CS3133 - A Term 2009: Foundations of Computer Science	Prof. Carolina Ruiz
Homework 4	
WPI	*By Li Feng, Shweta Srivastava, and Carolina Ruiz*

Chapter 7

Problem: **Chap 7.1**
Solution:

part a

This PDA accepts the language $L(G) = \left\{a^i b^j \mid 0 \leq j \leq i\right\}$.
part b

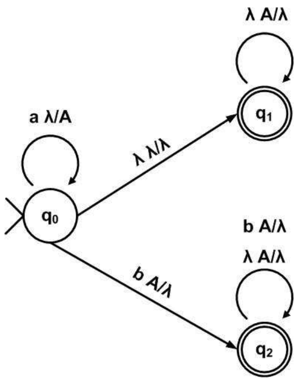

Figure 1: PDA for Chap7 1

part c

See Figures 2, 3, and 4.

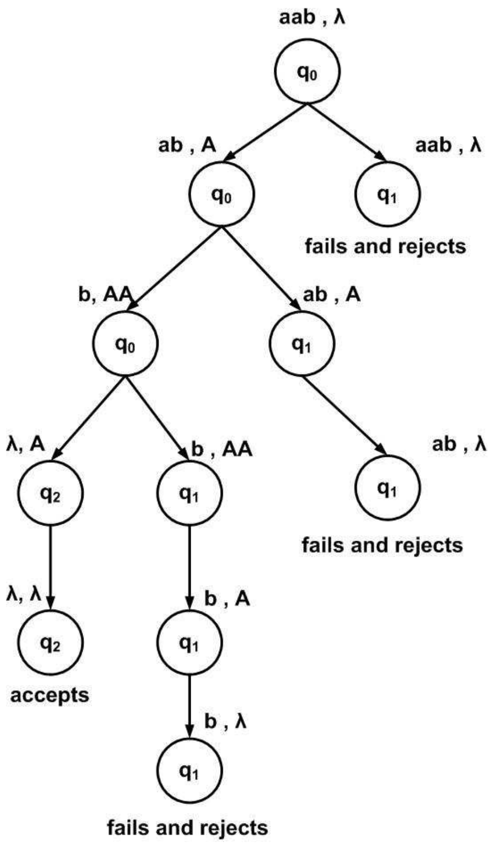

Figure 2: Execution Tree for string aab

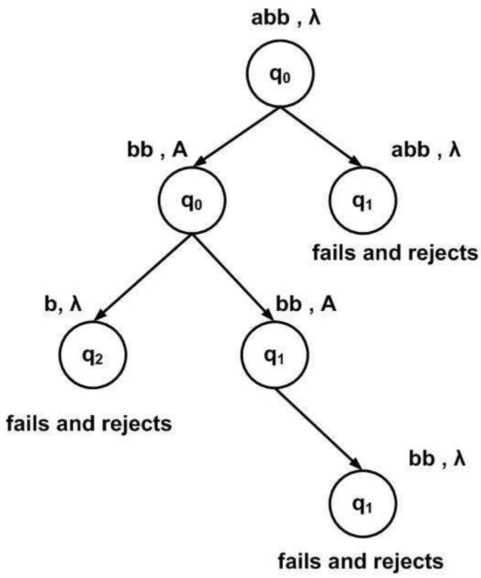

Figure 3: Execution Tree for string abb

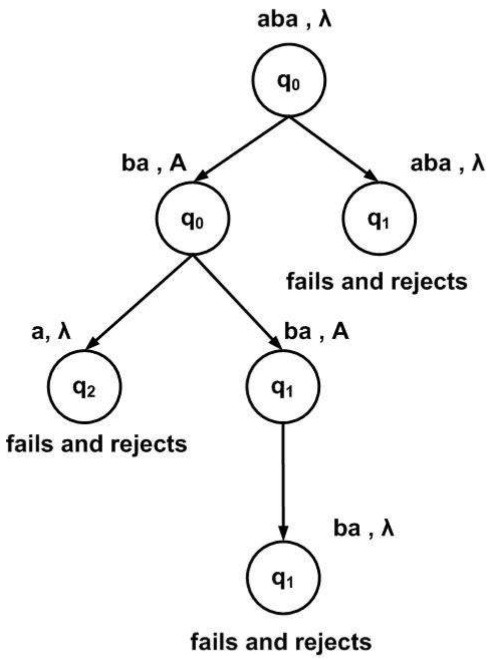

Figure 4: Execution Tree for string aba

part d To show $aabb$ and $aaab$ are in $L(M)$, we need to trace only a computation of M which accepts these strings.

