The Distribution of Student Errors Across Schools: An Initial Study

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Abstract. The little previous research comparing student errors across schools
indicates that student "bugs" do not transfer – that is, the distribution of students'
systematic errors in one school does not significantly match those in other schools.
The issue has practical implications as cognitive (or "model-tracing") tutors rely on the
modeling of student errors in order to provide targeted remediation. In this study we
examine the responses of students at three schools to a middle-school mathematics
problem. We find the same error is the most common error across all schools, and this
single error accounts for some half of all incorrect responses at each school. The top
five errors are similar across schools and account for some 2/3 of errors at each school.
We conclude that in this example, there appears to be considerable overlap of student
errors across schools.

Introduction

The study of student errors in problem solving has been an active area of research inquiry in
the fields of cognitive science, artificial intelligence and instructional technology for decades.
The little previous research done comparing student errors across schools indicates that student
"bugs" do not transfer – that is, the distribution of students' systematic errors in one school
does not significantly match those in other schools.

The idea that students in different schools might make substantially different errors on the
same problems is interesting in and of itself. From a practical perspective, the transferability of
errors has significant importance in the area of intelligent tutoring systems. The principal
method by which cognitive or model-tracing tutors [1] attempt to identify student errors is via
"bug" or "buggy" rules – that is, rules that capture expected, systematic student errors in
problem solving. As building cognitive tutors is a resource intensive enterprise, it would be
beneficial if a tutor built based on the behavior of students at one institution could be
implemented with little or no modification at other schools. Indeed, the proponents of
constraint-based modeling tutors (CBMTs) claim a distinct relative advantage for their
approach as, according to them, bug rules don't transfer well and CBMTs provide good
quality remediation without the need for bug rules [2]. The literature seems to indicate a single
study devoted to the study of the transfer of student errors across institutions. Payne and
Squibb (1990) [3] examined the errors made by 13-14 year-old students on algebra problems
at three (English) secondary schools. One conclusion they drew (p. 455) is that, "the rules that
do the most explanatory work in the three separate groups have surprisingly little overlap."

1. The Study

In this initial study we analyzed student responses to a single question from the Assistment e-learning and e-assessing system [4]. The question is provided below in figure 1.

Leila's Graduating Classes

The Venn diagram shows Leila's graduating classes from middle school, high school, and college. How many students graduated together from **both** Leila's middle school and high school?
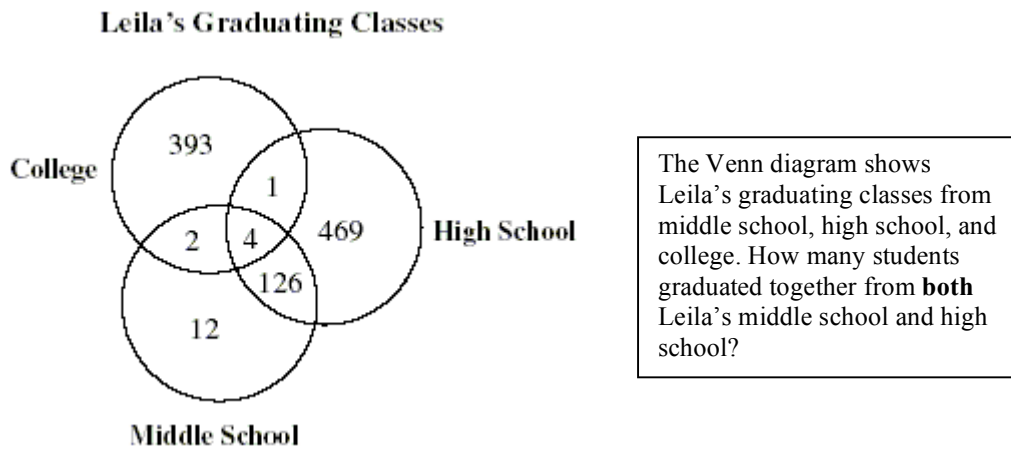
Figure 1: The Question Used In This Study

From the available data, we selected three schools from which there were more than 100 responses – that is, more than 100 students in each school had attempted the question. The three schools may be described as urban; they are all in the same general area of the same state. Additional data was available from three other schools that had fewer responses; additionally there were 418 responses that were inadvertently not associated with any school. We report the 10 most frequently occurring student responses from the three schools, and from all the data, in table 1 below.

Table 1: Top Ten Most Frequently Occurring Student Responses

| | School 1 | | | | School 2 | | |
Response	No. Students	%	Cum. %	Response	No. Students	%	Cum. %
126	54	38.85%	38.85%	126	110	34.06%	34.06%
130	17	12.23%	51.08%	130	87	26.93%	60.99%
607	11	7.91%	58.99%	607	23	7.12%	68.11%
611	5	3.60%	62.59%	614	20	6.19%	74.30%
481	5	3.60%	66.19%	611	8	2.48%	76.78%
614	3	2.16%	68.35%	481	7	2.17%	78.95%
4	3	2.16%	70.50%	4	6	1.86%	80.80%
471	3	2.16%	72.66%	1007	5	1.55%	82.35%
1007	2	1.44%	74.10%	133	4	1.24%	83.59%
715	2	1.44%	75.54%	612	3	0.93%	84.52%
Total Number of students:	139			Total Number of students:	323		
Number of unique responses:	40			Number of unique responses:	53		

| | School 3 | | | | All Schools | | |
Response	No. Students	%	Cum. %	Response	No. Students	%	Cum. %
126	36	32.73%	32.73%	126	415	39.15%	39.15%
130	31	28.18%	60.91%	130	238	22.45%	61.60%
614	9	8.18%	69.09%	607	70	6.60%	68.21%
607	7	6.36%	75.45%	614	58	5.47%	73.68%

133	6	5.45%	80.91%	481	35	3.30%	76.98%
481	4	3.64%	84.55%	611	25	2.36%	79.34%
744	2	1.82%	86.36%	4	20	1.89%	81.23%
4	2	1.82%	88.18%	133	13	1.23%	82.45%
132	2	1.82%	90.00%	1007	9	0.85%	83.30%
12732	1	0.91%	90.91%	132	9	0.85%	84.15%
Total number of students:			110	Total number of students:			1060
Number of unique responses:			20	Number of unique responses:			111

2. Discussion

A striking result is that in each school, and then across all the data, the most common student answer, 126, is both incorrect and provided by roughly 35% of the students. (The correct answer is 130.) Put another way, of the students who get this question wrong, approximately half get it wrong this way, at each school. Further we see that schools 1 and 2, as well as the data for all observations, share the same top five most occurring responses, though in slightly different orders. These five errors account for some 2/3 of the incorrect responses in school 1, school 2 and across all the data. Four of these five errors appear among the top five for school 3, and here these four also comprise approximately 2/3 of the incorrect student responses. We get similar results using the top five errors of school 3.

The distribution of student errors is highly positively skewed – that is there are many rarely occurring errors. Overlap of responses across schools seems to break down after the sixth response. The skewness of the responses concurs with previous work in the area [4, 5].

3. Conclusions and Future Work

In this case, building a cognitive tutor that recognizes the most commonly occurring error at one school allows for targeted remediation for roughly 50% of the students who get the problem wrong at any school. Capturing the top five errors at one school accounts for 66% of errors at any of them. It appears that in this case, in a practical sense, bug rules do transfer across schools. More work is needed, on more problems, across more schools, and in more domains in order to confirm these results and to explore what characteristics of the domain influence the transferability of student errors.

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