Transmission Errors

Error Detection and Correction

Advanced Computer Networks

D12

Transmission Errors Outline

- Error Detection versus Error Correction
- Hamming Distances and Codes
- Linear Codes - Parity
- Internet Checksum
- Polynomial Codes
- Cyclic Redundancy Checking (CRC)
- Properties for Detecting Errors with Generating Polynomials

Transmission Errors

- Transmission errors are caused by:
  - thermal noise {Shannon}
  - impulse noise (e..g, arcing relays)
  - signal distortion during transmission (attenuation)
  - crosstalk
  - voice amplitude signal compression (companding)
  - quantization noise (PCM)
  - jitter (variations in signal timings)
  - receiver and transmitter out of synch.

Error Detection and Correction

- error detection :: adding enough "extra" bits (redundancy) to deduce that there is an error but not enough bits to correct the error.
- If only error detection is employed in a network transmission → a retransmission is necessary to recover the frame (data link layer) or the packet (network layer).
- *At the data link layer, this is referred to as* ARQ (Automatic Repeat reQuest).

Error Detection and Correction

- **error correction ::** requires enough additional **redundant** bits to deduce what the correct bits must have been.

Examples

- Hamming Codes
- FEC = Forward Error Correction
  found in MPEG-4 for streaming multimedia.

Hamming Codes

- codeword :: a legal dataword consisting of m data bits and r redundant bits.
- Error detection involves determining if the received message matches one of the legal codewords.
- Hamming distance :: the number of bit positions in which two bit patterns differ.
- Starting with a complete list of legal codewords, we need to find the two codewords whose Hamming distance is the smallest. This determines the Hamming distance of the code.

Error Correcting Codes

Char.	ASCII	Check bits
H	1001000	00110010000
a	1100001	10111001001
m	1101101	11101010101
m	1101101	11101010101
i	1101001	01101011001
n	1101110	01101010110
g	1100111	01111001111
	0100000	10011000000
c	1100011	11111000011
o	1101111	10101011111
d	1100100	11111001100
e	1100101	00111000101

Order of bit transmission

Note
Check bits occupy power of 2 slots

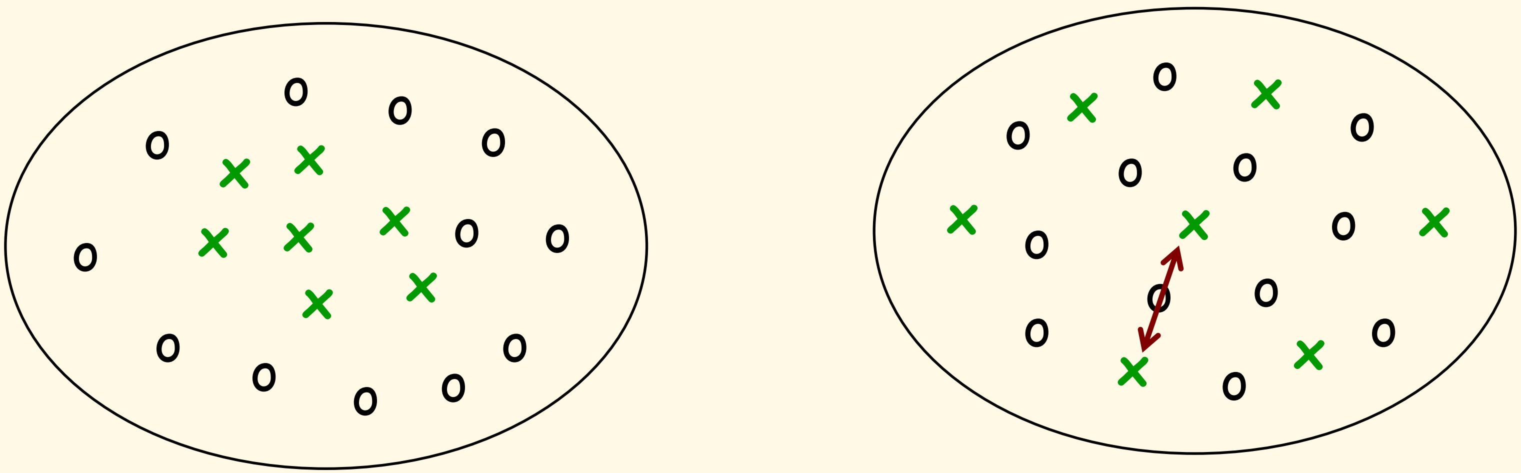
Figure 3-7. Use of a Hamming code to correct burst errors.

