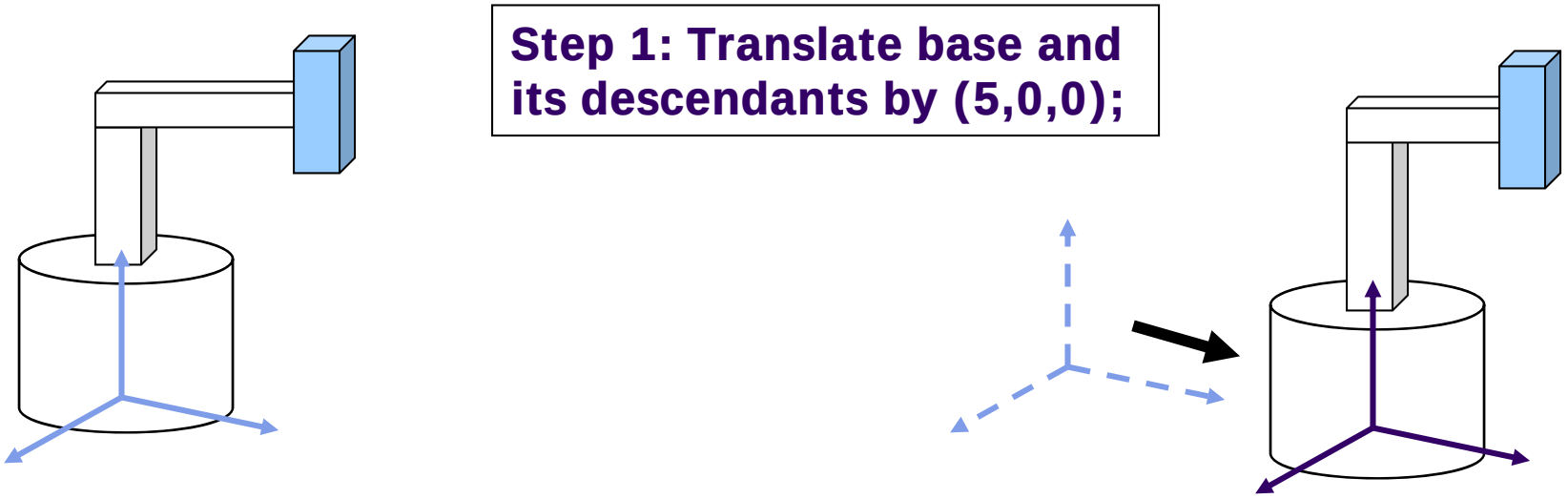


Relative Transformation

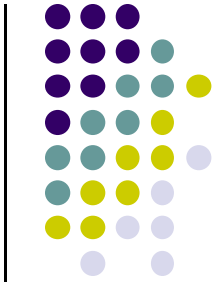


A better (and easier) way:

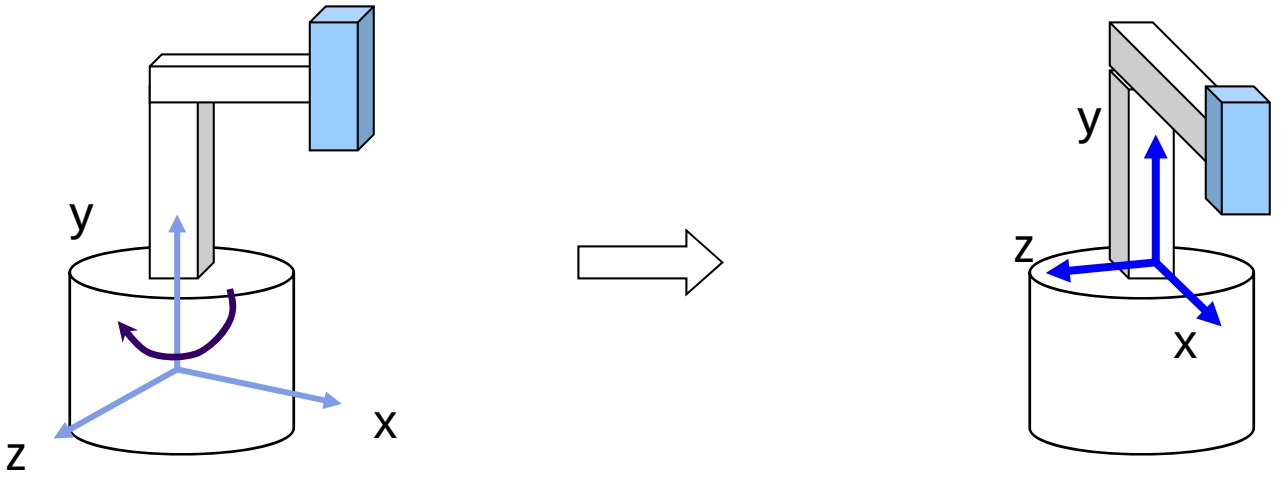
(2) **Relative transformation:** Specify transformation for each object relative to its parent



