Logic: Syntax and Semantics
Class 5
Application: Scheduling

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October 18, 2026 – 10 : 48

A Very Small Scheduling Problem

We have two people, **Alice** and **Bob**, and two possible meeting times:

1:00 **2:00**

A Very Small Scheduling Problem

We have two people, **Alice** and **Bob**, and two possible meeting times:

1:00 **2:00**

The constraints are:

- Alice must be scheduled at exactly one of the two times.
- Bob must be scheduled at exactly one of the two times.
- Alice and Bob cannot be scheduled at the same time.
- Alice cannot meet at 2:00.

Represent the Possibilities

Let

A_1 stand for: Alice is scheduled at 1:00

A_2 stand for: Alice is scheduled at 2:00

B_1 stand for: Bob is scheduled at 1:00

B_2 stand for: Bob is scheduled at 2:00

Represent the Possibilities

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A_1	stand for: Alice is scheduled at 1:00
A_2	stand for: Alice is scheduled at 2:00
B_1	stand for: Bob is scheduled at 1:00
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A_1, B_1, etc will be **atomic formulas**

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A **schedule** is a **possible world**.

Now Write the Constraints as Logic

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$\neg(A_1 \wedge B_1) \wedge \neg(A_2 \wedge B_2)$	they can't meet at the same time

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$\neg(A_1 \wedge B_1) \wedge \neg(A_2 \wedge B_2)$	they can't meet at the same time
$\neg A_2$	Alice cannot meet at 2:00

All of the constraints now form a set of **hypotheses**:

$$\mathcal{C} = \left\{ \begin{array}{l} A_1 \vee A_2, \\ \neg(A_1 \wedge A_2), \\ B_1 \vee B_2, \\ \neg(B_1 \wedge B_2), \\ \neg(A_1 \wedge B_1) \wedge \neg(A_2 \wedge B_2), \\ \neg A_2 \end{array} \right\}$$

The Logical Question

We now ignore the fact that these formulas came from a “scheduling problem.”

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We simply ask:

Is there a possible world in which

all of these formulas are true?

A Possible World That Works

One possible world is:

A_1 is **true** A_2 is **false**

B_1 is **false** B_2 is **true**

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So the schedule is:

Alice at 1:00 **Bob at 2:00**

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A_1 is **true** A_2 is **false**

B_1 is **false** B_2 is **true**

So the schedule is:

Alice at 1:00 **Bob at 2:00**

Every constraint is satisfied.

The Reduction

Scheduling problem



Formulas of logic



Is there a possible world
where they are all true?

A Less Obvious Scheduling Problem

Four people—**Alice, Bob, Carol, David**—must be assigned to four time slots:

1:00 **1:30** **2:00** **2:30**

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Their available slots are:

Alice	1:00, 1:30, or 2:30
Bob	1:00 or 2:00
Carol	1:30, 2:00, or 2:30
David	1:00, 1:30, or 2:30

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Bob	1:00 or 2:00
Carol	1:30, 2:00, or 2:30
David	1:00, 1:30, or 2:30

The constraints are:

- No two people can be scheduled in the same slot.
- Alice must be scheduled earlier than Carol.
- Bob must be scheduled earlier than David.

Make our atomic formulas

Let A_i mean that Alice is scheduled in slot i.

Similarly use B_i, C_i, and D_i for Bob, Carol, and David.

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For example:

$A_2 =$ Alice is scheduled at 1:30

$D_4 =$ David is scheduled at 2:30

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Let A_i mean that Alice is scheduled in slot i.

Similarly use B_i, C_i, and D_i for Bob, Carol, and David.

For example:

$A_2 =$ Alice is scheduled at 1:30

$D_4 =$ David is scheduled at 2:30

Again, a **schedule** is a **possible world**.

The availability constraints become:

$A_1 \vee A_2 \vee A_4$	Alice
$B_1 \vee B_3$	Bob
$C_2 \vee C_3 \vee C_4$	Carol
$D_1 \vee D_2 \vee D_4$	David

Exactly One Time for Each Person

For each person, **exactly one** time-slot atom must be true.

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“Exactly” comes from:

$$\neg(A_1 \wedge A_2) \quad \neg(A_1 \wedge A_3) \quad \neg(A_1 \wedge A_4)$$
$$\neg(A_2 \wedge A_3) \quad \neg(A_2 \wedge A_4) \quad \neg(A_3 \wedge A_4)$$

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“Exactly” comes from:

$$\neg(A_1 \wedge A_2) \quad \neg(A_1 \wedge A_3) \quad \neg(A_1 \wedge A_4)$$
$$\neg(A_2 \wedge A_3) \quad \neg(A_2 \wedge A_4) \quad \neg(A_3 \wedge A_4)$$

The same kind of formulas are required for Bob, Carol, and David.