Tanenbaum

Hamming Distance

(a) A code with poor distance properties

(b) A code with good distance properties



x = codewords   o = non-codewords

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Hamming Codes

- To detect *d* single bit errors, you need a *d+1* code distance.
- To correct *d* single bit errors, you need a *2d+1* code distance.

→ In general, the price for redundant bits is too expensive to do error correction for network messages.

Network protocols normally use error detection and ARQ.

Error Detection

Note - Errors in network transmissions are bursty.

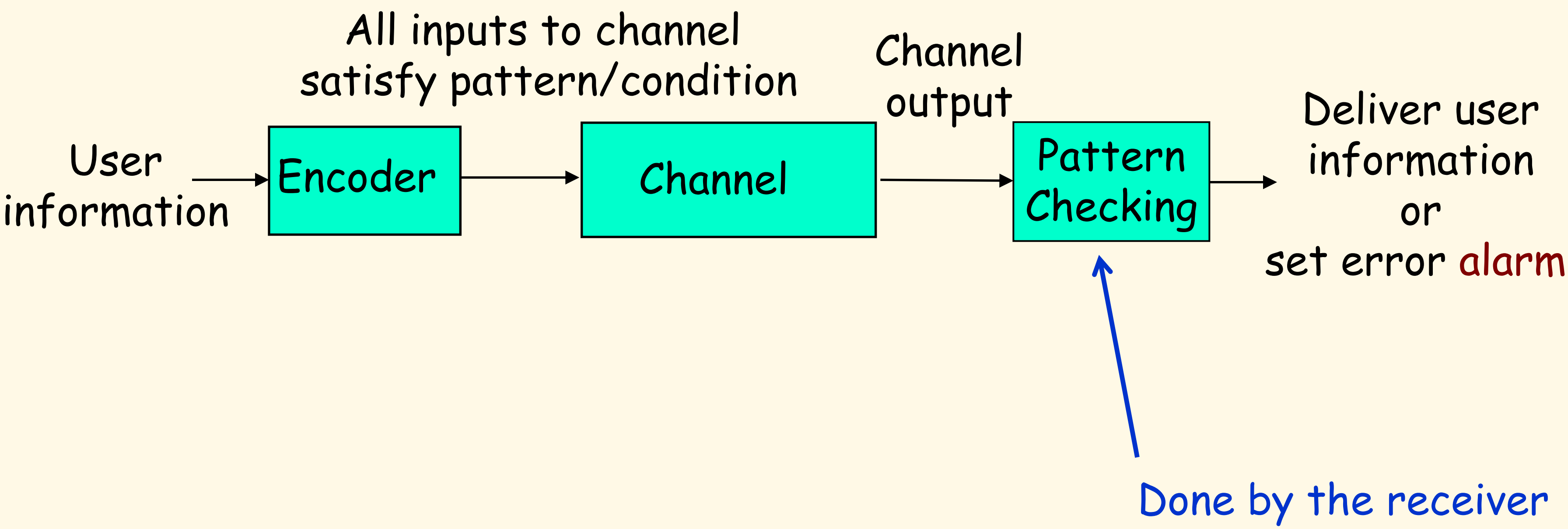
→ The percentage of damage due to errors is lower.

→ It is harder to detect and correct network errors.