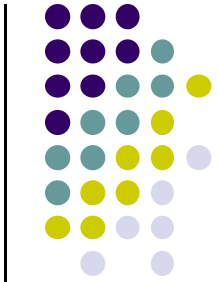
Relative Transformation



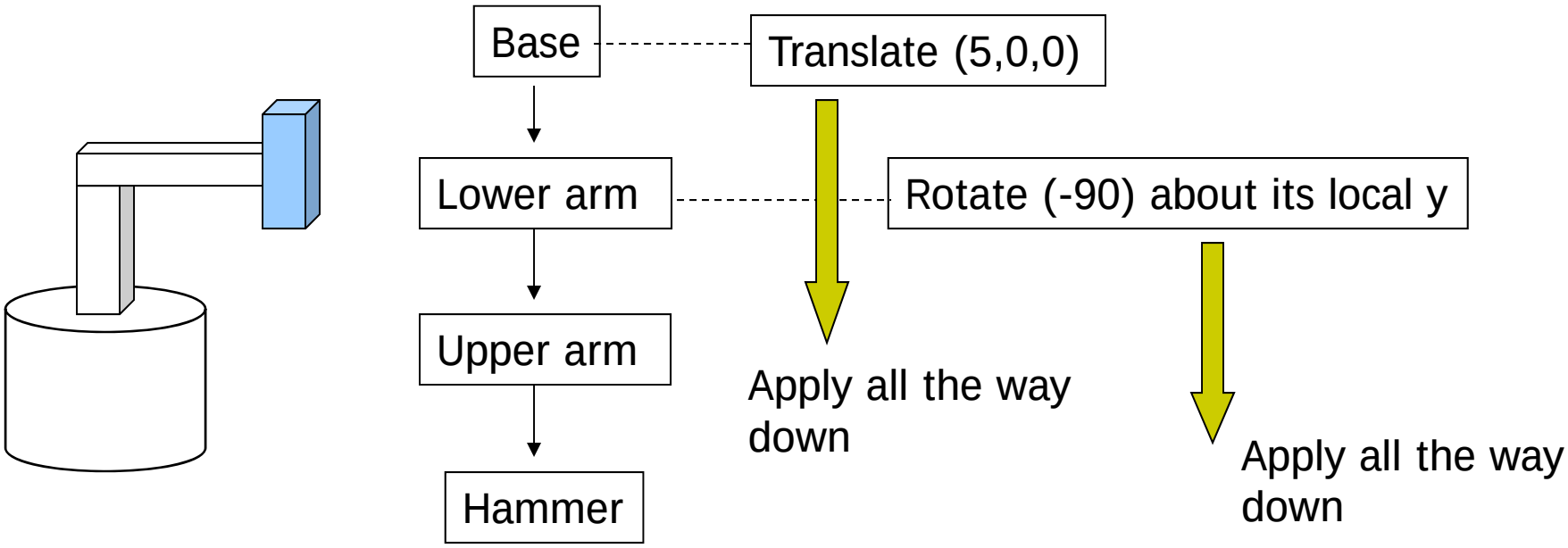
Step 2: Rotate the lower arm and all its descendants relative to the base's local y axis by -90 degree



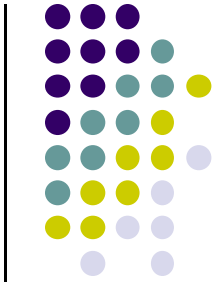
Relative Transformation



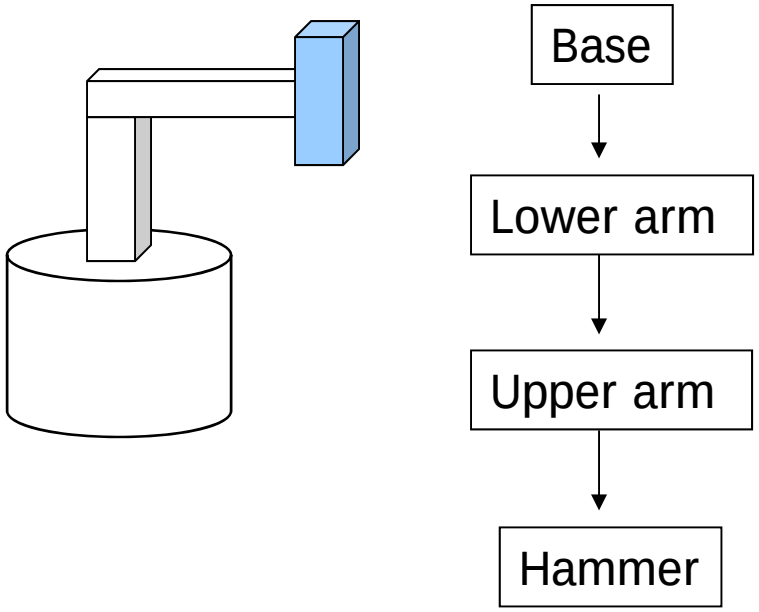
- Relative transformation using scene graph



Hierarchical Transforms Using OpenGL

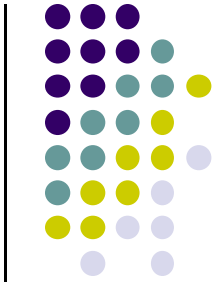


- Translate base and all its descendants by (5,0,0)
- Rotate lower arm and its descendants by -90 degree about local y