No Two People at the Same Time

No two people can occupy the same slot. For slot 1:

$$\neg(A_1 \wedge B_1) \quad \neg(A_1 \wedge C_1) \quad \neg(A_1 \wedge D_1)$$
$$\neg(B_1 \wedge C_1) \quad \neg(B_1 \wedge D_1) \quad \neg(C_1 \wedge D_1)$$

No Two People at the Same Time

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$$\neg(A_1 \wedge B_1) \quad \neg(A_1 \wedge C_1) \quad \neg(A_1 \wedge D_1)$$
$$\neg(B_1 \wedge C_1) \quad \neg(B_1 \wedge D_1) \quad \neg(C_1 \wedge D_1)$$

This pattern is imposed for $i = 1, 2, 3, 4$.

The “earlier than” constraints

“Alice is earlier than Carol” becomes:

$$(A_1 \rightarrow (C_2 \vee C_3 \vee C_4)) \wedge$$
$$(A_2 \rightarrow (C_3 \vee C_4)) \wedge$$
$$(A_3 \rightarrow C_4) \wedge \neg A_4$$

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$$(A_2 \rightarrow (C_3 \vee C_4)) \wedge$$
$$(A_3 \rightarrow C_4) \wedge \neg A_4$$

“Bob is earlier than David” becomes:

$$(B_1 \rightarrow (D_2 \vee D_3 \vee D_4)) \wedge$$
$$(B_2 \rightarrow (D_3 \vee D_4)) \wedge$$
$$(B_3 \rightarrow D_4) \wedge \neg B_4$$

The Constraint Set $\mathcal{C}$

Here's the whole set of constraints

$$
\begin{array}{llll}
A_1 \vee A_2 \vee A_4, & B_1 \vee B_3, & C_2 \vee C_3 \vee C_4, & D_1 \vee D_2 \vee D_4, \\
A_1 \vee A_2 \vee A_3 \vee A_4, & \neg(A_1 \wedge A_2), & \neg(A_1 \wedge A_3), & \neg(A_1 \wedge A_4), \\
\neg(A_2 \wedge A_3), & \neg(A_2 \wedge A_4), & \neg(A_3 \wedge A_4), & B_1 \vee B_2 \vee B_3 \vee B_4, \\
\neg(B_1 \wedge B_2), & \neg(B_1 \wedge B_3), & \neg(B_1 \wedge B_4), & \neg(B_2 \wedge B_3), \\
\neg(B_2 \wedge B_4), & \neg(B_3 \wedge B_4), & C_1 \vee C_2 \vee C_3 \vee C_4, & \neg(C_1 \wedge C_2), \\
\neg(C_1 \wedge C_3), & \neg(C_1 \wedge C_4), & \neg(C_2 \wedge C_3), & \neg(C_2 \wedge C_4), \\
\neg(C_3 \wedge C_4), & D_1 \vee D_2 \vee D_3 \vee D_4, & \neg(D_1 \wedge D_2), & \neg(D_1 \wedge D_3), \\
\neg(D_1 \wedge D_4), & \ldots &&
\end{array}
$$

The Constraint Set $\mathcal{C}$ (continued)

$\neg(D_2 \wedge D_3)$, $\neg(D_2 \wedge D_4)$, $\neg(D_3 \wedge D_4)$, $\neg(A_1 \wedge B_1)$,

$\neg(A_1 \wedge C_1)$, $\neg(A_1 \wedge D_1)$, $\neg(B_1 \wedge C_1)$, $\neg(B_1 \wedge D_1)$,

$\neg(C_1 \wedge D_1)$, $\neg(A_2 \wedge B_2)$, $\neg(A_2 \wedge C_2)$, $\neg(A_2 \wedge D_2)$,

$\neg(B_2 \wedge C_2)$, $\neg(B_2 \wedge D_2)$, $\neg(C_2 \wedge D_2)$, $\neg(A_3 \wedge B_3)$,

$\neg(A_3 \wedge C_3)$, $\neg(A_3 \wedge D_3)$, $\neg(B_3 \wedge C_3)$, $\neg(B_3 \wedge D_3)$,

$\neg(C_3 \wedge D_3)$, $\neg(A_4 \wedge B_4)$, $\neg(A_4 \wedge C_4)$, $\neg(A_4 \wedge D_4)$,

$\neg(B_4 \wedge C_4)$, $\neg(B_4 \wedge D_4)$, $\neg(C_4 \wedge D_4)$, $A_1 \rightarrow (C_2 \vee C_3 \vee C_4)$,

$A_2 \rightarrow (C_3 \vee C_4)$, $A_3 \rightarrow C_4$, $\neg A_4$, $B_1 \rightarrow (D_2 \vee D_3 \vee D_4)$,

$B_2 \rightarrow (D_3 \vee D_4)$, $B_3 \rightarrow D_4$, $\neg B_4$

Now Ask the Logic Question

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If there is, that possible world **is a schedule satisfying all the constraints**.

