C.S.504
H.W. #1

Due: September 17, 1992

1. (1 point) What is $(0.125)^{-2/3}$?

2. (4 points) Given array $A[1..n]$ of integers such that $|A[k+1]-A[k]| \leq 1$ for $1 \leq k < n$ and given integer $x$ such that $A[1] \leq x \leq A[n]$, we seek a $j$, $1 \leq j \leq n$, such that $A[j]=x$. Describe an algorithm to solve this problem which examines at most $1+\lceil \lg n \rceil$ elements of $A$. Precise pseudocode suffices.

3. (3 points) The Fibonacci numbers are defined (GKP, pg. 276) as

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1}+F_{n-2} & \text{if } n > 1 \end{cases}$$

They may be extended to negative values of $n$ by using the same definition. That is, $F_1 = 1=F_0+F_{-1} = 0+F_{-1}$ which implies that $F_{-1} = 1$. Likewise, $F_{-2} = -1$. Define $G_n = F_{-n}$ for $n \geq 0$, so that $G_0 = 0$, $G_1 = 1$, $G_2 = -1$,... Write a recurrence for $G_n$, $n \geq 0$, which does not involve $F_n$.

4. (4 points) (from Martin Gardner) Consider the four fair dice with faces

$D_1: 1,2,3,9,10,11$
$D_2: 0,1,7,8,8,9$
$D_3: 5,5,6,6,7,7$
$D_4: 3,4,4,5,11,12$

For each pair of dice $D_i, D_j$, $1 \leq i,j \leq 4$, $i \neq j$, compute the probability that the top face of $D_i$ will be larger than the top face of $D_j$. Notice that if you and another person were betting on who could throw the larger number and if the other person chose a die first, you could always have a higher expectation of winning.
1.) \[ (0.125)^{-2/3} = 1/(0.125)^{2/3} = 1/(1/8)^{2/3} = 1/[(1/8)^{1/3}]^2 = \\
1/(1/2)^2 = 1/(1/4) = 4 \]

2.) \textbf{function} \( \text{search}(lo, hi, x : \text{int}) : \text{int} \)

\begin{verbatim}
begin
mid := (lo + hi) \text{ div } 2;
if \( x = A[mid] \) then return(lo)
else if \( x < A[mid] \) then return(search(lo, mid-1, x))
else return(search(mid+1, hi, x))
end
\end{verbatim}

which is initially invoked by \text{search}(1,n,x).

3.) \( G_0 = 0, G_1 = 1, G_n = G_{n-2} - G_{n-1} \) for \( n \geq 2 \). In fact, \( G_n = (-1)^{n+1}F_n \).

4.) The probability that \( D_i \) will beat \( D_j \) is the entry of row \( i \) and column \( j \) of the array

\[
\begin{array}{cccc}
- & 22 & 18 & 12 \\
36 & 36 & 36 & \\
12 & - & 22 & 16 \\
36 & 36 & 36 & \\
18 & 12 & - & 22 \\
36 & 36 & 36 & \\
22 & 20 & 12 & - \\
36 & 36 & 36 & \\
\end{array}
\]

For any choice of die that your opponent makes (say \( D_i \)), you pick a die (say \( D_j \)) such that the corresponding entry in the array (the probability of you winning) is <1/2. Since every row has an entry 12/36, you are expected to win.