Higher-Order Functions and Loops

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Remember quicksort?

Answer: (list 1 2 3 6 7 9)
Warm Up: Sorting a List of Numbers

Let’s write quicksort.
As usual, start with the template for list[num]

;; quicksort : list[num] → list[num]
;; sorts a list of nums into increasing order
(define (quicksort alon)
  (cond [(empty? alon) …]
        [(cons? alon) …
          (first alon) …
          (quicksort (rest alon)) … ]))
Warm Up: Sorting a List of Numbers

What do the pieces in the cons? case give us?

;; quicksort : list[num] \rightarrow list[num]
;; sorts a list of nums into increasing order
(define (quicksort alon)
  (cond [(empty? alon) \ldots]
        [(cons? alon) \ldots
         (first alon) \ldots
         (quicksort (rest alon)) \ldots \ldots]]))

sorts the rest of the list
into increasing order
Warm Up: Sorting a List of Numbers

So, how do we combine them? We need to insert the first element into the sorted rest of the list …

;; quicksort : list[num] → list[num]
;; sorts a list of nums into increasing order
(define (quicksort alon)
  (cond [(empty? alon) …]
    [(cons? alon) …
      (first alon) …
      (quicksort (rest alon)) … ]))

But that’s insertion sort!

sorts the rest of the list into increasing order
Writing quicksort via templates

We got insertion-sort. What happened?

• With templates, you write programs according to the “natural” recursion
• Insertion-sort is the naturally recursive sort
• Quicksort uses recursion in a different way

Moral: some algorithms need different forms of recursion (“generative recursion” – see HTDP).

Templates aren’t a catch-all for program design (but they are still very useful for lots of programs)
Quicksort: Take 2

The template is fine until the natural recursion, so we’ll take that out and leave the rest intact …

;; quicksort : list[num] → list[num]
;; sorts a list of nums into increasing order
(define (quicksort alon)
  (cond [(empty? alon) …]
        [(cons? alon) …
         (first alon) …
         (quicksort (rest alon)) … ]))

How did quicksort work? Gather the elts smaller than (first alon); gather those larger; sort; and combine:
Quicksort: Take 2

How did quicksort work? Gather the elts smaller than 
(first alon); gather those larger; sort; and combine:

;; quicksort : list[num] → list[num]
;; sorts a list of nums into increasing order
(define (quicksort alon)
  (cond [(empty? alon) …
     [(cons? alon) …
        (smaller-than (first alon) (rest alon)) …
        (larger-than (first alon) (rest alon)) … ]]))

[we’ll write smaller-than, larger-than later]
Quicksort: Take 2

How did quicksort work? Gather the elts smaller than (first alon); gather those larger; sort; and combine:

;; quicksort : list[num] \(\rightarrow\) list[num]
;; sorts a list of nums into increasing order
(define (quicksort alon)
  (cond [(empty? alon) …]
        [(cons? alon) …
         (quicksort (smaller-than (first alon) (rest alon)))
         (quicksort (larger-than (first alon) (rest alon))))])

sort the smaller elts
sort the larger elts
Quicksort: Take 2

How did quicksort work? Gather the elts smaller than (first alon); gather those larger; sort; and combine:

;; quicksort : list[num] → list[num]
;; sorts a list of nums into increasing order
(define (quicksort alon)
  (cond [(empty? alon) …]
     [(cons? alon)
      (append
       (quicksort (smaller-than (first alon) (rest alon)))
       (list (first alon)) [don’t forget the pivot!]
       (quicksort (larger-than (first alon) (rest alon)))))
     ]))

[append (built in) takes any number of lists and “concatenates” them]
Quicksort: Take 2

The main quicksort program
(shown with a local name for the pivot)

;; quicksort : list[num] → list[num]
;; sorts a list of nums into increasing order
(define (quicksort alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
         [(local [(define pivot (first alon))]
          (append
           (quicksort (smaller-than pivot (rest alon)))
           (list pivot)
           (quicksort (larger-than pivot (rest alon))))])))
Quicksort: Take 2

The main quicksort program
But where are smaller-than and larger-than?