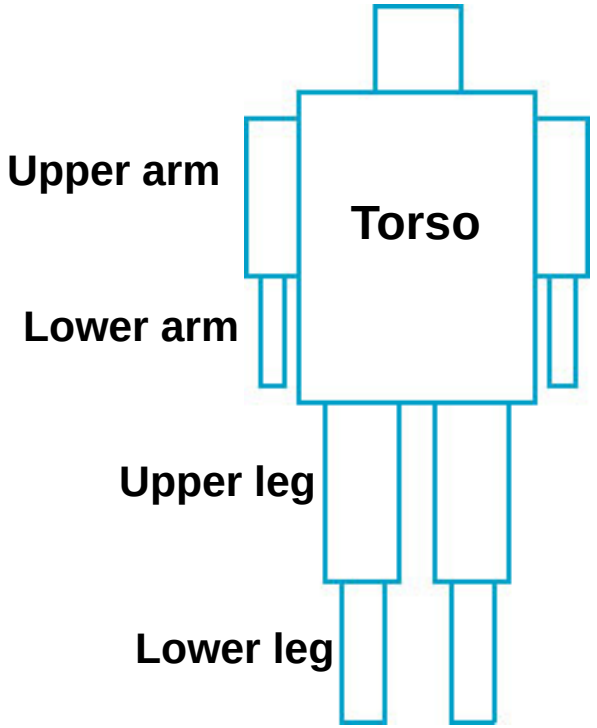



```
ctm = LoadIdentity();

... // setup your camera

ctm = ctm * Translatef(5,0,0);

Draw_base();

ctm = ctm * Rotatef(-90, 0, 1, 0);

Draw_lower _arm();
Draw_upper_arm();
Draw_hammer();
```

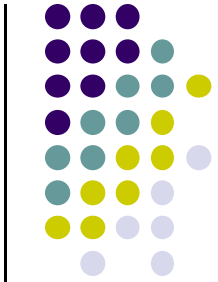
Hierarchical Modeling



- For large objects with many parts, need to transform **groups** of objects
- Need better tools



Hierarchical Modeling

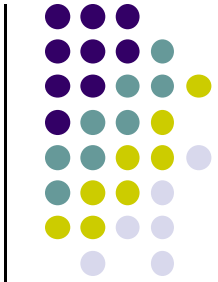


- Previous CTM had 1 level
- **Hierarchical modeling:** extend CTM to stack with multiple levels using linked list
- Manipulate stack levels using 2 operations
  - pushMatrix
  - popMatrix

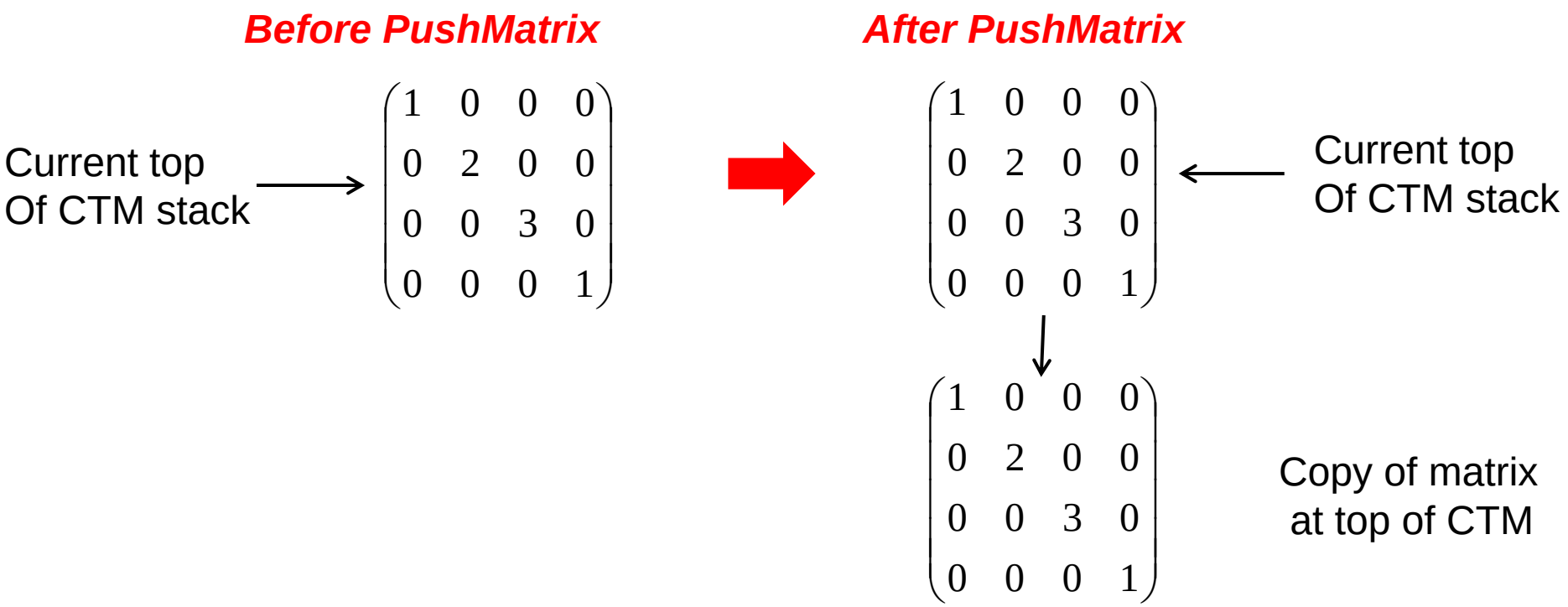
Current top
Of CTM stack

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

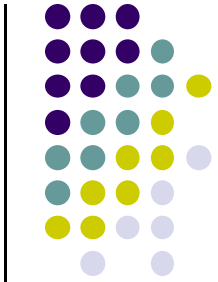
PushMatrix



- **PushMatrix( ):** Save current modelview matrix (CTM) in stack
- Positions 1 & 2 in linked list **are same** after PushMatrix



