C.S.504
H.W. #4

Due: October 8, 1992

1. (1 point) Evaluate \[ \sum_{0 \leq n^2 \leq 5} \frac{1}{2n^2 + 1} \]

2. (1 point) Evaluate \[ \sum_{0 \leq m \leq k \leq n} k \]

3. (4 points) Assume that you’re sequentially seeking a card in a perfectly shuffled deck of $10^6$ distinct cards. The card is in the deck.
   A) What is the probability that you’ll have to examine at least 900,000 cards?
   B) Use Chebyshev’s inequality to give a bound on the probability that you’ll have to examine at least 900,000 cards.
1. \[ \sum_{0 \leq n^2 \leq 5} \frac{1}{2n^2+1} = \frac{1}{9} + \frac{1}{3} + \frac{1}{3} + \frac{1}{9} = \frac{8}{9} \]

2. Replacing \( k \) by \( k+m \),

\[
\sum_{m \leq k \leq n} \sum_{m \leq k+m \leq n} \sum_{0 \leq k \leq n-m} \sum_{0 \leq k \leq n-m} (n-m)(n-m-1) \frac{1}{2} + (n-m+1)m
\]

3. Let \( A_{1,000,000} \) be a r.v. denoting the number of cards examined.

A) \( \Pr\{A_{1,000,000} \geq 900,000\} = 0.1 \)

B) For a perfect shuffle, \( \mathbb{E}[A_{1,000,000}] = 500,000.5 \) and \( \mathbb{V}[A_{1,000,000}] = \frac{(10^{12}-1)}{12} = 833333333333.3333 \), yielding \( \sigma(A_{1,000,000}) = 288675.1345948129 \) Since 900,000 is 1.385638914004294 standard deviations from the mean of \( A_{1,000,000} \), Chebyshev's inequality assures us that the probability of being at least 1.385638914004294*\( \sigma \) from the mean is at most \( \frac{1}{(1.385638914004294)^2} = 0.5208346354191083 \).