;; quicksort : list[num] → list[num]
;; sorts a list of nums into increasing order
(define (quicksort alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
         (local [(define pivot (first alon))]
           (append
            (quicksort (smaller-than pivot (rest alon)))
            (list pivot)
            (quicksort (larger-than pivot (rest alon))))])))
Smaller-than and Larger-than

;; smaller-than : num list[num] → list[num]
;; returns elts in input list that are smaller than given num
(define (smaller-than anum alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
         (cond [(< (first alon) anum)
                  (cons (first alon) (smaller-than anum (rest alon)))]
              [else (smaller-than anum (rest alon)))])))

;; larger-than : num list[num] → list[num]
;; returns elts in input list that are larger than given num
(define (larger-than anum alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
         (cond [ (> (first alon) anum)
                  (cons (first alon) (larger-than anum (rest alon)))]
              [else (larger-than anum (rest alon)))]))
Smaller-than and Larger-than

;; smaller-than : num list[num] → list[num]
;; returns elts in input list that are smaller than given num
(define (smaller-than anum alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
         (cond [(< (first alon) anum)
                (cons (first alon) (smaller-than anum (rest alon)))]
              [else (smaller-than anum (rest alon))])]))

;; larger-than : num list[num] → list[num]
;; returns elts in input list that are larger than given num
(define (larger-than anum alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
         (cond [(> (first alon) anum)
                (cons (first alon) (larger-than anum (rest alon)))]
              [else (larger-than anum (rest alon))])]))

these programs are identical aside from < and >; can’t we share the similar code?

Normally, we share similar code by creating parameters for the different parts
Sharing Smaller- and Larger-than code

;; extract-nums : num list[num] → list[num]
;; returns elts in input list that compare to the given num
(define (extract-nums anum alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
         (cond [(compare (first alon) anum)
                (cons (first alon) (extract-nums anum (rest alon)))
                [else (extract-nums anum (rest alon))])]))

;; larger-than : num list[num] → list[num]
;; returns elts in input list that are larger than given num
(define (larger-than anum alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
         (cond [(> (first alon) anum)
                (cons (first alon) (larger-than anum (rest alon)))
                [else (larger-than anum (rest alon))])]))

First, replace the different part with a new name
[larger-than here for reference]
Sharing Smaller- and Larger-than code

;; extract-nums : num list[num] → list[num]
;; returns elts in input list that compare to the given num
(define (extract-nums anum compare alon)
  (cond [(empty? alon) empty]
      [(cons? alon)
       (cond [(compare (first alon) anum)
              (cons (first alon) (extract-nums anum compare (rest alon)))]
            [else (extract-nums anum compare (rest alon))])]))

;; larger-than : num list[num] → list[num]
;; returns elts in input list that are larger than given num
(define (larger-than anum alon)
  (cond [(empty? alon) empty]
      [(cons? alon)
       (cond [(> (first alon) anum)
              (cons (first alon) (larger-than anum (rest alon)))]
            [else (larger-than anum (rest alon))])]))
Sharing Smaller- and Larger-than code

;;; extract-nums : num list[num] → list[num]
;;; returns elts in input list that compare to the given num
(define (extract-nums anum compare alon)
  (cond [(empty? alon) empty]
       [(cons? alon)
        (cond [(compare (first alon) anum)
               (cons (first alon) (extract-nums anum compare (rest alon)))
               [else (extract-nums anum compare (rest alon))])])
)

;;; larger-than : num list[num] → list[num]
;;; returns elts in input list that are larger than given num
(define (larger-than anum alon)
  (extract-nums anum ____ alon))

Next, redefine larger-than in terms of extract-nums ...

But what can we send as the argument to the compare parameter?
Sharing Smaller- and Larger-than code

;; extract-nums : num list[num] → list[num]
;; returns elts in input list that compare to the given num
(define (extract-nums anum compare alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
         (cond [(compare (first alon) anum)
                (cons (first alon) (extract-nums anum compare (rest alon)))]
              [else (extract-nums anum compare (rest alon))])]))

;; larger-than : num list[num] → list[num]
;; returns elts in input list that are larger than given num
(define (larger-than anum alon)
  (extract-nums anum > alon))

Next, redefine larger-than in terms of extract-nums …

But what can we send as the argument to the compare parameter?