	State	String	Stack
	$[q_0,$	$aabb,$	$\lambda]$
$\vdash$	$[q_0,$	$abb,$	$A]$
$\vdash$	$[q_0,$	$bb,$	$AA]$
$\vdash$	$[q_2,$	$b,$	$A]$
$\vdash$	$[q_2,$	$\lambda,$	$\lambda]$

	State	String	Stack
	$[q_0,$	$aaab,$	$\lambda]$
$\vdash$	$[q_0,$	$aab,$	$A]$
$\vdash$	$[q_0,$	$ab,$	$AA]$
$\vdash$	$[q_0,$	$b,$	$AAA]$
$\vdash$	$[q_2,$	$\lambda,$	$AA]$
$\vdash$	$[q_2,$	$\lambda,$	$A]$
$\vdash$	$[q_2,$	$\lambda,$	$\lambda]$

We can see that both $aabb$ and $aaab$ terminate at the accepting state q_2 with an empty stack. Both of them are in $L(M)$.

Problem: **Chap 7.3.f**

Solution:

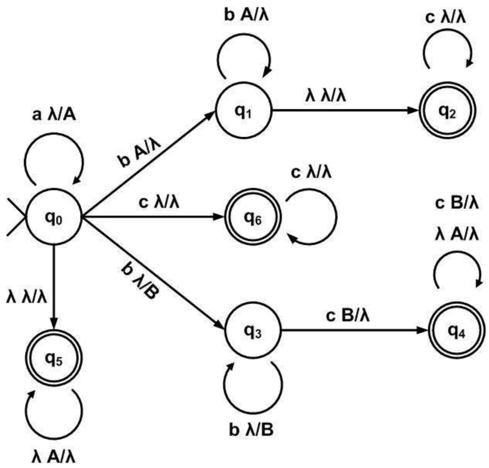

Figure 5: PDA for Chap7.3.f

Note, $L = \{a^i b^j c^k \mid i = j \text{ or } j = k\}$ contains some *special strings*:

- $\lambda, i = j = k = 0,$
- a^i, where $i > 0, j = k = 0,$ and
- c^k, where $k > 0, i = j = 0.$

Thus, our PDA should be able to accept these strings. As shown in Figure 5 , q_5 accepts λ and a^i, where $i > 0$; q_6 accepts string c^k, where $k > 0$; q_2 accepts string $a^i b^j c^k$, where $i = j$, and q_4 accepts string $a^i b^j c^k$, where $j = k$.

Problem: **Chap 7.3.j**
Solution:

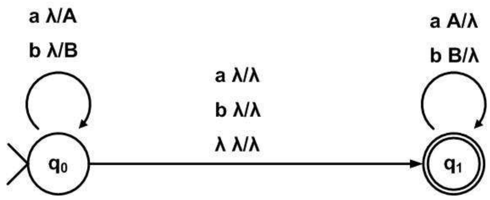

Figure 6: PDA for Chap7.3.j

Problem: **Chap 7.12**
Solution:

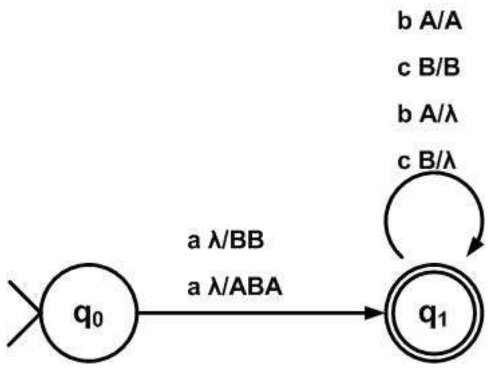

Figure 7: PDA for Chap7.12

Problem: **Chap 7.14**
Solution:

part a

See Figure 8.
part b

$$L(M) = \left\{a^i b^{2i} \mid i > 0\right\}$$

.

part c

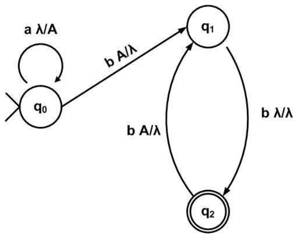

Figure 8: PDA for Chap7.14

Step 1: After adding transitions the new machine M' is as shown in Figure 9:

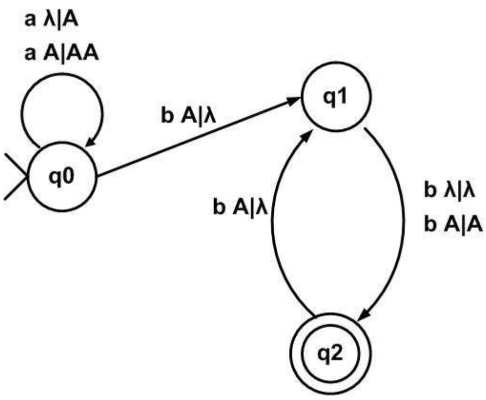

Figure 9: PDA M' for Chap7.14

Step 2: Generate grammar for every transition

Adding rules for S:
$S \rightarrow < q_0, \lambda, q_2 >$

Transition: $\delta(q_0, a, \lambda) = \{[q_0, A]\}$
Corresponding Rules in Grammar:
$< q_0, \lambda, q_0 > \rightarrow a < q_0, A, q_0 >$
$< q_0, \lambda, q_1 > \rightarrow a < q_0, A, q_1 >$
$< q_0, \lambda, q_2 > \rightarrow a < q_0, A, q_2 >$

Transition: $\delta(q_0, a, A) = \{[q_0, AA]\}$
Corresponding Rules in Grammar:
$< q_0, A, q_0 > \rightarrow a < q_0, A, q_0 >< q_0, A, q_0 >$
$< q_0, A, q_1 > \rightarrow a < q_0, A, q_0 >< q_0, A, q_1 >$
$< q_0, A, q_2 > \rightarrow a < q_0, A, q_0 >< q_0, A, q_2 >$

$< q_0, A, q_0 > \rightarrow a < q_0, A, q_1 >< q_1, A, q_0 >$
$< q_0, A, q_1 > \rightarrow a < q_0, A, q_1 >< q_1, A, q_1 >$
$< q_0, A, q_2 > \rightarrow a < q_0, A, q_1 >< q_1, A, q_2 >$

$< q_0, A, q_0 > \to a < q_0, A, q_2 >< q_2, A, q_0 >$
$< q_0, A, q_1 > \to a < q_0, A, q_2 >< q_2, A, q_1 >$
$< q_0, A, q_2 > \to a < q_0, A, q_2 >< q_2, A, q_2 >$

Transition: $\delta(q_0, b, A) = \{[q_1, \lambda]\}$
Corresponding Rules in Grammar:
$< q_0, A, q_0 > \to b < q_1, \lambda, q_0 >$
$< q_0, A, q_1 > \to b < q_1, \lambda, q_1 >$
$< q_0, A, q_2 > \to b < q_1, \lambda, q_2 >$

Transition: $\delta(q_1, b, \lambda) = \{[q_2, \lambda]\}$
Corresponding Rules in Grammar:
$< q_1, \lambda, q_0 > \to b < q_2, \lambda, q_0 >$
$< q_1, \lambda, q_1 > \to b < q_2, \lambda, q_1 >$
$< q_1, \lambda, q_2 > \to b < q_2, \lambda, q_2 >$

Transition: $\delta(q_1, b, A) = \{[q_2, A]\}$
Corresponding Rules in Grammar:
$< q_1, A, q_0 > \to b < q_2, A, q_0 >$
$< q_1, A, q_1 > \to b < q_2, A, q_1 >$
$< q_1, A, q_2 > \to b < q_2, A, q_2 >$

Transition: $\delta(q_2, b, A) = \{[q_1, \lambda]\}$
Corresponding Rules in Grammar:
$< q_2, A, q_0 > \to b < q_1, \lambda, q_0 >$
$< q_2, A, q_1 > \to b < q_1, \lambda, q_1 >$
$< q_2, A, q_2 > \to b < q_1, \lambda, q_2 >$

Finally:
$< q_0, \lambda, q_0 > \to \lambda$
$< q_1, \lambda, q_1 > \to \lambda$
$< q_2, \lambda, q_2 > \to \lambda$

part d

	State	String	Stack
$\vdash$	$[q_0,$	$aabbbb,$	$\lambda]$
$\vdash$	$[q_0,$	$abbbbb,$	$A]$
$\vdash$	$[q_0,$	$bbbb,$	$AA]$
$\vdash$	$[q_1,$	$bbb,$	$A]$
$\vdash$	$[q_2,$	$bb,$	$A]$
$\vdash$	$[q_1,$	$b,$	$\lambda]$
$\vdash$	$[q_2,$	$\lambda,$	$\lambda]$

part e

Derivation tree for the string $aabbbb$:

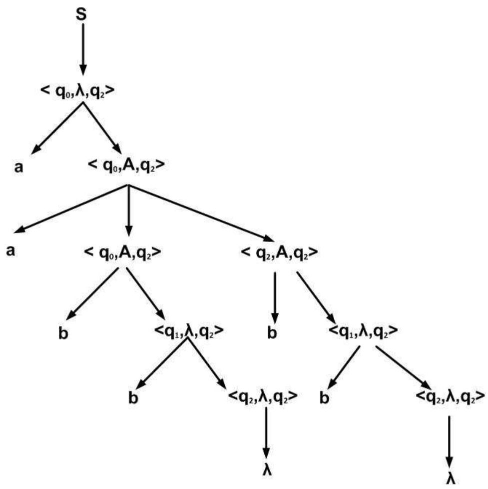

Figure 10: Derivation Tree for the string $aabbbb$

Derivation for the string $aabbbb$:

$$\begin{array}{l} \mathrm{DerivationSteps} \\ S \Rightarrow < q_0, \lambda, q_2 > \\ \Rightarrow a < q_0, A, q_2 > \\ \Rightarrow aa < q_0, A, q_2 >< q_2, A, q_2 > \\ \Rightarrow aab < q_1, \lambda, q_2 >< q_2, A, q_2 > \\ \Rightarrow aabb < q_2, \lambda, q_2 >< q_2, A, q_2 > \\ \Rightarrow aabb < q_2, A, q_2 > \\ \Rightarrow aabbb < q_1, \lambda, q_2 > \\ \Rightarrow aabbbb < q_2, \lambda, q_2 > \\ \Rightarrow aabbbb \end{array}$$

Problem: **Chap 7.17.c**
$L = \left\{a^i b^{2i} a^i\right\}$ is not context free.

Solution:

Proof

Assume L is a context free language, and let k be the number specified for L by the pumping lemma for context free languages. Let $z = a^k b^{2k} a^k$. Clearly, $z \in L$ and $length(z) > k$. By the pumping lemma z can be written as $uvwxy$, where:

1. $length(vwx) \leq k$
2. $length(v) + length(x) > 0$
3. $uv^i wx^i y \in L$, for all $i \geq 0$

For the string $z = a^k b^{2k} a^k$, consider the possibilities for the substring v and x. We will find that:

- Case 1:
  if either of v or x contains more than one type of terminal symbol, then uv^2wx^2y results in a string which contains as between bs. The result string would not be in L.

- Case 2:
  If both v and x contains only as, then uv^2wx^2y will have the same number of bs as string $uvwxy$ while it contains more as than string $uvwxy$. Thus, uv^2wx^2y is not in L. Similarly, we can see that uv^2wx^2y would not be in L when both v and x contains only bs.

- Case 3:
  If one of v and x is empty string(λ), and the other one contains only as. Condition $length(v) + length(x) > 0$ implies that v and x could not be λ at the same time. Then, similar to Case 3, uv^2wx^2y will have the same number of bs as string $uvwxy$ while it contains more as than string $uvwxy$. Thus, uv^2wx^2y is not in L. Similarly, we can see the uv^2wx^2y would not be in L when one of v and x is λ and the other contains only bs.

- Case 4:
  if v contains only as and x contains only bs. This implies that vwx is from the substring of $a^k b^k$. For the string uv^2wx^2y, the number of as before bs is larger than the number of as after bs. Thus, $uv^2wx^2y \notin L$.

- Case 5:
  if v contains only bs and x contains only as. This implies that vwx is from the substring of $b^k a^k$. For the string uv^2wx^2y, the number of as before bs is smaller than the number of as after bs. Thus, $uv^2wx^2y \notin L$.

Based on the discussion in Cases 1-5, there is no decomposition of z satisfying the condition of pumping lemma for context free languages. So, L is not context free language.

Problem: **Chap 7 18 a Solution:**

The language $L_1(G) = \left\{a^i b^{2i} c^j \mid i, j \geq 0\right\}$ can be accepted by the PDA shown in Figure 11.
Or one can construct a CFL grammar G, where $L(G) = L_1$:

$$S \to AC$$
$$A \to aAbb|\lambda$$
$$C \to cC \mid \lambda$$

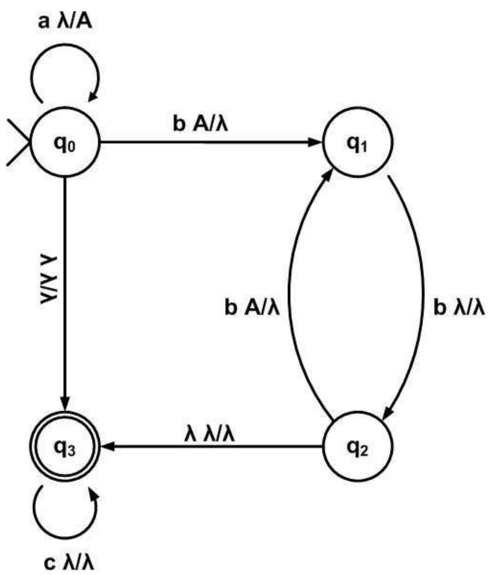

Figure 11: PDA for Chap7 18