

Logic: Syntax and Semantics

Class 3

Formalization

Dan Dougherty

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translation from English into logic formal language

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BTW we haven't finished tableaux

- quantifiers!
- will come back to that

Why work with formulas?

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Why work with formulas?

1. writing arguments out in English is tedious
2. formalizing things leads to algorithms (recall our “forty-three times eleven” example)
3. translation gives insights into ordinary language
 - ▶ this is the fun part

start with basic (boring) translations

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then tackle subtleties

English and Logical Symbols

not	is written as	$\neg$
and	is written as	$\wedge$
or	is written as	$\vee$
if ... then ...	is written as	$\rightarrow$
every / for all	is written as	$\forall$
some / there exists	is written as	$\exists$

We abbreviate atomic English sentences with capital letters.

Conjunction

Example 1

English: Alice is tired and Bob is hungry.

Key: T = Alice is tired; H = Bob is hungry

Example 1

English: Alice is tired and Bob is hungry.

Key: T = Alice is tired; H = Bob is hungry

Translation: $T \wedge H$

Example 2

English: The cat is asleep and the dog is awake.

Key: C = The cat is asleep; D = The dog is awake

Example 2

English: The cat is asleep and the dog is awake.

Key: C = The cat is asleep; D = The dog is awake

Translation: $C \wedge D$

Example 3

English: The soup is hot and the spoon is cold.

Key: H = The soup is hot; C = The spoon is cold

Example 3

English: The soup is hot and the spoon is cold.

Key: H = The soup is hot; C = The spoon is cold

Translation: $H \wedge C$

Try going in the reverse direction.

Use this key:

Key: R = It is raining; S = My shoes are wet

Start with this logic formula:

$R \wedge S$

What is the corresponding English sentence?

Try going in the reverse direction.

Use this key:

Key: R = It is raining; S = My shoes are wet

Start with this logic formula:

$R \wedge S$

What is the corresponding English sentence?

Answer: *It is raining and my shoes are wet.*

Highly recommended!
Make up a couple of your own examples

no subtleties there ...

Conditional

Example 4

English: If it rains, then the picnic is canceled.

Key: R = It rains; C = The picnic is canceled

Example 4

English: If it rains, then the picnic is canceled.

Key: R = It rains; C = The picnic is canceled

Translation: $R \rightarrow C$

Example 5

English: If Alice studies, then Alice passes.

Key: S = Alice studies; P = Alice passes

Example 5

English: If Alice studies, then Alice passes.

Key: S = Alice studies; P = Alice passes

Translation: $S \rightarrow P$

Example 6

English: If the alarm rings, then the dog barks.

Key: A = The alarm rings; B = The dog barks

Example 6

English: If the alarm rings, then the dog barks.

Key: A = The alarm rings; B = The dog barks

Translation: $A \rightarrow B$

Example 7

English: If the dragon sneezes, then the knight runs.

Key: S = The dragon sneezes; R = The knight runs

Example 7

English: If the dragon sneezes, then the knight runs.

Key: S = The dragon sneezes; R = The knight runs

Translation: $S \rightarrow R$

Again, try going in the reverse direction.

Use this key:

Key: R = It is raining; S = My shoes are wet

Start with this logic formula:

$$R \rightarrow S$$

What is the corresponding English sentence?

Again, try going in the reverse direction.

Use this key:

Key: R = It is raining; S = My shoes are wet

Start with this logic formula:

$$R \rightarrow S$$

What is the corresponding English sentence?

Answer: *If it is raining then my shoes are wet.*

Reverse direction... read carefully

Use this key:

Key: R = It is raining; S = My shoes are wet

Start with this logic formula:

$$S \rightarrow R$$

What is the corresponding English sentence?

Reverse direction... read carefully

Use this key:

Key: R = It is raining; S = My shoes are wet

Start with this logic formula:

$$S \rightarrow R$$

What is the corresponding English sentence?

Answer: *If my shoes are wet then it is raining then*

You should see that—in actual conversation—this says something different from the previous exercise.