We can send the > operator itself!
Sharing Smaller- and Larger-than code

;; extract-nums : num list[num] → list[num]
;; returns elts in input list that compare to the given num
(define (extract-nums anum compare alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
         (cond [(compare (first alon) anum)
                  (cons (first alon) (extract-nums anum compare (rest alon)))]
               [else (extract-nums anum compare (rest alon))])]))

;; larger-than : num list[num] → list[num]
;; returns elts in input list that are larger than given num
(define (larger-than anum alon)
  (extract-nums anum > alon))

;; smaller-than : num list[num] → list[num]
;; returns elts in input list that are smaller than given num
(define (smaller-than anum alon)
  (extract-nums anum < alon))
Sharing Smaller- and Larger-than code

;;; extract-nums : num (num num → bool) list[num] → list[num]
;;; returns elts in input list that compare to the given num
(define (extract-nums anum compare alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
         (cond [(compare (first alon) anum)
                (cons (first alon) (extract-nums anum compare (rest alon)))]
               [else (extract-nums anum compare (rest alon))])]))

;;; larger-than : num list[num] → list[num]
;;; returns elts in input list that are larger than given num
(define (larger-than anum alon)
  (extract-nums anum > alon))

;;; smaller-than : num list[num] → list[num]
;;; returns elts in input list that are smaller than given num
(define (smaller-than anum alon)
  (extract-nums anum < alon))

We need to fix the contract. What’s the contract on compare?
Functions are values in Scheme

This means we can pass them as arguments to functions

We can also return them from functions (but hold that thought for now)
Where else can we use extract-nums?

;; extract-nums : num (num num → bool) list[num] → list[num]
;; returns elts in input list that compare to the given num
(define (extract-nums anum compare alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
         (cond [(compare (first alon) anum)
                (cons (first alon) (extract-nums anum compare (rest alon))))]
              [else (extract-nums anum compare (rest alon))]]))

Extract-nums extracts numbers from lists of numbers

What if we wanted to extract all boas that eat pets or mice from a list of boas?
extract-eats-pets-or-mice

;; extract-nums : num (num num → bool) list[num] → list[num]
;; returns elts in input list that compare to the given num
(define (extract-nums anum compare alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
         (cond [(compare (first alon) anum)
                (cons (first alon) (extract-nums anum compare (rest alon)))]
               [else (extract-nums anum compare (rest alon))])]))

;; extract-eats-pets-or-mice : list[boa] → list[boa]
;; returns boas in input list that eat pets or mice
(define (extract-eats-pets-or-mice aloboa)
  (cond [(empty? aloboa) empty]
        [(cons? aloboa)
         (cond [(or (symbol=? 'pets (boa-food (first aloboa)))
                (symbol=? 'mice (boa-food (first aloboa))))
                (cons (first aloboa) (extract-eats-pets-or-mice (rest aloboa)))]
               [else (extract-eats-pets-or-mice (rest aloboa))])]))
extract-eats-pets-or-mice/extract-nums

;; extract-nums : num (num num → bool) list[num] → list[num]
;; returns els in input list that compare to the given num
(define (extract-nums anum compare alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
         (cond [(compare (first alon) anum)
                (cons (first alon) (extract-nums anum compare (rest alon)))]
               [else (extract-nums anum compare (rest alon))])]))

;; extract-eats-pets-or-mice : list[boa] → list[boa]
;; returns boas in input list that eat pets or mice
(define (extract-eats-pets-or-mice aloboa)
  (cond [(empty? aloboa) empty]
        [(cons? aloboa)
         (cond [(or (symbol=? 'pets (boa-food (first aloboa)))
                  (symbol=? 'mice (boa-food (first aloboa))))
                (cons (first aloboa) (extract-eats-pets-or-mice (rest aloboa)))]
               [else (extract-eats-pets-or-mice (rest aloboa))])]))
Let’s write one function that captures both

