1. **(Snake Oil Method)**

The following summations look dangerous, but are in fact entirely straightforward – similar to what was done in class. It will be easier if you happen to remember that the Catalan numbers \( b_n \), the number of binary trees of \( n \) nodes, are given by

\[
\frac{1}{n+1} \binom{2n}{n},
\]

and have the OGF \( \frac{1 - \sqrt{1 - 4z}}{2z} \).

It is a particular case of the binomial theorem, though.

\[
\sum_k \binom{n+k}{m+2k} \left( \frac{2k}{k} \right) \frac{(-1)^k}{k+1}.
\]

\( m, n \in \mathbb{N}_0 \)

(a) Explain why it is “obvious” that to use the Snake Oil method here you need to generate on \( n \), and not on \( m \).

(b) Find the sum; check by observing that the sum vanishes for \( n > m \), and via MAPLE (which I think cannot easily do it on its own).

2. **Recognize the following sum as a convolution** \( \sum_{k=0}^n a_k b_{n-k} \). evaluate the sum by computing the corresponding \( \left[ z^n \right] a(z)b(z) \):

\[
\sum_k \frac{1}{k+1} \binom{2k}{k} \frac{1}{n-k+1} \binom{2n-2k}{n-k}.
\]

\( n \in \mathbb{N}_0 \)

3. **(Binary trees)**

This is essentially Exercise 7.3.

Read §7.2. I think I did it in part in class, earlier, when we were discussing recurrences. Consider the set of all binary trees in which the left subtree is limited to have at most 10 nodes – without any other limitation.

(a) How many such trees with \( n \) nodes exist? Write a recurrence for this number, \( t_n \), and determine the integer \( R \) for which \( b_n = t_n \) for \( n \leq R \).

(b) Use MAPLE to compare \( t_n \) with the \( b_n \) above for \( n \in [0,100] \). In the process you will also find which \( R \) you have to argue for in part (a)....