PushMatrix



- Further Rotate, Scale, Translate affect only top matrix
- E.g. **`ctm = ctm * Translate (3,8,6)`**

After PushMatrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

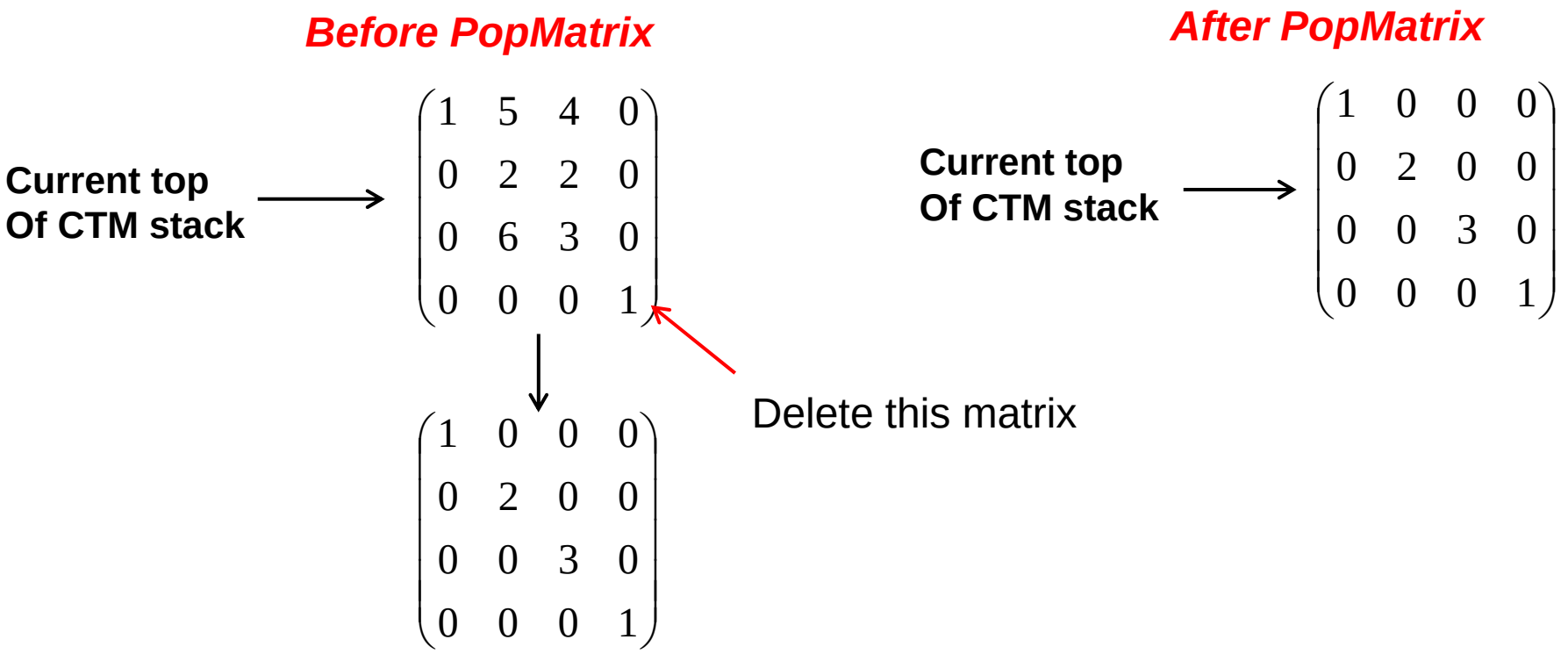
Translate(3,8,6) applied only to current top Of CTM stack

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Matrix in second position saved. Unaffected by Translate(3,8,6)