How are these two expressions similar?

```scheme
;; extract-nums : num (num num → bool) list[num] → list[num]
;; returns elts in input list that compare to the given num
(define (extract-nums anum compare alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
          (cond [(compare (first alon) anum)
                 (cons (first alon) (extract-nums anum compare (rest alon))))
                [else (extract-nums anum compare (rest alon))])])

;; extract-eats-pets-or-mice : list[boa] → list[boa]
;; returns boas in input list that eat pets or mice
(define (extract-eats-pets-or-mice aloboa)
  (cond [(empty? aloboa) empty]
        [(cons? aloboa)
          (cond [(or (symbol=? ‘pets (boa-food (first aloboa)))
                  (symbol=? ‘mice (boa-food (first aloboa))))
                 (cons (first aloboa) (extract-eats-pets-or-mice (rest aloboa))))
                [else (extract-eats-pets-or-mice (rest aloboa))])])
```

Let’s write one function that captures both

How are these two expressions similar?

```scheme
;; extract-nums : num (num num → bool) list[num] → list[num]
;; returns elts in input list that compare to the given num
(define (extract-nums anum compare alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
          (cond [(compare (first alon) anum)
                 (cons (first alon) (extract-nums anum compare (rest alon))))
                [else (extract-nums anum compare (rest alon))])])

;; extract-eats-pets-or-mice : list[boa] → list[boa]
;; returns boas in input list that eat pets or mice
(define (extract-eats-pets-or-mice aloboa)
  (cond [(empty? aloboa) empty]
        [(cons? aloboa)
          (cond [(or (symbol=? ‘pets (boa-food (first aloboa)))
                  (symbol=? ‘mice (boa-food (first aloboa))))
                 (cons (first aloboa) (extract-eats-pets-or-mice (rest aloboa))))
                [else (extract-eats-pets-or-mice (rest aloboa))])])
```
Both do a comparison on the first elt

Both expressions return booleans
extract-eats-pets-or-mice/extract-nuns

;; extract-nuns : num (num num → bool) list[num] → list[num]
;; returns elts in input list that compare to the given num
(define (extract-nuns anum compare alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
           (cond [(compare (first alon) anum)
                  (cons (first alon) (extract-nuns anum compare (rest alon)))]
                [else (extract-nuns anum compare (rest alon))])]))

;; extract-eats-pets-or-mice : list[boa] → list[boa]
;; returns boas in input list that eat pets or mice
(define (extract-eats-pets-or-mice aloboa)
  (cond [(empty? aloboa) empty]
        [(cons? aloboa)
           (cond [(or (symbol=? 'pets (boa-food (first aloboa)))
                  (symbol=? 'mice (boa-food (first aloboa))))
                  (cons (first aloboa) (extract-eats-pets-or-mice (rest aloboa)))]
                [else (extract-eats-pets-or-mice (rest aloboa))])]))

Compares first against one datum

Compares first against two data
Summary: What’s in common?

• Both expressions perform some comparison on the first elt of the list
• Both comparisons return booleans
• But, the expressions use different numbers of additional information in their comparisons

So, to collapse these expressions into a common definition, they need to take the first elt and return a boolean …
extract-eats-pets-or-mice/extract-nums

;; extract-nums : num (num num → bool) list[num] → list[num]
;; returns elts in input list that compare to the given num
(define (extract-nums anum compare alon)
  (cond [(empty? alon) empty]
       [(cons? alon) (cond [(compares-to-num? (first alon))
                               (cons (first alon) (extract-nums anum compare (rest alon)))]
                       [else (extract-nums anum compare (rest alon))])]

;; extract-eats-pets-or-mice : list[boa] → list[boa]
;; returns boas in input list that eat pets or mice
(define (extract-eats-pets-or-mice aloboa)
  (cond [(empty? aloboa) empty]
       [(cons? aloboa) (cond [(food-is-pets-or-mice? (first aloboa))
                                (cons (first aloboa) (extract-eats-pets-or-mice (rest aloboa)))]
                          [else (extract-eats-pets-or-mice (rest aloboa))])])

Rewritten in terms of functions from first → bool
extract-eats-pets-or-mice/extract-nums

