## SEARCH TREE

**Node:** State in state tree

Root node: Top of state tree

Children: Nodes that can be reached from a given

node in 1 step (1 operator)

**Expanding:** Generating the children of a node

Open: Node not yet expanded

Closed: Node after expansion

Queue: Ordered list of open nodes

#### **SEARCH**

BLIND SEARCH: Systematic Search

**Depth–1st:** Continue along current path looking for goal

Breadth-1st: Expand all nodes at current level before progressing to next level

**Depth-limited Search:** Depth-1st + depth-limit **Iterative Deepening Search:** limit=0,limit=1,...

**USING COST:**  $g(n) = cost \ from \ start \ to \ n$ 

Uniform-Cost Search (= Branch-and-bound): Select node n with best g(n).

**USING HEURISTIC:** h(n)= $Estimate\ cost\ to\ a\ goal$ 

**Greedy Search:** Select node n with best h(n)

**A\*:** Select node n with best f(n) = g(n) + h(n)

**IDA\*:**  $A^* + f$ -cost limit.

**Hill-Climbing:** Depth-1st exploring best h(n) first

Simulated Annealing: Hill-Climbing + RandomWalk

**Beam Search:** Breadth-1st keeping only m nodes with best h(n)'s per level

## DEPTH-1st SEARCH

- 1. Put start state onto queue
- 2. If queue is empty then fail
- 3. If head of queue is goal then succeed
- 4. Else remove head of queue, expand it, place children in front of queue
- 5. Recurse to 2

# DEPTH-1st (cont.)

#### When to use

- Depth limited or known beforehand
- All solutions at same depth
- Any solution will do
- Possibly fast

## When to avoid

- Large or infinite subtrees
- Prefer shallow solution

## BREADTH-1st SEARCH

- 1. Put start state onto queue
- 2. If queue is empty then fail
- 3. If head of queue is goal then succeed
- 4. Else remove head of queue, expand it, place children at end of queue
- 5. Recurse to 2

# BREADTH-1st (Cont.)

#### When to use

- Large or infinite search tree
- Solution depth unknown
- Prefer shallow solution

# When to avoid

- Very wide trees
- Generally slow
- May need a lot of space

# MODIFICATIONS TO DEPTH/BREADTH 1ST

# Depth-limited Search:

Limit the total depth of the depth 1st search.

# Iterative Deepening Search:

Repeat depth-limited search with limit 0, 1, 2, 3, ... until a solution is found.

#### Bidirectional Search:

Simultaneously search forward from initial state and backward from goal state until both paths meet.

# UNIFORM-COST SEARCH (= BRANCH-AND-BOUND)

- 1. Put start state onto queue
- 2. If queue is empty then fail
- 3. If head of queue is goal then succeed
- 4. Else
  - remove head of queue,
  - expand it,
  - place in queue, and
  - sort entire queue with least cost-so-far nodes in front
- 5. Recurse to 2

#### UNIFORM-COST SEARCH SUMMARY

# Advantages

- Optimal (when costs are non-negative)
- Complete

# Disadvantages

• Can be inefficient

#### When to use

- Desire best solution
- Keep track of cost so far

#### When to avoid

- May not work with negative costs
- May be overly conservative
- Any solution will do

# Potential improvement

• Dynamic Programming

# UNIFORM-COST SEARCH + DYNAMIC PROG.

- 1. Put start state onto queue
- 2. If queue is empty then fail
- 3. If head of queue is goal then succeed
- 4. Else
  - remove head of queue,
  - expand it,
  - place in queue,
  - \* remove redundant paths:

    Paths that reach the same node as other paths but are more expensive, and
  - sort entire queue with least cost-so-far nodes in front
- 5. Recurse to 2

# GREEDY SEARCH (= called BEST-1st SEARCH in other textbooks)

- 1. Put start state onto queue
- 2. If queue is empty then fail
- 3. If head of queue is goal then succeed
- 4. Else
  - remove head of queue,
  - expand it,
  - place in queue, and
  - sort entire queue with least estimated-costto-goal nodes in front
- 5. Recurse to 2