Highly recommended!
Make up a couple of your own examples

no real subtleties if-then, except

no real subtleties if-then, except
don't confuse "if-then" with "implies"
(we'll discuss why later)

Negation

Example 8

English: It is not the case that Alice is asleep.

Key: A = Alice is asleep

Example 8

English: It is not the case that Alice is asleep.

Key: A = Alice is asleep

Translation: $\neg A$

Example 9

The English in that example was stilted, though it shows off exactly what the “not” is doing. In more colloquial language:

English: Alice is not asleep.

Key: A = Alice is asleep

Example 9

The English in that example was stilted, though it shows off exactly what the “not” is doing. In more colloquial language:

English: Alice is not asleep.

Key: A = Alice is asleep

Translation: $\neg A$

Different English, same logic.

(But we *know* different English sentences can mean the same thing, right?)

Example 10

English: The moon isn't made of cheese.

Key: C = The moon is made of cheese

Example 10

English: The moon isn't made of cheese.

Key: C = The moon is made of cheese

Translation: $\neg C$

Example 11

English: It was not Bert who won the race.

Key: W = Bert won the race

Example 11

English: It was not Bert who won the race.

Key: W = Bert won the race

Translation: $\neg W$

Reverse direction.

Use this key:

Key: R = It is raining

Start with this logic formula:

$\neg R$

What is the corresponding English sentence?

Reverse direction.

Use this key:

Key: R = It is raining

Start with this logic formula:

$\neg R$

What is the corresponding English sentence?

Answer: *It is not raining.*

Reverse direction.

Use this key:

Key: R = It is raining

Start with this logic formula:

$\neg\neg R$

What is the corresponding English sentence?

Reverse direction.

Use this key:

Key: R = It is raining

Start with this logic formula:

$\neg\neg R$

What is the corresponding English sentence?

Answer: *It is not not raining.*

Reverse direction.

Use this key:

Key: R = It is raining

Start with this logic formula:

$\neg\neg R$

What is the corresponding English sentence?

Answer: *It is not not raining.*

Or more colloquially : *It isn't true that it isn't raining.*

Reverse direction.

Use this key:

Key: R = It is raining

Start with this logic formula:

$\neg\neg R$

What is the corresponding English sentence?

Answer: *It is not not raining.*

Or more colloquially : *It isn't true that it isn't raining.*

Or more dramatically: *What to do mean, it's not raining, are you nuts?*

Ok, I'm just playing around there....

Highly recommended!
Make up a couple of your own examples

Negation is a little subtle

The English word “not” is very flexible!

Sometimes, in the English, the “not” is applied to the whole sentence (as in, “It is not the case that ...”

Sometimes, in the English, the “not” is applied to the verb (as in the green cheese example)

Sometimes (in the Bert example) it's applied to a noun!

But at the **logic** level it's always applied to the whole sentence.

Disjunction

Two flavors!

Example 12

English: Alice is in Paris or Bob is in Rome.

Key: P = Alice is in Paris; R = Bob is in Rome

Example 12

English: Alice is in Paris or Bob is in Rome.

Key: P = Alice is in Paris; R = Bob is in Rome

If the speaker means “or maybe both” (aka “and/or”) :

Translation: $P \vee R$

If the speaker means “but not both”:

Translation: $P \oplus R$

SUPER IMPORTANT

It is not logic's job to read the mind of the speaker!

Logic comes into the picture **after** any English ambiguity is resolved.

Example 13

English: The monster is friendly or the monster is imaginary.

Key: F = The monster is friendly; I = The monster is imaginary

Example 13

English: The monster is friendly or the monster is imaginary.

Key: F = The monster is friendly; I = The monster is imaginary

If the speaker means “or maybe both”

Translation: $F \vee I$

If the speaker means “but not both”:

Translation: $F \oplus I$

Example 14

English: Either Alice gets the last cookie or Bob gets it.

Key: A = Alice gets the last cookie; B = Bob gets the last cookie

Example 14

English: Either Alice gets the last cookie or Bob gets it.