;; extract-nums : list[num] → list[num]
;; returns elts in input list that compare to the given num
(define (extract-nums alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
         (cond [(compares-to-num? (first alon))
            (cons (first alon) (extract-nums (rest alon)))]
            [else (extract-nums (rest alon))])]))

;; extract-eats-pets-or-mice : list[boa] → list[boa]
;; returns boas in input list that eat pets or mice
(define (extract-eats-pets-or-mice aloboa)
  (cond [(empty? aloboa) empty]
        [(cons? aloboa)
         (cond [(food-is-pets-or-mice? (first aloboa))
            (cons (first aloboa) (extract-eats-pets-or-mice (rest aloboa)))]
            [else (extract-eats-pets-or-mice (rest aloboa))])]))

Remove compare and anum parameters since extract-nums no longer uses them.
Now, these two functions look identical minus the name of the comparison function …
extract-elts

;; extract-elts : list[num] → list[num]
;; returns elts in input list that satisfy keep? predicate
(define (extract-elts keep? alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
           (cond [(keep? (first alon))
                      (cons (first alon) (extract-elts keep? (rest alon)))]
                 [else (extract-elts keep? (rest alon))]]))

extract-eats-pets-or-mice

;; extract-eats-pets-or-mice : list[boa] → list[boa]
;; returns boas in input list that eat pets or mice
(define (extract-eats-pets-or-mice aloboa)
  (cond [(empty? aloboa) empty]
        [(cons? aloboa)
           (cond [(food-is-pets-or-mice? (first aloboa))
                      (cons (first aloboa) (extract-eats-pets-or-mice (rest aloboa)))]
                 [else (extract-eats-pets-or-mice (rest aloboa))]]))

Make the name of the comparison function a parameter. We use keep?
Since the comparison determines whether we keep an elt in the output.
extract-elts and extract-eats

;; extract-elts : list[num] \toypes list[num]
;; returns els in input list that satisfy keep? predicate
(define (extract-elts keep? alon)
  (cond [(empty? alon) empty]
      [(cons? alon)
        (cond [(keep? (first alon))
          (cons (first alon) (extract-elts keep? (rest alon)))]
          [else (extract-elts keep? (rest alon))])]))

;; extract-eats-pets-or-mice : list[boa] \toypes list[boa]
;; returns boas in input list that eat pets or mice
(define (extract-eats-pets-or-mice aloboa)
  (extract-elts food-is-pets-or-mice? aloboa))

;; food-is-pets-or-mice? : boa \toypes boolean
;; determines whether boa’s food is ‘pets or ‘mice
(define (food-is-pets-or-mice? aboa)
  (or (symbol=? (boa-food aboa) ‘pets)
      (symbol=? (boa-food aboa) ‘mice))))
extract-elts and extract-eats

;; extract-elts : list[num] → list[num]
;; returns els in input list that satisfy keep? predicate
(define (extract-elts keep? alon)
  (cond [(empty? alon) empty]
    [(cons? alon)
      (cond [(keep? (first alon))
            (cons (first alon) (extract-elts keep? (rest alon)))]
            [else (extract-elts keep? (rest alon))])]
    ))

;; extract-eats-pets-or-mice : list[boa] → list[boa]
;; returns boas in input list that eat pets or mice
(define (extract-eats-pets-or-mice aloboa)
  (extract-elts food-is-pets-or-mice? aloboa))

;; food-is-pets-or-mice? : boa → boolean
;; determines whether boa’s food is ‘pets or ‘mice
(define (food-is-pets-or-mice? aboa)
  (or (symbol=? (boa-food aboa) ‘pets)
      (symbol=? (boa-food aboa) ‘mice))))
extract-elts and extract-eats

;; extract-elts : list[α] → list[num]
;; returns elts in input list that satisfy keep? predicate
(define (extract-elts keep? alon)
  (cond [(empty? alon) empty]
       [(cons? alon)
        (cond [(keep? (first alon))
               (cons (first alon) (extract-elts keep? (rest alon)))]
              [else (extract-elts keep? (rest alon))])]))