A Possible World That Works

Here is one:

1:00	Alice
1:30	Carol
2:00	Bob
2:30	David

A Possible World That Works

Here is one:

1:00	Alice
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Every formula is true in this possible world.

A Possible World That Works

Here is one:

1:00	Alice
1:30	Carol
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2:30	David

Every formula is true in this possible world.

So this schedule is a solution.

Another Possible World That Works

But that is not the only possible world satisfying all the formulas.

1:00	Bob
1:30	Alice
2:00	Carol
2:30	David

Another Possible World That Works

But that is not the only possible world satisfying all the formulas.

1:00	Bob
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Every formula is true in this possible world too.

Another Possible World That Works

But that is not the only possible world satisfying all the formulas.

1:00	Bob
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Every formula is true in this possible world too.

So there can be **more than one schedule** satisfying the constraints.

Imagine scheduling a Major League Baseball season.

A Bigger Scheduling Problem

Imagine scheduling a Major League Baseball season.

- 30 teams
- 162 games for each team
- 81 home games and 81 away games for each team
- **2,430 games altogether**

And there are constraints involving:

- which teams must play which other teams
- home versus away games
- series of consecutive games against the same opponent
- travel and days off
- stadium availability
- special dates and events

How Big Does the Logic Get?

Suppose we represented just

$G_{g,d}$ = game g is played on day d.

There are 2,430 games and roughly 180 days in the season.

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$G_{g,d}$ = game g is played on day d.

There are 2,430 games and roughly 180 days in the season.
So this alone gives roughly

$$2430 \times 180 = 437,400$$

atomic propositions.

How big, cont'd

And we have not yet written the constraints.

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For the 4-person scheduling, we had 16 atomic propositions, and 36 constraints …

How big, cont'd

And we have not yet written the constraints.

For the 4-person scheduling, we had 16 atomic propositions, and 36 constraints …

So we have hundreds of thousands of constraints

Yikes.

How do we actually *build* a possible world?

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tableaux would work!

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in practice specialized fast algorithms are used, fine-tuned for scheduling

Summary

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A scheduling problem can be approached as follows:

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2. Observe that a **schedule is a possible world**.
3. Search for a possible world making **all** the formulas true.

   If you find such a possible world, you have found a schedule.

Question:

Does this have anything to do with valid arguments?

Remember Our Main Idea from Lecture 1

valid arguments vs possible worlds

Consider an argument

$$A_1,\ A_2,\ \ldots,\ A_n \quad \text{therefore} \quad B.$$

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Remember Our Main Idea from Lecture 1

valid arguments vs possible worlds

Consider an argument

$$A_1,\ A_2,\ \ldots,\ A_n \quad \text{therefore} \quad B.$$

We said: such an argument is **valid** if:

*There is no **possible world** making*

$$A_1,\ A_2,\ \ldots,\ A_n,\ \neg B$$

all true.

For Instance: Scheduling

Suppose the scheduling constraints are

$$C_1,\ C_2,\ \ldots,\ C_n.$$

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If there is **no possible world** satisfying all of them, then

$$C_1,\ C_2,\ \ldots,\ C_{n-1} \quad \text{therefore} \quad \neg C_n$$

is a **valid argument.**

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The constraints cannot all hold together.

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Suppose the patient's symptoms are

$$A_1,\ A_2,\ \ldots,\ A_n$$

and the proposed diagnosis is D.

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If the symptoms do **not** validly imply D, then there is a possible world in which

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are all true.

For Instance: Diagnosis

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and the proposed diagnosis is D.

If the symptoms do **not** validly imply D, then there is a possible world in which

$$A_1,\ A_2,\ \ldots,\ A_n,\ \neg D$$

are all true.

In that possible world, all the symptoms are present—but the patient does **not** have disease D.

The Basic Connection

A **valid argument** is about having

NO possible worlds of a certain kind.

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A **valid argument** is about having

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A **possible world** can be evidence that

an argument is NOT valid.