Key: A = Alice gets the last cookie; B = Bob gets the last cookie

Since it is pretty clear that the speaker means “but not both”:

Translation: $A \oplus B$

Example 15

English: The Red Sox will win or the Yankees will win

Key: R = The Red Sox will win; Y = The Yankees will win

Example 15

English: The Red Sox will win or the Yankees will win

Key: R = The Red Sox will win; Y = The Yankees will win

If the speaker means “and/or” :

Translation: $R \vee Y$

E.g., suppose the Red Sox are playing the Rays and the Yankees are playing the Orioles that day.

If the speaker means “either/or” :

Translation: $R \oplus Y$

E.g., suppose the Red Sox and Yankees are playing each other that day.

Example 16

English: The switch is on or it is off.

Key: O = The switch is on; F = The switch is off

Example 16

English: The switch is on or it is off.

Key: O = The switch is on; F = The switch is off

It's a good bet that the speaker means "either/or":

Translation: $O \oplus F$

Reverse direction.

Use this key:

Key: U = I'm going to understand logic; D = I'm going to go crazy

Start with these logic formulas:

1. $U \oplus S$

2. $U \vee S$

What are the corresponding English sentences?

Reverse direction.

Use this key:

Key: U = I'm going to understand logic; D = I'm going to go crazy

Start with these logic formulas:

1. $U \oplus S$

2. $U \vee S$

What are the corresponding English sentences?

Answers:

1. Either I'm going to understand logic or I'm going to go crazy!
2. I'm going to understand logic or I'm going to go crazy, or maybe both!

Highly recommended!
Make up a couple of your own examples

The speaker's meaning has to be determined *before* we can do a logic translation

If all we have is a transcript, a text, there is no true right or wrong for a translation

a very short digression about the idea of “ambiguity”

A subtlety about ambiguity

Often “ambiguity” is confused with “being vague”

But no, almost always an “ambiguous” sentence has two possible meanings, and **these meanings are each in themselves perfectly clear**

It's not “vagueness”. It's just a choice between two well-defined alternatives.

The metaphor for ambiguity

- is NOT: “an unmarked field where you can walk in any direction you want”
- but RATHER: “a fork in the road: choose this path or that path”

Two or More Propositional Connectives

Example 17

English: Alice is tired and Bob is hungry and Carol is asleep.

Key: T = Alice is tired; H = Bob is hungry; S = Carol is asleep

Example 17

English: Alice is tired and Bob is hungry and Carol is asleep.

Key: T = Alice is tired; H = Bob is hungry; S = Carol is asleep

Translation: $T \wedge H \wedge S$

Example 18

English: Alice is in Paris or Bob is in Rome or Carol is in London.

Key: P = Alice is in Paris; R = Bob is in Rome; L = Carol is in London

Example 18

English: Alice is in Paris or Bob is in Rome or Carol is in London.

Key: P = Alice is in Paris; R = Bob is in Rome; L = Carol is in London

Translation: $P \vee R \vee L$

Example 19

English: Alice is not tired and Bob is hungry.

Key: T = Alice is tired; H = Bob is hungry

Example 19

English: Alice is not tired and Bob is hungry.

Key: T = Alice is tired; H = Bob is hungry

Translation: $\neg T \wedge H$

Example 20

English: It is not the case that Alice is tired and Bob is hungry.

Key: T = Alice is tired; H = Bob is hungry

Example 20

English: It is not the case that Alice is tired and Bob is hungry.

Key: T = Alice is tired; H = Bob is hungry

Translation: $\neg(T \wedge H)$

Why do we use those parentheses? To make it clear that is the whole expression “Alice is tired and Bob is hungry” that is being negated.

If we wrote $\neg T \wedge H$ it would look like the previous Example 19, where the English meant “Alice is not tired, and (also) Bob is hungry”

To *precisely* explicate the difference between these two formulas

$$\neg T \wedge H$$

$$\neg(T \wedge H)$$

Describe two possible worlds:

1. a world where the first formula is true and the second is false
2. a world where the first formula is false and the second is true

To “describe a possible world” here simply means:

say whether Alice is tried or not and then say whether Bob is hungry or not

Example 21

English: If Alice is tired, then Bob is not hungry.