;; extract-eats-pets-or-mice : list[boa] → list[boa]
;; returns boas in input list that eat pets or mice
(define (extract-eats-pets-or-mice aloboa)
  (extract-elts food-is-pets-or-mice? aloboa))

;; food-is-pets-or-mice? : boa → boolean
;; determines whether boa’s food is ‘pets or ‘mice
(define (food-is-pets-or-mice? aboa)
  (or (symbol=? (boa-food aboa) ‘pets)
      (symbol=? (boa-food aboa) ‘mice))))

Nothing in the defn of extract-elts requires numbers, so we can relax the contract to allow input lists of any type. α is just a variable over types.
extract-elts and extract-eats

;; extract-elts : list[α] → list[α]
;; returns els in input list that satisfy keep? predicate
(define (extract-elts keep? alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
         (cond [(keep? (first alon))
               (cons (first alon) (extract-elts keep? (rest alon)))]
               [else (extract-elts keep? (rest alon))])]))

;; extract-eats-pets-or-mice : list[boa] → list[boa]
;; returns boas in input list that eat pets or mice
(define (extract-eats-pets-or-mice aloboa)
  (extract-elts food-is-pets-or-mice? aloboa))

;; food-is-pets-or-mice? : boa → boolean
;; determines whether boa’s food is ‘pets or ‘mice
(define (food-is-pets-or-mice? aboa)
  (or (symbol=? (boa-food aboa) ‘pets)
      (symbol=? (boa-food aboa) ‘mice))))
extract-elts and extract-eats

;; extract-elts : list[α] → list[α]
;; returns elts in input list that satisfy keep? predicate
(define (extract-elts keep? alon)
  (cond [(empty? alon) empty]
        [(cons? alon)
         (cond [(keep? (first alon))
                (cons (first alon) (extract-elts keep? (rest alon)))]
               [else (extract-elts keep? (rest alon))])))

;; extract-eats-pets-or-mice : list[boa] → list[boa]
;; returns boas in input list that eat pets or mice
(define (extract-eats-pets-or-mice aloboa)
  (extract-elts food-is-pets-or-mice? aloboa))

;; food-is-pets-or-mice? : boa → boolean
;; determines whether boa’s food is ‘pets or ‘mice
(define (food-is-pets-or-mice? aboa)
  (or (symbol=? (boa-food aboa) ‘pets)
      (symbol=? (boa-food aboa) ‘mice))))
We also never added `keep?` to the contract.

What is `keep?'s type?

Keep? takes an elt of the list and returns a boolean.
Filter

extract-elts is built-in. It’s called filter

;; filter : (α → bool) list[α] → list[α]
;; returns list of elts in input list that satisfy keep? predicate
(define (filter keep? alst)
  (cond [(empty? alst) empty]
        [(cons? alst)
           (cond [(keep? (first alst))
                      (cons (first alst) (filter keep? (rest alst)))]
                 [else (filter keep? (rest alst))])]))

Use filter whenever you want to extract elts from a list according to some predicate
Back to smaller-than and larger-than

Rewrite these in terms of filter ...

;; filter : (α → bool) list[α] → list[α]

;; larger-than : num list[num] → list[num]
;; returns elts in input list that are larger than given num
(define (larger-than anum alon)
  (extract-nums anum > alon))

;; smaller-than : num list[num] → list[num]
;; returns elts in input list that are smaller than given num
(define (smaller-than anum alon)
  (extract-nums anum < alon))

Must replace the calls to extract-nums with calls to filter
Back to smaller-than and larger-than

Rewrite these in terms of filter …

;; filter : (α → bool) list[α] → list[α]

;; larger-than : num list[num] → list[num]
;; returns elts in input list that are larger than given num
;; was (extract-nums anum > alon)
;; (filter _______ alon)

;; smaller-than : num list[num] → list[num]
;; returns elts in input list that are smaller than given num
;; was (extract-nums anum < alon)

What do we pass as keep?

