Logic-Based Systems

AI Lecture

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Using Theorem Provers

AS REASONING SYSTEMS

- to implement independent agents that make decisions and act on their own.

AS ASSISTANTS

tool for mathemathicians

- Proof-Checkers:
  - mathematician provides a sketch of the proof and TP checks it and fills in the details.
- Socratic Reasoners:
  - (e.g. ONTIC). Mathematician and TP construct proof together.

Practical uses of Theorem Provers (TPs)

SAM	Semi-automated math. Guard et al, 1969	Lattice theory
AURA	Wos & Winker, 1983	Open questions in several areas of mathematics
BOYER & MOORE	Boyer & Moore, 1979	Produced 1^{st} fully computerized proof of Godel's incompleteness thm
OTTER	Organized techniques for theorem proving McCure 1992	Open questions in combinatorical logic

CS/ECE: Verification of Systems

- SOFTWARE

procedure swap(x,y)

var t;

{Pre: x = C1, y = C2}

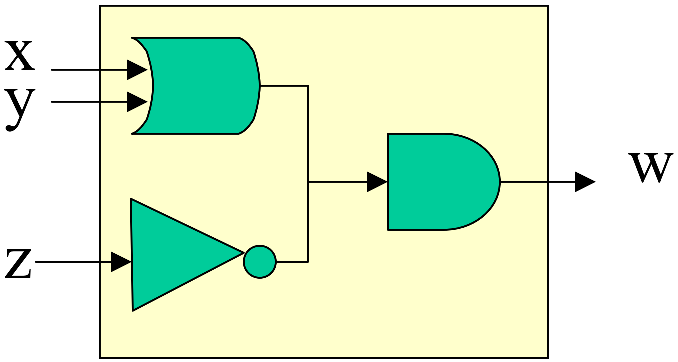
t := x;

x:= y;

y:= t

{Post: x = C2, y = C1}

- HARDWARE



w = (x OR y) and ~z

CS/ECE: Verification of Systems

- SOFTWARE
  - Boyer & Moore:
    - verified the RSA public key encryption algorithm
    - verified the Boyer & Moore string matching algorithm
- HARDWARE
  - Aura:
    - Verifies design of a 10-bit adder
  - MRS:
    - performs diagnosis of computer systems

CS/ECE: Synthesis of Systems

- SOFTWARE

procedure swap(x,y)

{Pre: x = C1, y = C2}

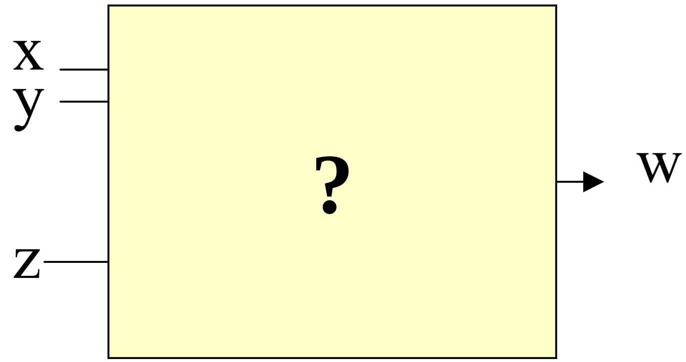
?

{Post: x = C2, y = C1}

Prove that there exists a program satisfying the specification.

If the proof is constructed, a program can be extracted.

- HARDWARE



w = (x OR y) and ~z

AURA: used to design circuits more compact than before

Inside a Logic-based System

Knowledge Representation First order logic
Problem Solving Strategy Refutation using resolution

Knowledge representation
1st order logic

- Everybody who can read is literate
  - $\forall$ x, r(x) -> l(x)
- Dolphins are not literate
  - $\forall$ x, d(x) -> !l(x)
- Some dolphins are intelligent
  - $\exists$ x, [d(x) & i(x)]
- Some who are intelligent cannot read
  - $\exists$ x, [i(x) & !r(x)]

Problem Solving
Problem Statement

- A1: Everybody who can read is literate
  - $\forall$ x, r(x) -> l(x)
- A2: Dolphins are not literate
  - $\forall$ x, d(x) -> !l(x)
- A3: Some dolphins are intelligent
  - Э x, [d(x) & i(x)]
- Conclusion: Some who are intelligent cannot read
  - Э x, [i(x) & !r(x)]

Problem Solving

Proof by Refutation

- A1: Everybody who can read is literate
  - ∀ x, r(x) -> l(x)
- A2: Dolphins are not literate
  - ∀ x, d(x) -> !l(x)
- A3: Some dolphins are intelligent
  - ∃ x, [d(x) & i(x)]
- ! Conclusion: it is not the case that some who are intelligent cannot read
  - !∃ x, [i(x) & !r(x)] = ∀x, [!i(x) || !!r(x)] = ∀x, [!i(x) || r(x)]

Problem Solving

Proof by Refutation using Resolution

translating formulas into clausal form

- A1: ∀x,   r(x) -> l(x)
- A2: ∀x, d(x) -> !l(x)
- A3: Э x, [d(x) & i(x)]
- !C: ∀x, [!i(x) || r(x)]

- A1: ∀x, !r(x) || l(x)
- A2: ∀x, !d(x) || !l(x)
- A3: Э x, [d(x) & i(x)]
- !C: ∀x, [!i(x) || r(x)]

Problem Solving

Proof by Refutation using Resolution

translating formulas into clausal form – done!

- A1: ∀x, !r(x) || l(x)
- A2: ∀x, !d(x) || !l(x)
- A3: Ǝ x, [d(x) & i(x)]
- !C: ∀x, [!i(x) || r(x)]

- A1: !r(x) || l(x)
- A2: !d(x) || !l(x)
- A3.1: d(**a**)
- A3.2: i(**a**)
- !C: !i(x) || r(x)

Problem Solving

Resolution

- A1: !r(x) || l(x)
- A2: !d(x) || !l(x)
- A4: !r(x) || !d(x)
- A3.1: d(**a**)
- A5: !r(**a**)
- !C: !i(x) || r(x)
- A6: !i(**a**)
- A3.2: i(**a**)
- A7: □ ← Contradiction!!! ☺

Hence C is a logical consequence of A1,A2,A3

- A1: !r(x) || l(x)
- A2: !d(x) || !l(x)
- A3.1: d(**a**)
- A3.2: i(**a**)
- !C: !i(x) || r(x)