Due: November 6/11, 1996

1. (2 points) For the queueing model discussed in class, what is the expected queue length if the queue is empty 50% of the time? What is the expected queue length if the queue is empty 10% of the time?

2. (4 points) Find as simple a form as possible for the recurrence 
\[ n_{a_n} = (n + 1)a_{n-1} + 4n^2, \text{ if } n > 0 \]
\[ 0, \text{ if } n = 0 \]
We note that the first few terms are \( a_0 = 0, a_1 = 4, a_2 = 14 \).

3. (8 points) A set of vertices of a graph is an independent set if there are no edges between any pair of vertices in the set. For example, in the following graph

\[
\begin{array}{c}
\text{G:} \\
\text{a - b - c - f - e - d - g}
\end{array}
\]

\( \{a, e, f\} \) is an independent set, though \( \{a, b, f\} \) is not. We want an algorithm to compute the cardinality of a maximum independent set. For the above graph, our algorithm should return 4 corresponding to the independent set \( \{b, d, e, f\} \). Let \( \Gamma(v) \) denote the neighbors of \( v \), and for \( S \) a set of vertices of \( G \), let \( G-S \) denote the graph obtained by removing all vertices of \( S \) plus all incident edges, and let \( A(G) \) denote the size of a largest independent set in \( G \). An algorithm to compute \( A(G) \) is suggested by the observation that any fixed vertex \( v \) either belongs or doesn’t belong to an independent set. If \( v \) belongs to an independent set, then no vertices of \( \Gamma(v) \) belong to the set.

\[
\text{function } A(G) : \text{integer} \\
\quad \text{if } G \text{ has no edges then return the number of vertices in } G \\
\quad \text{else select some vertex } v \text{ with at least one incident edge} \\
\quad \text{return max}(A(G-\{v\}), A(1+G-\{v\}-\Gamma(v)))
\]

A) What kind of graph causes function \( A(G) \) to have worst-case execution time?
B) Develop a recurrence to describe the worst-case execution time of \( A(G) \). Assume that all of the operations of \( A(G) \) (aside from the recursive calls) require time \( n^2 \), where \( n \) is the number of vertices of \( G \).
C) Solve the recurrence of part B). A solution using \( \Theta \)-notation suffices.