Logic: Syntax and Semantics
Class 5
Application: Clinical Decision Support

Dan Dougherty

October 18, 2026 – 10 : 25

Logic in Medicine

A sore throat can have many causes.

Some combinations of symptoms and test results point to strep throat.

What does this have to do with logic?

A Patient with a Sore Throat

Suppose the patient's electronic medical record says:

> ***The patient has a sore throat and fever.***

The examination and testing also find:

> ***Swollen neck lymph nodes and a positive rapid strep test.***

What can be inferred?

A Clinical Decision-Support Rule

Suppose the system has been given a (simplified!) rule:

If

1. the patient has a sore throat and fever
2. and has swollen neck lymph nodes
3. and a positive rapid strep test

Then

the patient has strep throat.

Formalizing the Rule

Let

T = the patient has a sore throat
F = the patient has a fever
L = neck lymph nodes are swollen
R = a rapid strep test is positive
S = the patient has strep throat

The rule then becomes the logic formula

$$(T \wedge F \wedge L \wedge R) \rightarrow S$$

A Particular Patient

We have the medical rule we just formalized.

$$(T \wedge F \wedge L \wedge R) \rightarrow S$$

This is “static”, it lives in a database somewhere.

A Particular Patient

We have the medical rule we just formalized.

$$(T \wedge F \wedge L \wedge R) \rightarrow S$$

This is “static”, it lives in a database somewhere.

Then suppose we see a particular patient, and we learn

A Particular Patient

We have the medical rule we just formalized.

$$(T \wedge F \wedge L \wedge R) \rightarrow S$$

This is “static”, it lives in a database somewhere.

Then suppose we see a particular patient, and we learn

T *(this patient has a sore throat)*

A Particular Patient

We have the medical rule we just formalized.

$$(T \wedge F \wedge L \wedge R) \rightarrow S$$

This is “static”, it lives in a database somewhere.

Then suppose we see a particular patient, and we learn

T *(this patient has a sore throat)*

L *(their neck lymph nodes are swollen)*

A Particular Patient

We have the medical rule we just formalized.

$$(T \wedge F \wedge L \wedge R) \rightarrow S$$

This is “static”, it lives in a database somewhere.

Then suppose we see a particular patient, and we learn

T *(this patient has a sore throat)*

L *(their neck lymph nodes are swollen)*

F *(this patient has a fever)*

A Particular Patient

We have the medical rule we just formalized.

$$(T \wedge F \wedge L \wedge R) \rightarrow S$$

This is “static”, it lives in a database somewhere.

Then suppose we see a particular patient, and we learn

T *(this patient has a sore throat)*

L *(their neck lymph nodes are swollen)*

F *(this patient has a fever)*

R *(their rapid strep test is positive)*

Now We Have an Argument

$(T \wedge F \wedge L \wedge R) \rightarrow S$	*(the medical rule)*
T	*(patient has a sore throat)*
L	*(neck lymph nodes are swollen)*
F	*(patient has a fever)*
R	*(rapid strep test is positive)*
S	*(patient has strep throat)*

Now We Have an Argument

$(T \wedge F \wedge L \wedge R) \rightarrow S$	*(the medical rule)*
T	*(patient has a sore throat)*
L	*(neck lymph nodes are swollen)*
F	*(patient has a fever)*
R	*(rapid strep test is positive)*
S	*(patient has strep throat)*

The argument is valid.

Now We Have an Argument

$(T \wedge F \wedge L \wedge R) \rightarrow S$	*(the medical rule)*
T	*(patient has a sore throat)*
L	*(neck lymph nodes are swollen)*
F	*(patient has a fever)*
R	*(rapid strep test is positive)*
S	*(patient has strep throat)*

The argument is valid.

All the hypotheses are true in the real world

Now We Have an Argument

$(T \wedge F \wedge L \wedge R) \rightarrow S$	*(the medical rule)*
T	*(patient has a sore throat)*
L	*(neck lymph nodes are swollen)*
F	*(patient has a fever)*
R	*(rapid strep test is positive)*
S	*(patient has strep throat)*

The argument is valid.

All the hypotheses are true in the real world

So this patient has strep

More interesting rules

The simple rule we just saw had the form

$$(A \wedge B \wedge C) \rightarrow D.$$

But clinical rules can use richer combinations of conditions:

$$((A \vee B) \wedge \neg C) \rightarrow D$$
$$E \wedge (F \vee G) \rightarrow H$$
$$T \wedge F \wedge L \wedge R \rightarrow S$$

Real rule engines combine conditions using **AND, OR, and NOT**, and can test numerical thresholds.