PopMatrix

- **PopMatrix( ):** Delete position 1 matrix, position 2 matrix becomes top



PopMatrix and PushMatrix Illustration

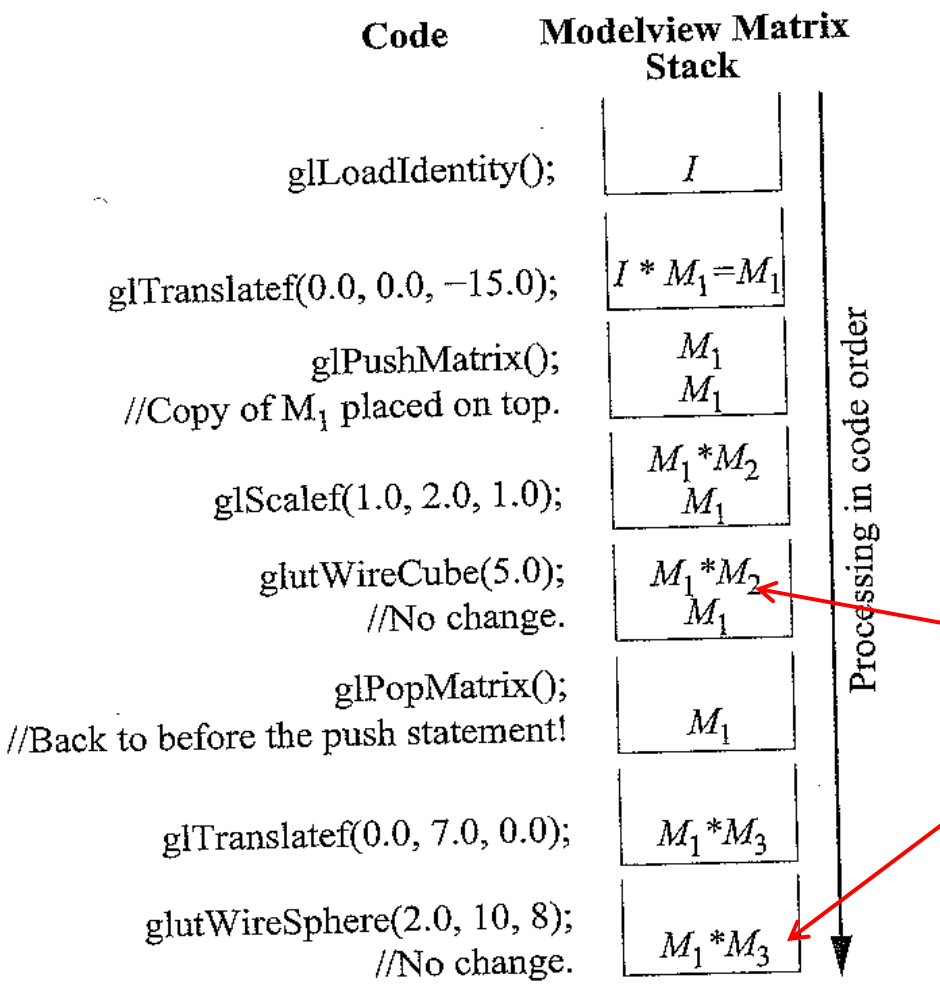


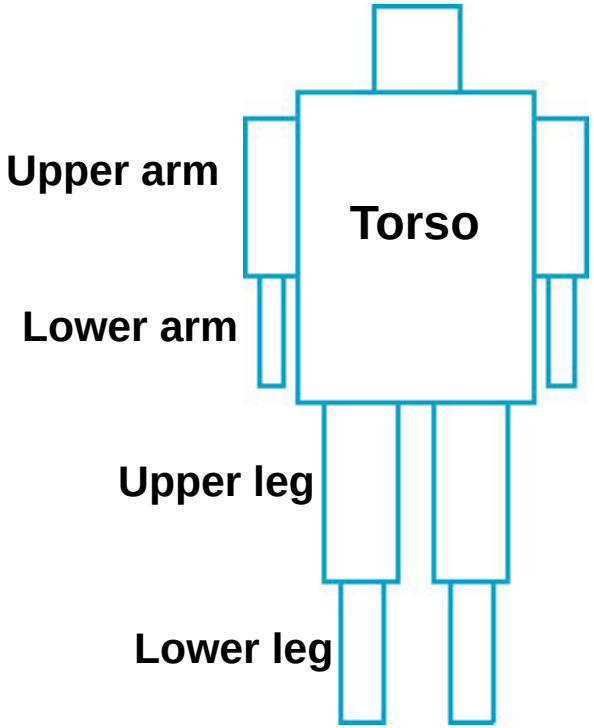
Figure 4.19: Transitions of the modelview matrix stack.

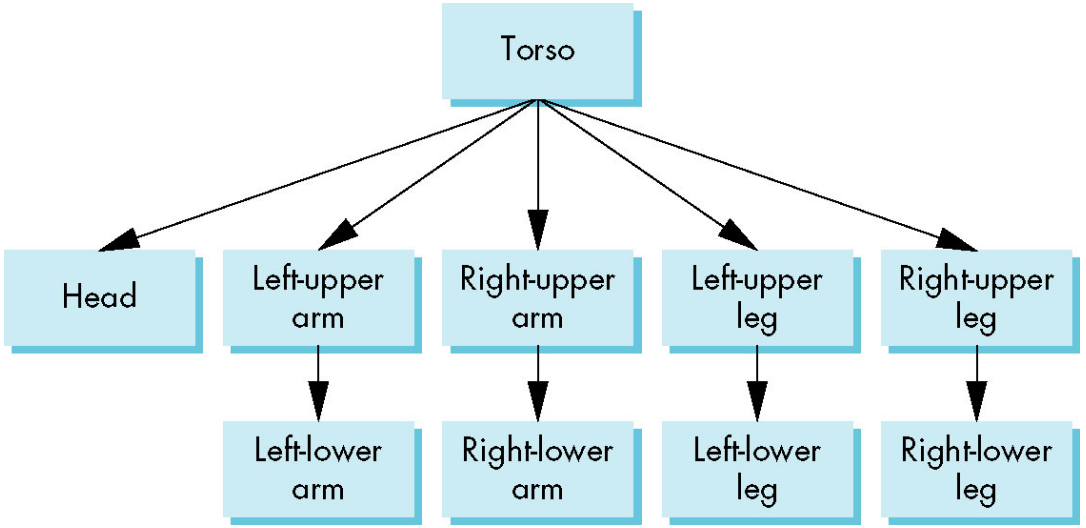
- **Note:** Diagram uses old glTranslate, glScale, etc commands
- We want same behavior though

