

Logic: Syntax and Semantics

Week 1

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Two examples

Either Pat is here or Kim is here
If Kim is here then Sam is not happy
Sam is happy.

Pat is here

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Pat is here

A *Valid* argument

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A *Valid* argument

Either Pat is here or Kim is here
If Kim is here then Sam is not happy
Sam is not happy.

Kim is here

Not Valid

Justify those intuitions?

...a main topic of the course

Two more examples

All cats have 4 legs

Fluffy is a cat

Hence, Fluffy has 4 legs

Two more examples

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Fluffy is a cat

Hence, Fluffy has 4 legs

A *Valid* argument

Two more examples

All cats have 4 legs

Fluffy is a cat

Hence, Fluffy has 4 legs

A *Valid* argument

All fish can swim

Flounder is a fish

Hence, Flounder can swim

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A *Valid* argument

These are the **SAME ARGUMENT**

What does “same argument” mean?

...a main topic of the course

Revisiting “not valid”

Either Pat is here or Kim is here
If Kim is here then Sam is not happy
Sam is not happy.

Kim is here

We said “not valid” ... Defend that?

A state of affairs:

1. Pat is here
2. Kim is not here
3. Sam is not happy

This is a **countermodel**

All cats have 4 legs

Charlie has 4 legs

Charlie is a cat

Not Valid

A countermodel:

Charlie is a dog

Now: definitions of valid / not valid

Definition

An argument is *not valid* if there is some possible state of affairs in which the premises are all true, yet the conclusion is false

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So :

Definition

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So :

Definition

An argument is *valid* if in every possible state of affairs in which the premises are all true, the conclusion is true

These ideas are subtle. An essential requirement for not getting confused is to commit to using certain words only in a precise technical sense.

- Use the words **true** and **false** only when referring to single sentences, and having a single “world” in mind.
- Use the words **valid** and **not valid** only when referring to *arguments*

In ordinary language we often use the word “valid” to mean “true.” Try not to do that when discussing logic.

Making careful distinctions is a core skill in thinking clearly.

Jump forward to some Examples

A really good exercise

Make up some **not valid** arguments.

Explain why each one is not valid, by giving a counter-model.

Suppose you see an argument with a conclusion that is false in the real world. Then EITHER

- it was not a valid argument (in our logic sense)
- OR at least one premise is false in the real world

Suppose you see an argument with a conclusion that is true in the real world. Then

- the argument is NOT NECESSARILY a valid logic argument
- (So by the way, you're not allowed to be frustrated if it doesn't convince me!)

Closing thoughts 3

Logic has a subtle relationship with “truth in the real world”
this is a strength, an essential feature, of logic
otherwise we couldn't use it as a tool
logic is NOT THE SAME as observation

Examples for practice

Decide whether the following arguments are valid or not valid
For those arguments following you feel are not valid, try to make counter-models.

The first several are purely “propositional” arguments; the later ones involve quantifiers.

To get you started, here are some answers:

Numbers 1,3,5, 13, 15, 17 are valid

Numbers 2,4,6, 14, 16, 18 are not valid

Acknowledgments: Some of the following examples were generated with assistance from

- ChatGPT (2026)
- Lewis Carroll (1896)

Example 1

All dogs are mammals.

Fido is a dog.

Fido is a mammal.

Example 2

All dogs are mammals.

Fido is a mammal.

Fido is a dog.

Example 3

If Kim is in Worcester then Kim is in Massachusetts.
Kim is in Worcester.

Kim is in Massachusetts.

Example 4

If Kim is in Worcester, then Kim is in Massachusetts.
Kim is in Massachusetts.

Kim is in Worcester.

Example 5

Either Pat is at home or Pat is at work.

Pat is not at home.

Pat is at work.

Example 6

Either Pat is at home or Pat is at work.

Pat is at home.

Pat is not at work.

Example 7

If the dragon is awake, then the princess is hiding.
The dragon is awake.

The princess is hiding.

Example 8

If the dragon is awake, then the princess is hiding.
The dragon is not awake.

The princess is not hiding.

Example 9

If Alice misses the bus, she will be late.
If Alice is late, the professor will notice.
Alice misses the bus.

The professor will notice.

Example 10

If Alice misses the bus, she will be late.
If Alice is late, the professor will notice.
The professor notices.

Alice missed the bus.

Example 11

Either the squirrel stole my sandwich or the raccoon stole my sandwich.
The squirrel did not steal my sandwich.

The raccoon stole my sandwich.

Example 12

Either the squirrel stole my sandwich or the raccoon stole my sandwich.
The squirrel stole my sandwich.

The raccoon did not steal my sandwich.

Examples with quantifiers

Example 13

All penguins are birds.

All birds are animals.

All penguins are animals.

Example 14

All penguins wear tuxedos.
Fred wears a tuxedo.

Fred is a penguin.

Example 15

No cats are vegetables.

Mr. Pickles is a cat.

Mr. Pickles is not a vegetable.