Need a function that consumes a num and returns a bool; function must compare input to anum …
Back to smaller-than and larger-than

Rewrite these in terms of filter …

We’d like something like

```
(make-function (elt) (> elt anum))
```

that takes a list of parameters and the body of the function

Need a function that consumes a num and returns a bool; function must compare input to anum …
Back to smaller-than and larger-than

Rewrite these in terms of filter …

;; filter : (α → bool) list[α] → list[α]

;; larger-than : num list[num] → list[num]
;; returns elts in input list that are larger than given num
(define (larger-than anum alon)
  (filter (lambda (elt) (> elt anum)) alon))
;; was (extract-nums anum > alon))

;; smaller-than : num list[num] → list[num]
;; returns elts in input list that are smaller than given num
(define (smaller-than anum alon)
  (extract-nums anum < alon))

make-function
exists in Scheme …

It’s called \texttt{lambda}
Back to smaller-than and larger-than

Rewrite these in terms of filter ...  

;; filter : (α → bool) list[α] → list[α]

;; larger-than : num list[num] → list[num]
;; returns elts in input list that are larger than given num
(define (larger-than anum alon)
   (filter (lambda (elt) (> elt anum)) alon))

;; smaller-than : num list[num] → list[num]
;; returns elts in input list that are smaller than given num
(define (smaller-than anum alon)
   (filter (lambda (elt) (< elt anum)) alon))

We can rewrite smaller-than in the same way
Back to smaller-than and larger-than

Rewrite these in terms of filter …

;; filter : (α → bool) list[α] → list[α]

;; larger-than : num list[num] → list[num]
;; returns elts in input list that are larger than given num
(define (larger-than anum alon)
  (local [(define (compare elt) (> elt anum))]
    (filter compare alon)))

;; smaller-than : num list[num] → list[num]
;; returns elts in input list that are smaller than given num
(define (smaller-than anum alon)
  (local [(define (compare elt) (< elt anum))]
    (filter compare alon)))

We could also write this without using lambda by using local

Either lambda or local is fine
Summary: What have we seen?

• Functions are values and can be passed as arguments to other functions
  – This lets us share code between similar functions
• Scheme provides lambda to make new functions
• We can pass functions created with define or functions created with lambda as arguments

Actually, (define (square n) (* n n)) is a shorthand for (define square (lambda (n) (* n n)))
Summary: What have we seen?

• We’ve also seen filter, which takes a function (predicate) and a list and returns a list of elements in the list for which the function returns true.

• Filter provides a nice, compact way of writing certain Scheme functions
Using Filter

;; filter : (α → bool) list[α] → list[α]
;; returns list of elts in input list that satisfy keep? predicate
(define (filter keep? alst)
  (cond [(empty? alst) empty]
        [(cons? alst)
         (cond [(keep? (first alst))
                (cons (first alst) (filter keep? (rest alst)))]
               [else (filter keep? (rest alst))]])
  )

Let’s use filter to get the list of all foods that a list of boas will eat

Example: (all-foods (list (make-boa ‘Slinky 10 ‘pets)
                   (make-boa ‘Curly 55 ‘rice)
                   (make-boa ‘Slim 15 ‘lettuce))
  = (list ‘pets ‘rice ‘lettuce)
Using Filter

;; filter : (α → bool) list[α] → list[α]
;; returns list of elts in input list that satisfy keep? predicate
(define (filter keep? alst)
  (cond [(empty? alst) empty]
    [(cons? alst)
      (cond [(keep? (first alst))
            (cons (first alst) (filter keep? (rest alst)))]
          [else (filter keep? (rest alst))])]))

;; all-foods : list[boa] → list[symbol]
;; given a list of boas, extracts a list of the foods that the boas eat
(define (all-foods aloboa)
  (filter (lambda (aboa) …) aloboa))

[What goes in the body of the lambda?]
Using Filter

;; filter : (α → bool) list[α] → list[α]
;; returns list of elts in input list that satisfy keep? predicate
(define (filter keep? alst)
  (cond [(empty? alst) empty]
        [(cons? alst)
           (cond [(keep? (first alst))
                  (cons (first alst) (filter keep? (rest alst)))]
                 [else (filter keep? (rest alst))])])

;; all-foods : list[boa] → list[symbol]
;; given a list of boas, extracts a list of the foods that the boas eat
(define (all-foods aloboa)
  (filter (lambda (aboa) (boa-food aboa)) aloboa))