Apply matrix at top of CTM to vertices of object created

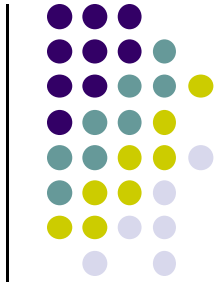
Ref: Computer Graphics Through OpenGL by Guha

Humanoid Figure

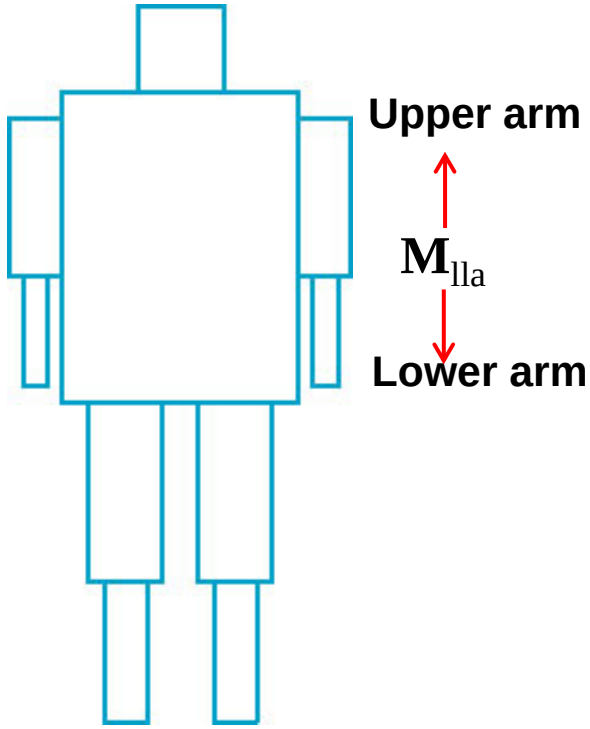




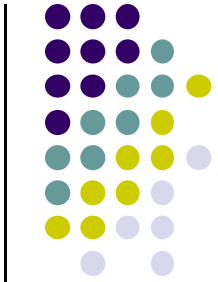
Building the Model



- Draw each part as a function
  - `torso()`
  - `left_upper_arm(), etc`
- **Transform Matrices:** transform of node wrt its parent
  - $\mathbf{M}_{lla}$ positions left lower arm with respect to left upper arm
- Stack based traversal (push, pop)



Draw Humanoid using Stack

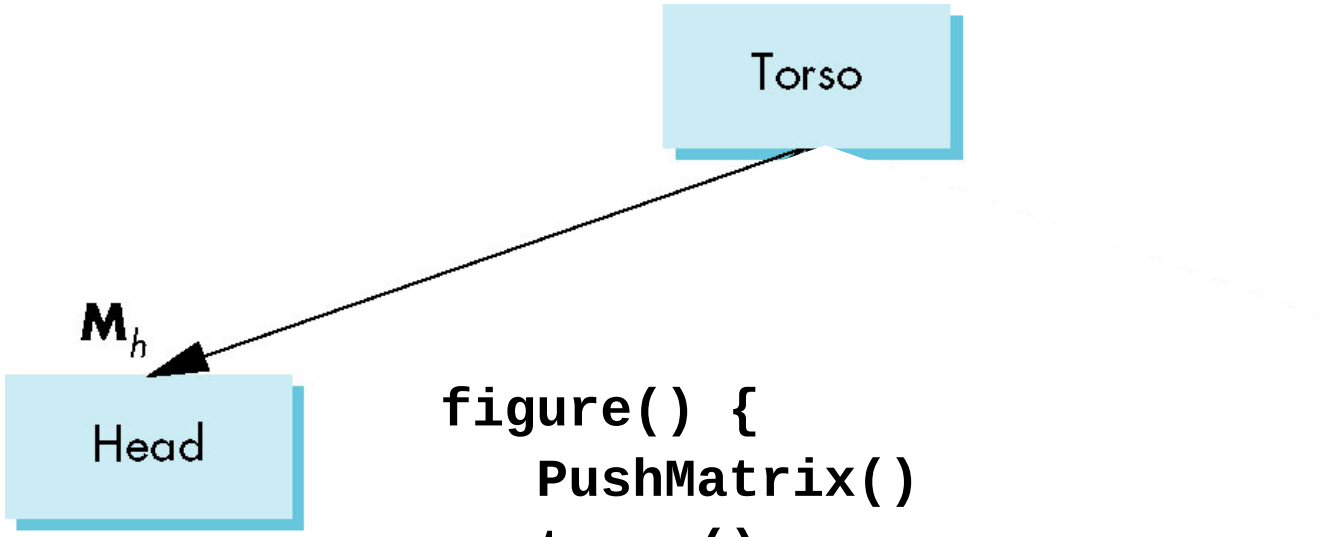



```
figure() {
    PushMatrix()
    torso();
```