- Linear codes
  - Single parity check code :: take **k** information bits and appends a single check bit to form a codeword.
  - Two-dimensional parity checks
- IP Checksum
- Polynomial Codes

  Example: CRC (Cyclic Redundancy Checking)

General Error Detection System



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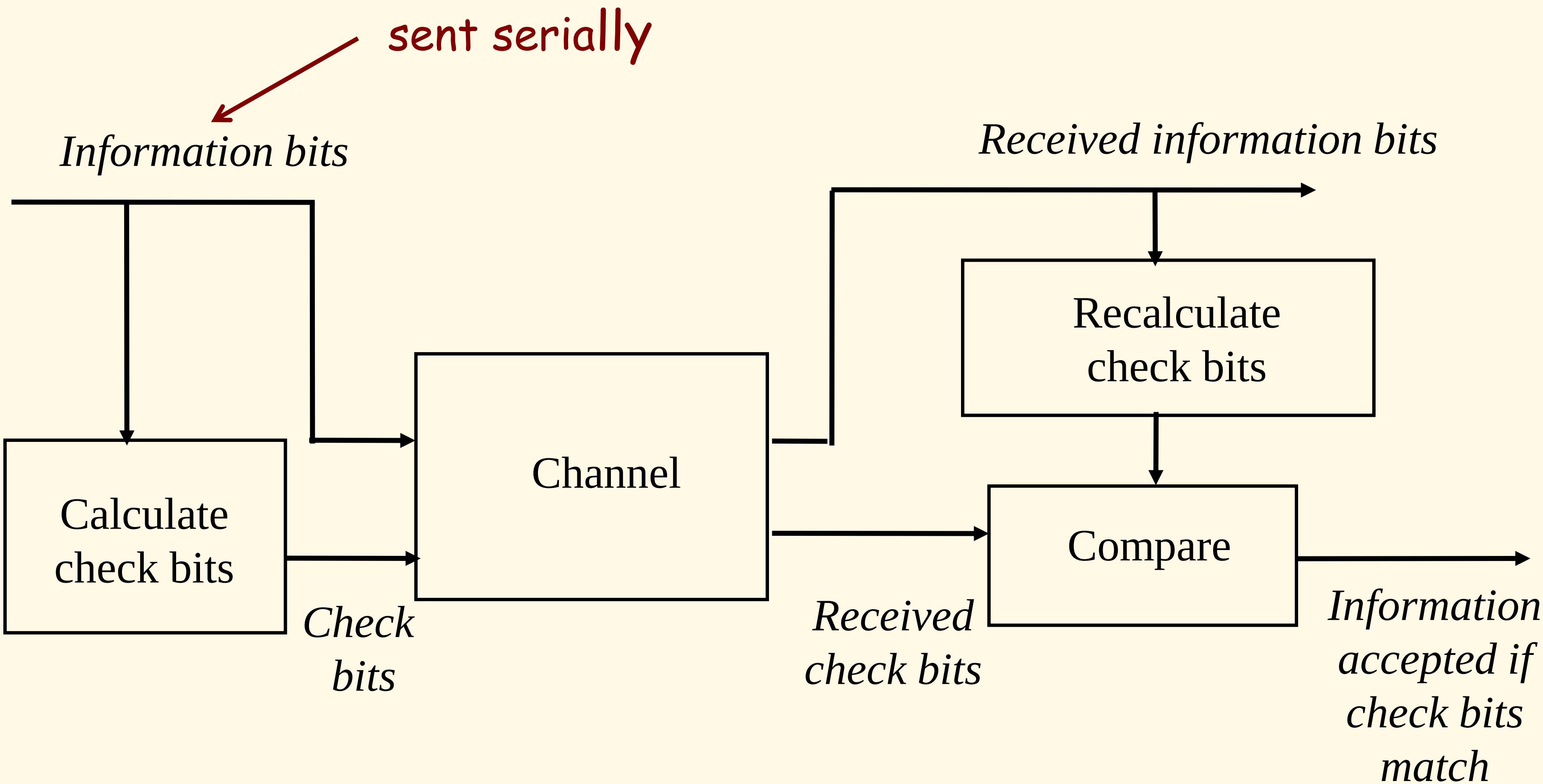
Error Detection System Using Check Bits



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Two-dimensional Parity Check Code