How about we simply extract the boa’s food?
Using Filter

;; filter : (α → bool) list[α] → list[α]
;; returns list of elts in input list that satisfy keep? predicate
(define (filter keep? alst)
  (cond [(empty? alst) empty]
       [(cons? alst)
         (cond [(keep? (first alst))
               (cons (first alst) (filter keep? (rest alst)))]
               [else (filter keep? (rest alst)))]))

;; all-foods : list[boa] → list[symbol]
;; given a list of boas, extracts a list of the foods that the boas eat
(define (all-foods aloboa)
  (filter (lambda (aboa) (boa-food aboa)) aloboa))

How about we simply extract the boa’s food?

Look at the contract – does boa-food return a boolean?

No, it returns a symbol …
Also, filter would return list[boa], not list[symbol] …
Using Filter

;; filter : (α → bool) list[α] → list[α]
;; returns list of elts in input list that satisfy keep? predicate
(define (filter keep? alst)
  (cond [(empty? alst) empty]
        [(cons? alst)
          (cond [(keep? (first alst))
                (cons (first alst) (filter keep? (rest alst)))]
                [else (filter keep? (rest alst))])]))

Filter returns a list of the same type as the input list, and is designed to leave some elements out.

all-foods must return a list of a different type, but with information gathered from every element of the input list

We need another function that takes a function and a list (like filter does), but with slightly different behavior
Map: Transforms a list

;; map : (α → β) list[α] → list[β]
;; returns list of results from applying function to every elts in input list
(define (map f alst)
  (cond [(empty? alst) empty]
        [(cons? alst) (cons (f (first alst)) (map f (rest alst)))]))

Filter returns a list of the same type as the input list, and is designed to leave some elements out.

all-foods must return a list of a different type, but with information gathered from every element of the input list

We need another function (map) that takes a function and a list (like filter does), but with slightly different behavior
Implementing all-foods with map

;; map : (α → β) list[α] → list[β]
;; returns list of results from applying function to every elts in input list
(define (map f alst)
    (cond [(empty? alst) empty]
          [(cons? alst) (cons (f (first alst)) (map f (rest alst)))]))

;; all-foods : list[boa] → list[symbol]
;; given a list of boas, extracts a list of the foods that the boas eat
(define (all-foods aloboa)
    (map __________ aloboa))

What function do we want to apply to each boa?
Implementing all-foods with map

;; map : (α → β) list[α] → list[β]
;; returns list of results from applying function to every elts in input list
(define (map f alst)
   (cond [(empty? alst) empty]
         [(cons? alst) (cons (f (first alst)) (map f (rest alst)))]))

;; all-foods : list[boa] → list[symbol]
;; given a list of boas, extracts a list of the foods that the boas eat
(define (all-foods aloboa)
   (map (lambda (aboa) (boa-food aboa)) aloboa))

What function do we want to apply to each boa?
boa-food
Implementing all-foods with map

;; map : (α → β) list[α] → list[β]
;; returns list of results from applying function to every elts in input list
(define (map f alst)
  (cond [(empty? alst) empty]
        [(cons? alst) (cons (f (first alst)) (map f (rest alst))))])

;; all-foods : list[boa] → list[symbol]
;; given a list of boas, extracts a list of the foods that the boas eat
(define (all-foods aloboa)
  (map boa-food aloboa))

Actually, we could write this more concisely
(since boa-food is already a function from boa → symbol)
Using map and filter together

Given a zoo (a list of boas and armadillos), how can we get the list of all foods eaten by the boas (ignoring the armadillos)?

;; all-boa-foods : list[animal] → list[symbol]
;; given a list of animals, extracts a list of the foods that the boas eat
(define (all-boa-foods aloboa)
  (map boa-food (filter boa? aloboa)))
Summary

• map and filter are Scheme’s looping constructs
• Each loops over the elements of a list
  – filter extracts elements according to a predicate
  – map applies a function to every element
• Their names are descriptive, in that they tell you what kind of operation the loop performs (in contrast to **while** versus **repeat** versus **for** loops in other languages)
• From now on, use map and filter in your programs