Key: T = Alice is tired; H = Bob is hungry

Example 21

English: If Alice is tired, then Bob is not hungry.

Key: T = Alice is tired; H = Bob is hungry

Translation: $T \rightarrow \neg H$

Example 22

English: If Alice is not tired, then Bob is hungry.

Key: T = Alice is tired; H = Bob is hungry

Example 22

English: If Alice is not tired, then Bob is hungry.

Key: T = Alice is tired; H = Bob is hungry

Translation: $\neg T \rightarrow H$

Example 23

English: It is not the case that if Alice studies, then Bob passes.

Key: S = Alice studies; P = Bob passes

Example 23

English: It is not the case that if Alice studies, then Bob passes.

Key: S = Alice studies; P = Bob passes

Translation: $\neg(S \rightarrow P)$

Example 24

English: If Alice doesn't study, then Bob passes.

Key: S = Alice studies; P = Bob passes

Example 24

English: If Alice doesn't study, then Bob passes.

Key: S = Alice studies; P = Bob passes

Translation: $\neg S \rightarrow P$

Compare with Example 23

Example 25

English: If Alice studies, then Bob passes or Carol complains.

Key: S = Alice studies; P = Bob passes; C = Carol complains

Example 25

English: If Alice studies, then Bob passes or Carol complains.

Key: S = Alice studies; P = Bob passes; C = Carol complains

Translation: $S \rightarrow (P \vee C)$

Example 26

English: If Alice studies then Bob passes, or Carol complains.

Key: S = Alice studies; P = Bob passes; C = Carol complains

Example 26

English: If Alice studies then Bob passes, or Carol complains.

Key: S = Alice studies; P = Bob passes; C = Carol complains

Translation: $(S \rightarrow P) \vee C$

Compare to the Example 26.

Look at the work the comma does in the English sentences in these last two examples!

It's not a "logic" thing: it is there to help convey the speaker's intention (outside of language)

Example 27

English: Either Alice gets the cookie or Bob gets the cookie, and Carol gets the milk.

Key: A = Alice gets the cookie; B = Bob gets the cookie; C = Carol gets the milk

Example 27

English: Either Alice gets the cookie or Bob gets the cookie, and Carol gets the milk.

Key: A = Alice gets the cookie; B = Bob gets the cookie; C = Carol gets the milk

Translation: $(A \oplus B) \wedge C$

Example 28

English: It is not the case that exactly one of Alice and Bob is asleep.

Key: A = Alice is asleep; B = Bob is asleep

Example 28

English: It is not the case that exactly one of Alice and Bob is asleep.

Key: A = Alice is asleep; B = Bob is asleep

Translation: $\neg(A \oplus B)$

Example 29

English: Alice is tired or Bob is hungry, and Carol is asleep.

Key: T = Alice is tired; H = Bob is hungry; S = Carol is asleep

Example 29

English: Alice is tired or Bob is hungry, and Carol is asleep.

Key: T = Alice is tired; H = Bob is hungry; S = Carol is asleep

Translation: $(T \vee H) \wedge S$

Example 30

English: Alice is tired, and Bob is hungry or Carol is asleep.

Key: T = Alice is tired; H = Bob is hungry; S = Carol is asleep

Example 30

English: Alice is tired, and Bob is hungry or Carol is asleep.

Key: T = Alice is tired; H = Bob is hungry; S = Carol is asleep

Translation: $T \wedge (H \vee S)$

Example 31

English: Alice is tired or Bob is hungry and Carol is asleep.

Key: T = Alice is tired; H = Bob is hungry; S = Carol is asleep

Example 31

English: Alice is tired or Bob is hungry and Carol is asleep.

Key: T = Alice is tired; H = Bob is hungry; S = Carol is asleep

Translation: $T \vee (H \wedge S)$

Example 32

English: Alice is tired or Bob is hungry, and Carol is asleep.

Key: T = Alice is tired; H = Bob is hungry; S = Carol is asleep

Example 32

English: Alice is tired or Bob is hungry, and Carol is asleep.

Key: T = Alice is tired; H = Bob is hungry; S = Carol is asleep

Translation: $(T \vee H) \wedge S$