COMP 408: Exercises in CTL

These exercises are intended to check your understanding of CTL. You do not need to turn in your answers, but you should make sure you can do all the problems.

Exercise 1 Draw computation trees that satisfy the following formulas:

1. $p \wedge \mathsf{AXAX}(q \vee r)$
2. $\mathsf{AFEG}p$
3. $\mathsf{AG}(p \vee (q \to r))$
4. $\mathsf{A}[p \ \mathsf{U} \ \mathsf{AG}q]$
5. $\mathsf{EX}(p \wedge \mathsf{EF}q)$
6. $\mathsf{E}[p \ \mathsf{U} \ \mathsf{A}[q \ \mathsf{U} \ r]]$

Exercise 2 Which of the following statements can be translated into CTL? If a statement can be translated, give the translation. If you feel it cannot be translated, explain why.

1. p is true in the current state and some path has q true in some future state.
2. There exists a path such that p is trye in the next state and q is true in the state after that.
3. If p is true in two consecutive states, there exists a future state in which q is true.
4. If p is true, then p remains true until q is true. Else, p must be true after at least two steps.
5. No path along which p is eventually true can ever have q or r true.
6. Whenever p is true, it cannot be true again until both q and r have been true (q and r need not have been true simultaneously).
7. It is possible to reach a state in which p is true and q is always true thereafter.
8. There does not always exist a future state in which p is true.
9. There exists a path on which q is true before p is true.
10. No state will ever be reached in which every subsequent appearance pf p is immediately followed by an appearance of q.
11. p is true infinitely often.
12. There exist at most three paths on which p is eventually true.
13. Every path on which q is eventually true at least twice has p true with the second appearance of q.
14. Along all paths, p alternates between being true and being false in each successive state.
15. Along all paths, p is true in every other state (its value is unspecified elsewhere).

Exercise 3 For each of the following pairs of formulas, determine whether the two formulas are true of the same structures $\langle S, R, L \rangle$. If you think they are true of the same structures, give an informal argument based on the semantics. If you think they are not true of the same structures, give an example of a structure in which one is true and the other is false.

1. $\mathsf{EF}p \wedge \mathsf{EG}q$ and $\mathsf{EF}(p \wedge \mathsf{EG}q)$
2. $\mathsf{AF}p \wedge \mathsf{AG}q$ and $\mathsf{AF}(p \wedge \mathsf{AG}q)$
3. $\mathsf{AF}p \wedge \mathsf{AG}q$ and $\mathsf{AG}(\mathsf{AF}p \wedge q)$
4. $\mathsf{AFAG}p \wedge \mathsf{AFAG}q$ and $\mathsf{AF}(\mathsf{AG}p \wedge \mathsf{AG}q)$
5. $\mathsf{E}[p \ \mathsf{U} \ q] \wedge \mathsf{E}[q \ \mathsf{U} \ r]$ and $\mathsf{E}[p \ \mathsf{U} \ r]$
6. $\mathsf{A}[p \ \mathsf{U} \ q] \wedge \mathsf{A}[q \ \mathsf{U} \ r]$ and $\mathsf{A}[p \ \mathsf{U} \ r]$