A Fragment of a Rule Base

Illustrative fragment — not literal diagnostic rules

$$(\mathit{SoreThroat} \wedge \mathit{Fever} \wedge \mathit{PositiveStrepTest}) \rightarrow \mathit{StrepThroat}$$

$$(\mathit{ItchyEyes} \wedge \mathit{Sneezing} \wedge \mathit{PollenSeason}) \rightarrow \mathit{SeasonalAllergy}$$

$$(\mathit{EarPain} \wedge \mathit{BulgingEardrum}) \rightarrow \mathit{MiddleEarInfection}$$

$$(\mathit{Heartburn} \wedge \mathit{AfterMeals} \wedge \neg \mathit{Exertional}) \rightarrow \mathit{AcidReflux}$$

$$(\mathit{PainfulRash} \wedge \mathit{OneSided} \wedge \mathit{Blisters}) \rightarrow \mathit{Shingles}$$

A Fragment of a Rule Base

More of the same illustrative rule base

$$(Fatigue \wedge LowHemoglobin \wedge LowFerritin) \rightarrow IronDeficiencyAnemia$$
$$(Congestion \wedge FacialPressure \wedge Symptoms > 10 Days) \rightarrow Sinusitis$$
$$(Cough \wedge RunnyNose \wedge \neg PositiveStrepTest) \rightarrow ViralSoreThroat$$
$$(RedEye \wedge Discharge \wedge Itching) \rightarrow Conjunctivitis$$
$$(JointPain \wedge RecentInjury \wedge Swelling) \rightarrow Sprain$$

Now imagine thousands of active rules.

Clinical Decision Support

A real clinical knowledge base can contain a very large number of rules covering many different symptoms, findings, tests, and diagnoses.

The task is to check, in real time:

- many kinds of rules,
- against the relevant facts in one patient's record,
- whenever new information appears.

The reasoning is ordinary logic.

Medical Diagnosis

valid arguments vs true facts

Medical Diagnosis

valid arguments vs true facts

Our familiar sometimes-confusing distinction:

Medical Diagnosis

valid arguments vs true facts

Our familiar sometimes-confusing distinction:

truth in the real world

versus

valid argument

Our familiar sometimes-confusing distinction:

truth in the real world

versus

valid argument

A valid argument can still have a false conclusion—if one of its hypotheses is false.

The argument (repeated)

This argument is valid. (That doesn't depend on the real world)

$(T \wedge F \wedge L \wedge R) \rightarrow S$	*(the medical rule)*
T	*(patient has a sore throat)*
L	*(neck lymph nodes are swollen)*
F	*(patient has a fever)*
R	*(rapid strep test is positive)*
S	*(patient has strep throat)*

The argument (repeated)

This argument is valid. (That doesn't depend on the real world)

$(T \wedge F \wedge L \wedge R) \rightarrow S$	*(the medical rule)*
T	*(patient has a sore throat)*
L	*(neck lymph nodes are swollen)*
F	*(patient has a fever)*
R	*(rapid strep test is positive)*
S	*(patient has strep throat)*

But if one or more of the T, L, F, R were false (in the real world)
This patient might not have strep

The argument (repeated)

This argument is valid. (That doesn't depend on the real world)

$(T \wedge F \wedge L \wedge R) \rightarrow S$	*(the medical rule)*
T	*(patient has a sore throat)*
L	*(neck lymph nodes are swollen)*
F	*(patient has a fever)*
R	*(rapid strep test is positive)*
S	*(patient has strep throat)*

But if one or more of the T, L, F, R were false (in the real world)
This patient might not have strep

But we've reduced the problem of the diagnosis
to the question of whether the simpler facts T, L, F, R are true

Sources and Acknowledgements

Centers for Disease Control and Prevention. “Clinical Guidance for Group A Streptococcal Pharyngitis.” 2025.
Used for the strep-throat symptoms, clinical findings, and role of rapid testing in the simplified example.
https://www.cdc.gov/group-a-strep/hcp/clinical-guidance/strep-throat.html

SNOMED International. *SNOMED CT Clinical Decision Support Guide*. SNOMED International Documents, 2026. Especially “Introduction,” “Knowledge Base: Rules,” and “Inference Engine.” Accessed October 2026.
https://docs.snomed.org/snomed-ct-practical-guides/snomed-ct-clinical-decision-support-guide