save present model-view matrix

draw torso

Draw Humanoid using Stack

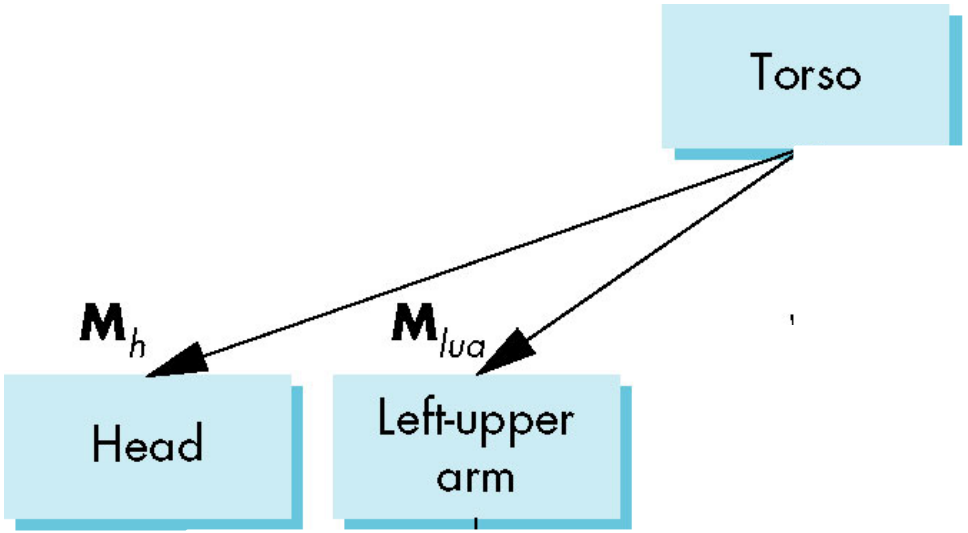



```
figure() {
    PushMatrix()
    torso();
    Rotate (…);
    head();
```

(M_h) Transformation of head Relative to torso

draw head

Draw Humanoid using Stack



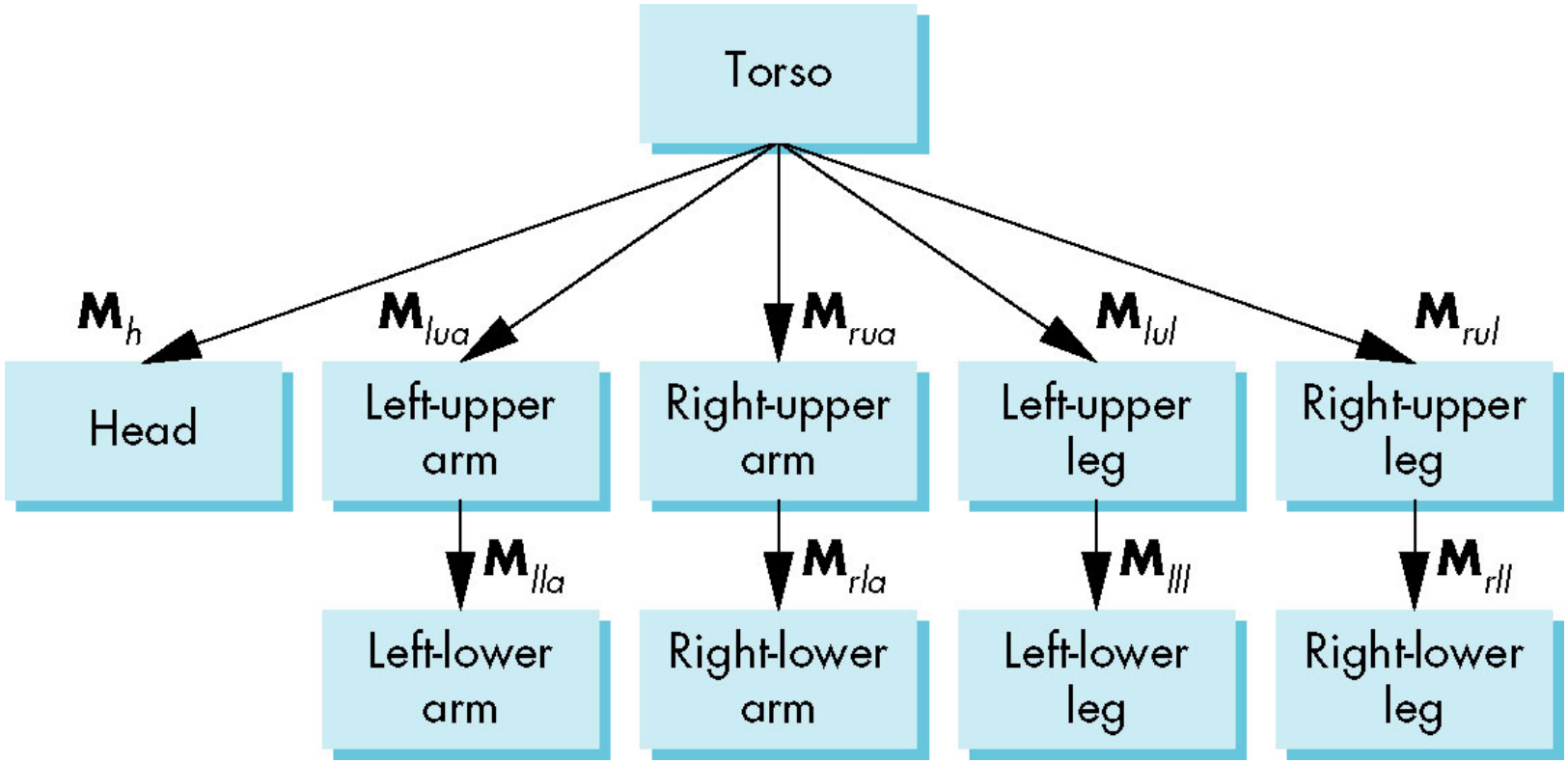
Go back to torso matrix, and save it again

