Due: March 25/26, 1998

Read Sections 5.1, 5.2, 5.3 (Trick 1)

Do Exercises 5.1, 5.2, 5.4, 5.15 from GKP. Do not submit your solutions, but check them with the answers from the back of the text.

1. (3 points) Find a closed form for \( \sum_k \binom{n}{k} \binom{k}{m} \), where closed forms can contain binomial coefficients.

2. (7 points) For any \( n \geq 0 \), define the sum of numbers along the \( n \)th diagonal (lower-left \( \rightarrow \) upper-right) of Pascal’s triangle to be \( f_n = \sum_k \binom{n-k}{k} \).

   (A) What is \( f_0 \)? What is \( f_1 \)?
   (B) Prove the recurrence \( f_n = f_{n-1} + f_{n-2} \) for \( n \geq 2 \).
   (C) Find a closed form for \( f_n \).

3. (2 points) Prove that for any integer \( n \), \( 3^n = \sum_k \binom{n}{k} 2^k \).

4. (3 points) When sending the sequence of bits 0110 down a noisy channel, if one of the bits is flipped, it will be received as some member of the set \{1110,0010,0100,0111\}. Define the input alphabet \( \Sigma \) to be the sequences of bits that could have been sent. If \( \Sigma = \{0110,1011\} \) and we assume that at most one bit was flipped and 0100 is received, then we may assume that 0110 was sent. In fact, \( \Sigma = \{0110,1011\} \) admits “1-error correction” since if any element \( \sigma \in \Sigma \) is sent and at most one bit of \( \sigma \) is flipped, then the received sequence could only have come from one member of \( \Sigma \). If \( \Sigma \) consists of sequences of \( n \) bits, then say that it is \( k \)-error correcting if when any element \( \sigma \in \Sigma \) is sent and at most \( k \) bits of \( \sigma \) is flipped, then the received sequence could only have come from one member of \( \Sigma \). Prove that under these conditions,

\[
|\Sigma| \leq \frac{2^n}{\sum_{0 \leq l \leq k} \binom{n}{l}}
\]