Homework 10

Due: Beginning of class, November 30, 2000

1. (Partitions of integers)
Define a pile to be an arrangement of identical square tiles on the plane. The arrangement of a pile of \( r \) rows is determined by the \( 2r \) numbers

\[
0 = a_1 \leq a_2 \leq \cdots \leq a_{r-1} \leq a_r < b_r \leq b_{r-1} \leq \cdots \leq b_2 \leq b_1.
\]

Row \( i \) has \( b_i - a_i \) tiles. You can imagine all tiles in each row to be left-justified (hence they create columns, with height that decreases as we go from left to right). Two piles of \( n \) tiles in \( r \) rows differ only if their \( \{a_i\} \) and \( \{b_i\} \) sequences differ, while in both \( n = \sum_{i=1}^{r} (b_i - a_i) \). Let \( t_{n,r} \) be the number of distinct piles of \( n \) tiles in \( r \) rows, and \( t_n \) the total number of \( n \)-tile piles. Develop one- and two-variable enumerators for the set of all piles.

(Principle of Inclusion and Exclusion) — Use the PIE for the following problems

2. Consider all the distinguishable permutations of \( \text{aabccddeee} \). How many of them have the two ‘a’s together? How many do not?

3. Consider Example 9.4 in the handout, about repeated letters in words... Your task is to design an essentially identical problem, except dealing with six letter words. How many words? how many letters shared by each grouping? — you need to decide all those, and then show how the problem would be solved.

For a special bonus – find such words (either in the English dictionary, or fairly well known names) that fit your problem!

4. How many positive integers in \([0..100,000]\) have the digits 3, 6 and 9 in their decimal representation?