1	0	0	1	0	0
0	1	0	0	0	1
1	0	0	1	0	0
1	1	0	1	1	0
1	0	0	1	1	1

Last column consists of check bits for each row

Bottom row consists of check bit for each column

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Multiple Errors

One error

1	0	0	1	0	0
0	0	0	0	0	1
1	0	0	1	0	0
1	1	0	1	1	0
1	0	0	1	1	1

Two errors

1	0	0	1	0	0
0	0	0	0	0	1
1	0	0	1	0	0
1	0	0	1	1	0
1	0	0	1	1	1

Three errors

1	0	0	1	0	0
0	0	0	1	0	1
1	0	0	1	0	0
1	0	0	1	1	0
1	0	0	1	1	1

Four errors

1	0	0	1	0	0
0	0	0	1	0	1
1	0	0	1	0	0
1	0	0	0	1	0
1	0	0	1	1	1

Arrows indicate failed check bits

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Internet Checksum

```
unsigned short cksum(unsigned short *addr, int count)
    {
        /*Compute Internet Checksum for "count" bytes
         *  beginning at location "addr".
        */
    register long sum = 0;
    while ( count > 1 ) {
        /* This is the inner loop*/
             sum += *addr++;
             count -=2;
        }

        /*  Add left-over byte, if any  */
    if ( count > 0 )
        sum += *addr;

        /* Fold 32-bit sum to 16 bits */
    while (sum >>16)
        sum = (sum & 0xffff) + (sum >> 16) ;

    return ~sum;
}
```

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Polynomial Codes

- Used extensively.
- Implemented using shift-register circuits for speed advantages.
- Also called CRC (cyclic redundancy checking) because these codes generate check bits.
- Polynomial codes :: bit strings are treated as representations of polynomials with ONLY binary coefficients (0's and 1's).

Polynomial Codes

- The *k bits* of a message are regarded as the coefficient list for an information polynomial of degree *k-1*.

$$I :: i(x) = i_{k-1} x^{k-1} + i_{k-2} x^{k-2} + \ldots + i_1 x + i_0$$

Example:

$$i(x) = x^6 + x^4 + x^3$$

1 0 1 1 0 0 0

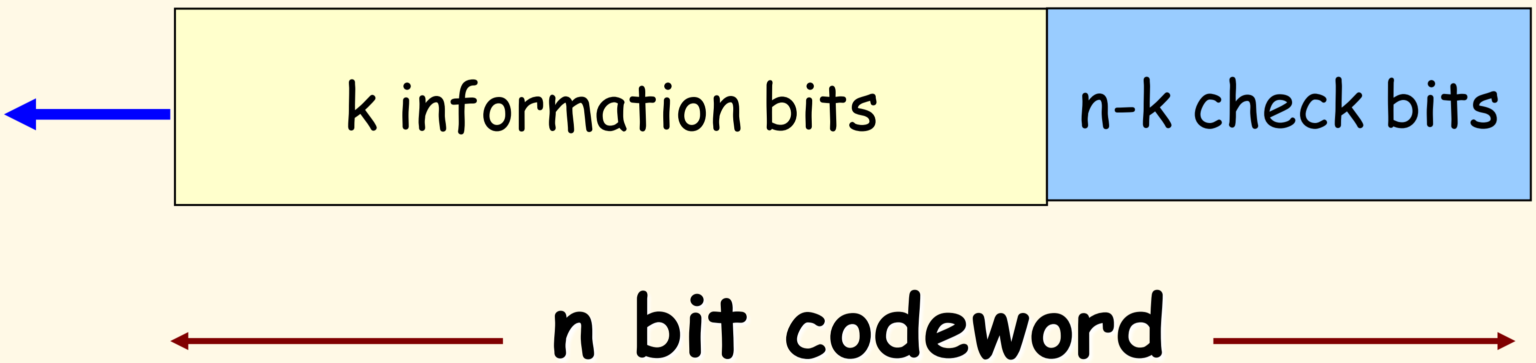
Polynomial Notation

- Encoding process takes i(x) produces a codeword polynomial b(x) that contains information bits and additional check bits that satisfy a pattern.
- Let the codeword have n bits with k information bits and n-k check bits.
- We need a generator polynomial of degree n-k of the form

$$G = g(x) = x^{n-k} + g_{n-k-1} x^{n-k-1} + \dots + g_1 x + 1$$

Note – the first and last coefficient are always 1.