(M_{lua}) Transformation(s) of left upper arm relative to torso

draw left-upper arm

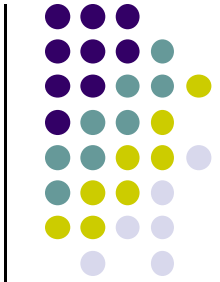
```
PushMatrix()
torso();
Rotate (…);
head();
PopMatrix();
PushMatrix();
Translate(…);
Rotate(…);
left_upper_arm();
……..
// rest of code()
```

Complete Humanoid Tree with Matrices



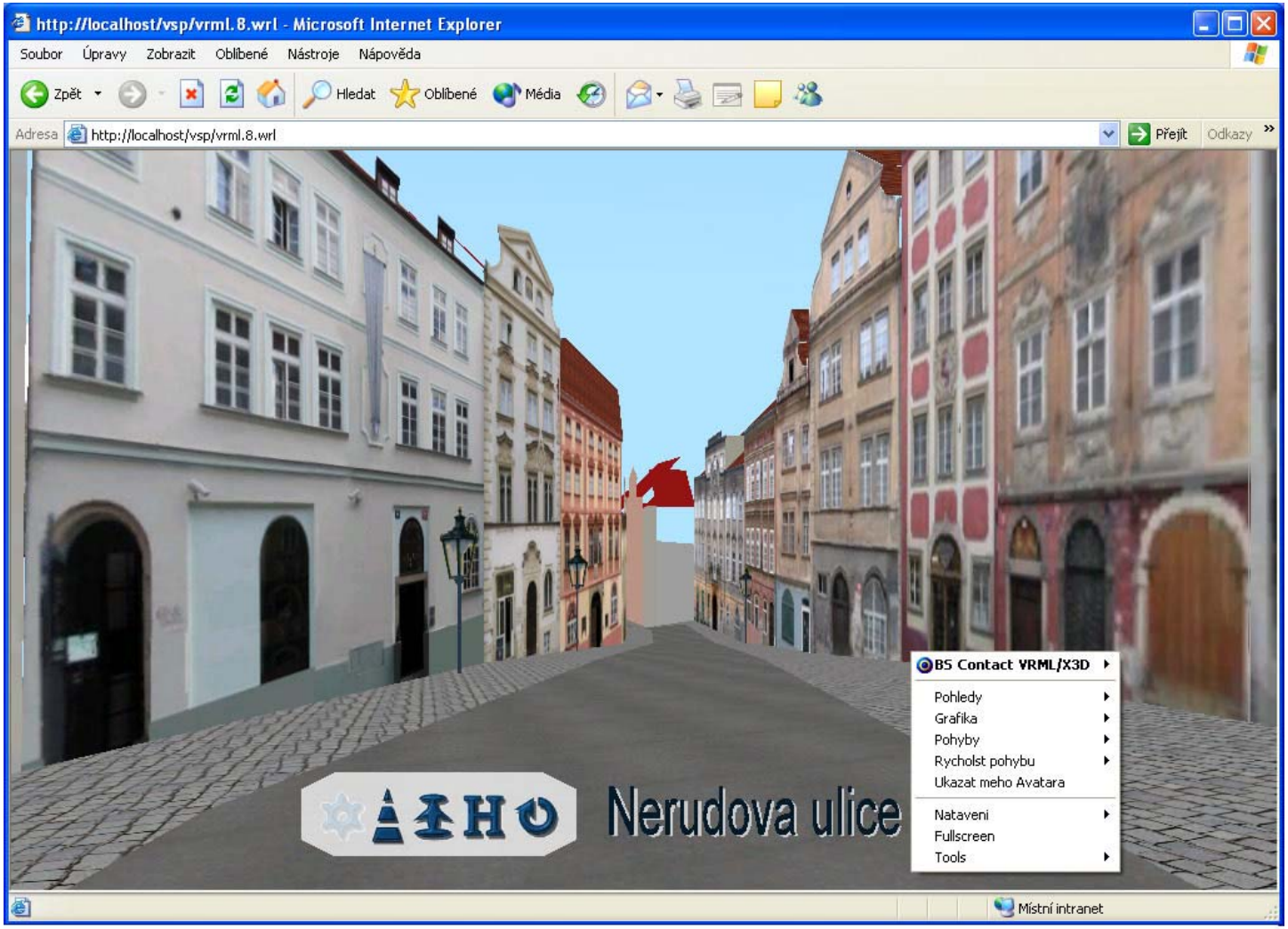
Scene graph of Humanoid Robot

VRML

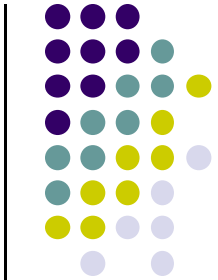


- Scene graph introduced by SGI Open Inventor
- Used in many graphics applications (Maya, etc)
- Want scene graph for World Wide Web
- Need links scene parts in distributed data bases
- Virtual Reality Markup Language
  - Based on Inventor data base
  - Implemented with OpenGL

VRML World Example



References



- Angel and Shreiner, Interactive Computer Graphics (6^{th} edition), Chapter 8