Example 16

No cats are vegetables.

Mr. Pickles is not a vegetable.

Mr. Pickles is a cat.

Example 17

All poodles are dogs.

All dogs are mammals.

All mammals are animals.

All poodles are animals.

Example 18

All poodles are dogs.

All beagles are dogs.

All poodles are beagles.

Example 19

Some professors are musicians.

All musicians are creative.

Some professors are creative.

Example 20

Some professors are musicians.

Some musicians are golfers.

Some professors are golfers.

Example 21

Some dogs are enormous.
All enormous things frighten Kevin.

Some dogs frighten Kevin.

Example 22

Some dogs are enormous.

Some dogs are friendly.

Some friendly dogs are enormous.

Example 23

No philosophers are kangaroos.
Socrates is a philosopher.

Socrates is not a kangaroo.

Example 24

No philosophers are kangaroos.

Socrates is not a kangaroo.

Socrates is a philosopher.

Example 25

Every Martian who owns a bicycle wears a helmet.
Zog is a Martian who owns a bicycle.

Zog wears a helmet.

Example 26

Every Martian who owns a bicycle wears a helmet.
Zog wears a helmet.

Zog is a Martian who owns a bicycle.

Example 27

All invisible purple cows are excellent dancers.

Bessie is an invisible purple cow.

Bessie is an excellent dancer.

Example 28

All excellent dancers eat oatmeal.

Bessie eats oatmeal.

Bessie is an excellent dancer.

Example 29

Every student who sleeps through logic class becomes a billionaire.

Sam did not sleep through logic class.

Sam will not become a billionaire.

Example 30

Every person who owns a giraffe needs a tall garage.
Nobody with a tall garage lives in this apartment building.
Chris lives in this apartment building.

Chris does not own a giraffe.

Example 31

Everyone who owns a giraffe needs a tall garage.

Chris does not own a giraffe.

Chris does not have a tall garage.

Example 32

All surgeons are doctors.
No plumbers are surgeons.

No plumbers are doctors.

Example 33

Everyone who likes Bach likes music.

Everyone who likes Beyonce likes music.

Everyone who likes Bach likes Beyonce.

Example 34

Some students passed the exam.

Some students fell asleep during the exam.

Some students who fell asleep during the exam passed it.

Example 35

Everyone who understands this argument understands logic.
Nobody in the room understands logic.

Nobody in the room understands this argument.

from Lewis Carroll

Example 36

Lewis Carroll

All my friends have colds.
No one who has a cold can sing.

None of my friends can sing.

Example 37

Lewis Carroll

All my friends have colds.

No one who has a cold can sing.

Everyone who cannot sing is one of my friends.

Example 38

No medicine is nice.

Senna is a medicine.

Senna is not nice.

Example 39

Lewis Carroll

No medicine is nice.
Senna is not nice.

Senna is a medicine.

Example 40

Lewis Carroll

Some oysters are silent.

No silent creatures are amusing.

Some oysters are not amusing.

Example 41

Lewis Carroll

Some oysters are silent.

No silent creatures are amusing.

No oysters are amusing.

Example 42

Lewis Carroll

All ducks waddle.

Nothing that waddles is graceful.

No ducks are graceful.

Example 43

Lewis Carroll

All ducks waddle.

Nothing that waddles is graceful.

Everything that waddles is a duck.

Example 44

Lewis Carroll

No bride-cakes are wholesome.

All unwholesome food should be avoided.

All bride-cakes should be avoided.

Example 45

Lewis Carroll

No bride-cakes are wholesome.

All unwholesome food should be avoided.

Everything that should be avoided is a bride-cake.

Example 46

Lewis Carroll

Bores are terrible.

You are a bore.

You are terrible.

Example 47

Lewis Carroll

Bores are terrible.

You are terrible.

You are a bore.

Example 48

Lewis Carroll

No lobsters are unreasonable.

No reasonable creatures expect impossibilities.

No lobsters expect impossibilities.

Example 49

Lewis Carroll

No lobsters are unreasonable.

No reasonable creatures expect impossibilities.

Everything that does not expect impossibilities is a lobster.

Example 50

Lewis Carroll

All spiders spin webs.

Some savage creatures do not spin webs.

Some savage creatures are not spiders.

Example 51

Lewis Carroll

No shark ever doubts that it is well fitted out.

A fish that cannot dance a minuet is contemptible.

No fish is quite certain that it is well fitted out, unless it has three rows of teeth.

All fishes, except sharks, are kind to children.

No heavy fish can dance a minuet.

A fish with three rows of teeth is not to be despised.

All heavy fish are kind to children.

Example 52

Lewis Carroll

All the human race, except my footmen, have a certain amount of common-sense.

No one who lives on barley-sugar can be anything but a mere baby.

None but a hop-scotch player knows what real happiness is.

No mere baby has a grain of common sense.

No engine-driver ever plays hop-scotch.

No footman of mine is ignorant of what true happiness is.

No engine-driver lives on barley-sugar.