CRC Codeword



Polynomial Arithmetic

Addition:
$$(x^7 + x^6 + 1) + (x^6 + x^5) = x^7 + (1+1)x^6 + x^5 + 1$$
$$= x^7 + x^5 + 1$$

Multiplication:
$$(x+1)(x^2 + x + 1) = x^3 + x^2 + x + x^2 + x + 1 = x^3 + 1$$

Division:

$$x^3 + x^2 + x \quad = \; q(x) \; \text{quotient}$$

$$\begin{array}{r|l}
x^3 + x + 1 & x^6 + x^5 \\
 & x^6 + \qquad x^4 + x^3 \\
\hline
 & \qquad x^5 + x^4 + x^3 \\
 & \qquad x^5 + \qquad x^3 + x^2 \\
\hline
 & \qquad\qquad x^4 + \qquad x^2 \\
 & \qquad\qquad x^4 + \qquad x^2 + x \\
\hline
 & \qquad\qquad\qquad\qquad x \; = r(x) \; \text{remainder}
\end{array}$$

divisor

dividend

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CRC Algorithm

CRC Steps:

1) Multiply $i(x)$ by x^{n-k} (puts zeros in $(n-k)$ low order positions)

quotient remainder

$$x^{n-k} i(x) = g(x)\, q(x) + r(x)$$

2) Divide $x^{n-k}\, i(x)$ by $g(x)$
3) Add remainder $r(x)$ to $x^{n-k}\, i(x)$

(puts check bits in the $n-k$ low order positions):

$$b(x) = x^{n-k} i(x) + r(x)$$ transmitted codeword

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CRC Example

Information: (1,1,0,0) $\Longrightarrow$ $i(x) = x^3 + x^2$

Generator polynomial: $g(x) = x^3 + x + 1$

Encoding: $x^3 i(x) = x^6 + x^5$

$$
\begin{array}{r@{\,}l}
 & x^3 + x^2 + x \\ \cline{2-2}
x^3 + x + 1\ ) & x^6 + x^5 \\
 & x^6 + \ x^4 + x^3 \\ \cline{2-2}
 & \ x^5 + x^4 + x^3 \\
 & \ x^5 + \ x^3 + x^2 \\ \cline{2-2}
 & \ x^4 + \ x^2 \\
 & \ x^4 + \ x^2 + x \\ \cline{2-2}
 & \ x
\end{array}
\qquad\qquad
\begin{array}{r@{\,}r}
 & 1110 \\ \cline{2-2}
1011\ ) & 1100000 \\
 & 1011 \\ \cline{2-2}
 & 1110 \\
 & 1011 \\ \cline{2-2}
 & 1010 \\
 & 1011 \\ \cline{2-2}
 & \boxed{010}
\end{array}
$$

Transmitted codeword:

$b(x) = x^6 + x^5 + x$

$\Longrightarrow$ $\underline{b} = (1,1,0,0,0,1,0)$

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CRC Long Division

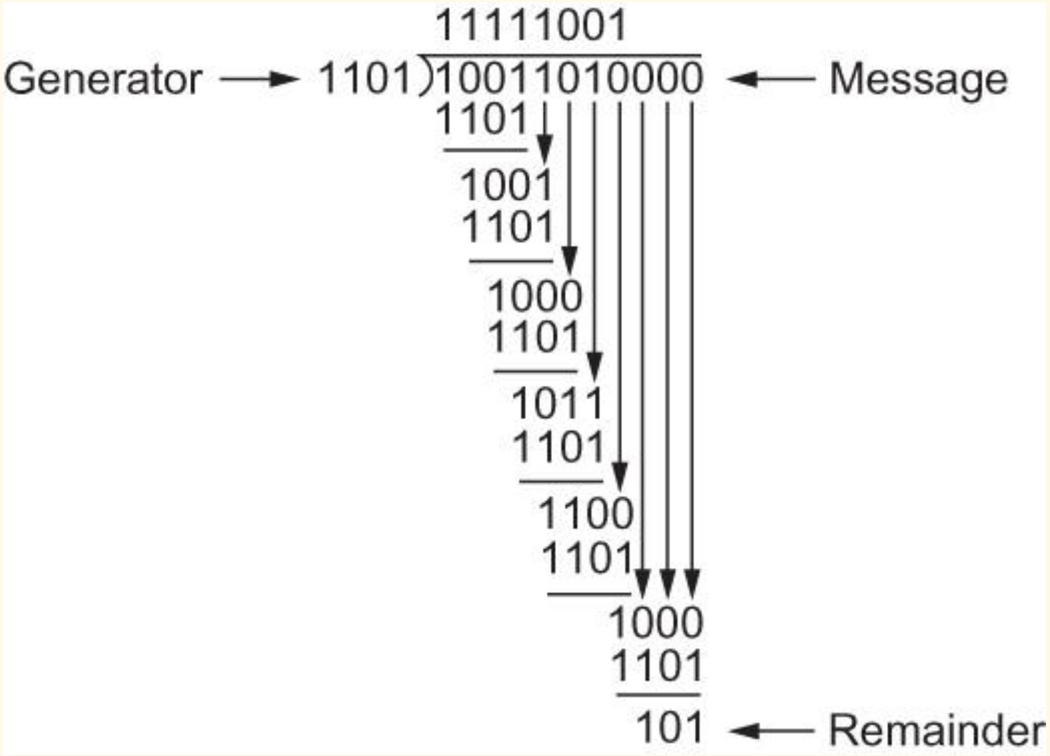

Figure 2.15 CRC Calculation using Polynomial Long Division

P&D

Generator Polynomial Properties for Detecting Errors

GOAL :: minimize the occurrence of an error going undetected.

Undetected means:

$E(x) / G(x)$ has no remainder.

GP Properties for Detecting Errors

1. **Single bit errors:** $e(x) = x^i \qquad 0 \leq i \leq n\text{-}1$

   If *g(x)* has more than one non-zero term, it cannot divide *e(x)*

2. **Double bit errors:** $e(x) = x^i + x^j \quad 0 \leq i < j \leq n\text{-}1$

   $= x^i\,(1 + x^{j-i})$

   If *g(x)* is primitive polynomial, it will not divide $(1 + x^{j-i})$ for $j\text{-}i \leq 2^{n-k} \leq 1$

3. **Odd number of bit errors: $e(1) = 1$**

   If number of errors is odd.

   If *g(x)* has $(x+1)$ as a factor, then $g(1) = 0$ and all codewords have an even number of 1s.

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GP Properties for Detecting Errors

i^{th} position

$\overbrace{\quad\quad\quad}^{L}$

4. Error bursts of length L: 000011 · $\underbrace{0001101100}_{\text{error pattern } d(x)}$ · · 0

$e(x) = x^{i}\ d(x)$ where $\deg(d(x)) = L-1$

$g(x)$ has degree $n-k$;

$g(x)$ cannot divide $d(x)$ if $\deg(g(x)) > \deg(d(x))$

if L = (n-k) or less: all will be detected

if L = (n-k+1) : deg(*d(x)*) = deg(*g(x)*)

i.e. d(x) = g(x) is the only undetectable error pattern,

fraction of bursts which are undetectable = $1/2^{L-2}$

if L > (n-k+1) : fraction of bursts which are undetectable = $1/2^{n-k}$

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Standard Generating Polynomials

Six generator polynomials that have become international standards are:

CRC-8 = x^8+x^2+x+1

CRC-10 = $x^{10}+x^9+x^5+x^4+x+1$

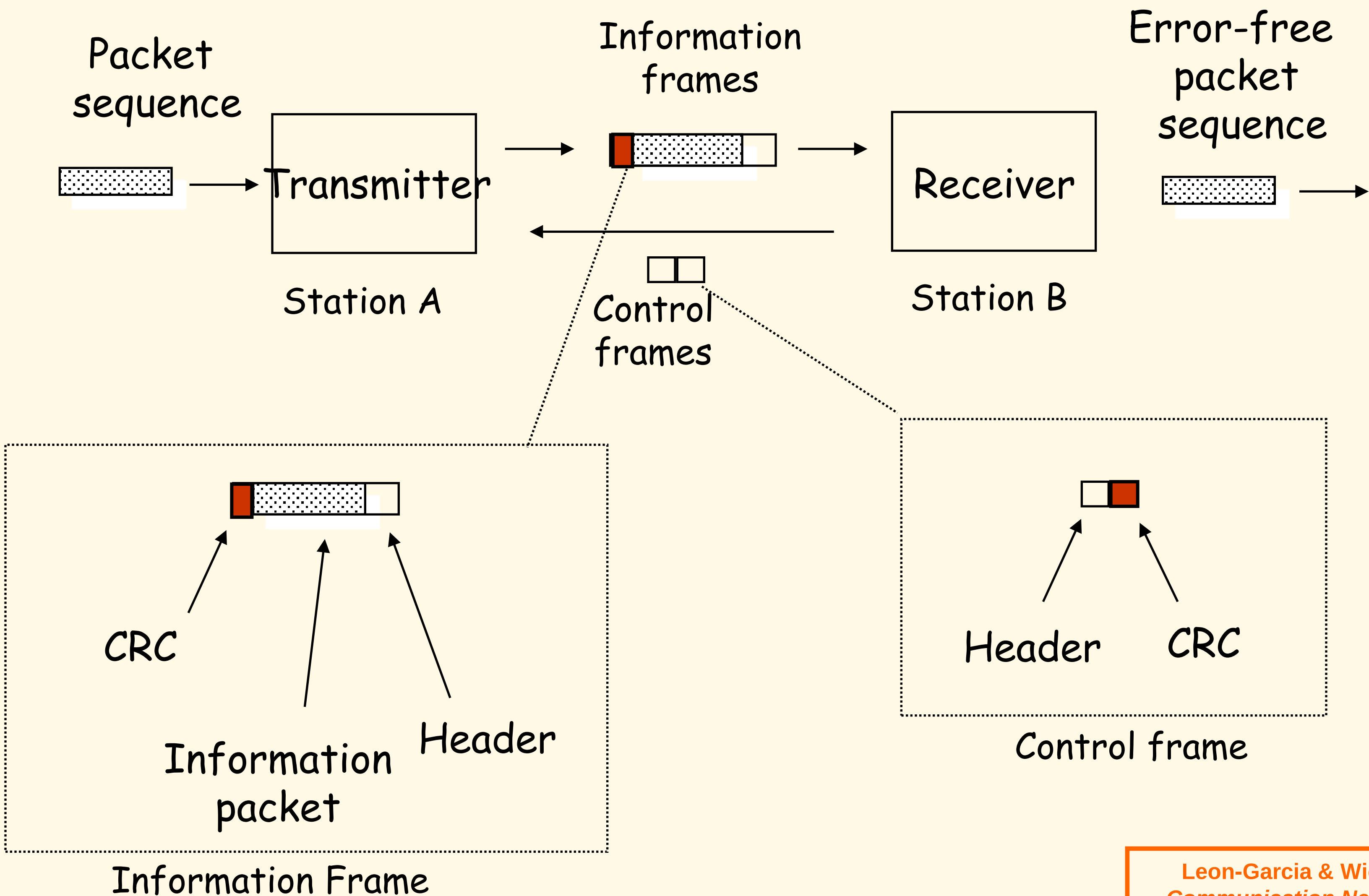
CRC-12 = $x^{12}+x^{11}+x^3+x^2+x+1$

CRC-16 = $x^{16}+x^{15}+x^2+1$

CRC-CCITT = $x^{16}+x^{12}+x^5+1$

CRC-32 = $x^{32}+x^{26}+x^{23}+x^{22}+x^{16}+x^{12}+x^{11}$
$+x^{10}+x^8+x^7+x^5+x^4+x^2+x+1$

Basic ARQ with CRC



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