#### GREEDY SEARCH SUMMARY

# Advantages

- Can be very efficient
- Paths found are likely to be short

# Disadvantages

• Neither optimal nor complete

#### When to use

• Desire "short" solution

#### When to avoid

• When an optimal solution is required

#### $\mathbf{A}^*$

- 1. Put start state onto queue
- 2. If queue is empty then fail
- 3. If head of queue is goal then succeed
- 4. Else remove head of queue, expand it, place in queue, and sort entire queue with least cost-so-far + estimated-cost-remaining nodes in front
- 5. If multiple paths reach a common goal, keep only lowest cost-so-far path
- 6. Recurse to 2
- f(node) = g(node) + h(node), where
  - -f(node) = estimated total cost
  - -g(node) = cost-so-far to node
  - -h(node) = estimated-cost-remaining (heuristic).
- Properties of h:
  - Lower bound ( $\leq$  actual cost)
  - Nonnegative

#### A\* SUMMARY

# Advantages

- Complete
- $\bullet$  Optimal, when h is an underestimate
- Optimally efficient among all optimal search algorithms

# Disadvantages

• Very high space complexity

#### When to use

- Desire best solution
- Keep track of cost so far
- Heuristic information available

#### When to avoid

• No good heuristics available

# HILL CLIMBING SEARCH version 1: with backtracking

- 1. Put start state onto queue
- 2. If queue is empty then fail
- 3. If head of queue is goal then succeed
- 4. Else remove head of queue, expand it, place **children** sorted by h(n) in front of queue
- 5. Recurse to 2

# HILL CLIMBING SEARCH version 2: without backtracking arguably this is the most common version of hill climbing

- 1. Put start state onto queue
- 2. If queue is empty then fail
- 3. If head of queue is goal then succeed
- 4. Else remove head of queue, expand it, sort the children by h(n), and place only the child with the best h(n) in (front of) queue
- 5. Recurse to 2

#### HILL CLIMBING SUMMARY

## Advantages

• Complete if backtracking is allowed (like in Winston's book) and the graph is finite

# Disadvantages

- Not optimal
- Not complete if backtracking is not allowed

#### When to use

- Depth limited or known beforehand
- All solutions at same depth
- Desire good solution
- Reliable estimate of remaining distance to goal
- Fast if good estimate

#### When to avoid

- If optimal solution is required
- Large or infinite subtrees
- No good estimate
- Difficult terrain

## **BEAM SEARCH**

- 1. Put start state onto queue
- 2. If queue is empty then fail
- 3. If head of queue is goal then succeed
- 4. Else remove head of queue, expand it, place children at end of queue
- 5. If finishing a level, keep only w best nodes in queue
- 6. Recurse to 2

#### BEAM SEARCH SUMMARY

# Advantages

• Saves space

# Disadvantages

• Neither optimal nor complete

## When to use

- Large or infinite search tree
- Solution depth unknown
- Prefer shallow solution
- Possibly fast
- No more than wb nodes stored

#### When to avoid

- Can't tell which solutions to prune
- Prefer conservative

#### SEARCH STRATEGIES -

Completeness; Optimality; and Time and Space Complexity

| Search          | Complete? | Optimal? | Time                 | Space |
|-----------------|-----------|----------|----------------------|-------|
| Depth-1st       | N         | N        | $b^d$                | bd    |
| Breadth-1st     | Y         | Y*       | $b^s$                | $b^s$ |
| Depth-limited   | N         | N        | $b^l$                | bl    |
| Iter. deepening | Y         | Y*       | $b^s$                | bs    |
| Branch-&-bound  | Y         | Y        | $b^s$                | $b^s$ |
| Greedy          | N         | N        | $b^d$                | $b^d$ |
| A*              | Y         | Y        | exp                  | exp   |
| Hill-climbing   | N         | N        | $\operatorname{dep}$ | dep   |
| Beam            | N         | N        | ms                   | 2m    |

(adapted from Russell & Norvig's book)

• Y\*: Yes, IF cost of a path is equal to its length. Otherwise No.

• b: branching factor

• s: depth of the solution

• d: maximum depth of the search tree

• *l*: depth limit

• m: beam size

 $\bullet$  exp: exponential depending on heuristic h

 $\bullet$  dep: depends on heuristic h

# SEARCH STRATEGIES Summary

- Depth 1st: Continue along current path looking for goal
- **Breadth 1st:** Expand all nodes at current level before progressing to next level
- Hill Climbing: Like depth 1st, but explore most promising children first (if allowing backtracking) or just the most promising child only (if not allowing backtracking)
- **Beam:** Like breadth 1st, but prune unpromising children
- **Greedy:** Expand best open node regardless of its depth
- **Uniform:** Expand the least-cost-so-far node until goal reached
- **A\*:** Like uniform search, but with heuristic information