COMP 280 : Assignment 5 - Sample Solutions

1. (a) Transitive graph with cyles: any fully connected graph.
   (b) Transitive graph without cycles: $\{\langle a, b\rangle, \langle b, c\rangle, \langle a, c\rangle\}$.

2. A graph with a single node is both symmetric and anti-symmetric.

3. (a) *PathInR* is reflexive if the nodes of A all belong to at least one cycle, as defined by R.
   (b) *PathInR* is symmetric if the edges defined by R all belong to at least one cycle.
   (c) *PathInR* is transitive by definition.

4. This is a simple transitive closure of R:
   - $\langle a, b\rangle \in R \rightarrow \langle 1, \langle a, b\rangle\rangle \in \mathit{ReachInSteps}$
   - $\langle a, b\rangle \in R \wedge \langle n, \langle b, c\rangle\rangle \in \mathit{ReachInSteps} \rightarrow \langle n+1, \langle a, c\rangle\rangle \in \mathit{ReachInSteps}$

5. (a) We have a set F of faculty members, a set G of research groups, a set S of students, and a set C of courses. We then define four relations on these sets: $A : S \rightarrow F$ to relate students to their advisors, $B : (S \cup F) \rightarrow G$ for research group membership, $T : F \rightarrow C$ for teaching. Then:

   $$s \in S \wedge f \in F \wedge g \in G \wedge \langle s, f\rangle \in A \wedge \langle f, g\rangle \in B \rightarrow \langle s, g\rangle \in B$$

   (b) Let $L : (F \cup G \cup S \cup C) \rightarrow (F \cup G \cup S \cup C \cup \{d\})$ (where d is the department home page) be the relation describing links. Then:
   - $f \in F \wedge c \in C \wedge \langle f, c\rangle \in T \rightarrow \langle f, c\rangle \in L \wedge \langle c, f\rangle \in L$
   - $s \in S \wedge f \in F \wedge \langle s, f\rangle \in A \rightarrow \langle s, f\rangle \in L$
   - $f \in F \wedge g \in G \wedge \langle f, g\rangle \in B \rightarrow \langle f, g\rangle \in L$
   - $x \in (S \cup F) \wedge g \in G \wedge \langle x, g\rangle \in B \rightarrow \langle g, x\rangle \in L$
   - $g \in G \rightarrow \langle g, d\rangle \in L$

   (c) Every student's home page is accessible from the department home page if every student has at least one advisor, and all the faculty members who advise some students each belong to at least one research group that has at least one teaching faculty among its members.

   Justification: if all the students have at least one adviser, and all these advisers belong to at least one research group each, then, by transitivity of B, all the students will belong to at least one research group. Then, by definition of L, all the students will have at least one link from one of these research groups back to their page.

   But if all these advisers are each in a research group that has at least one teaching faculty among its members, then, by definition of L, there is at least one link from some course page to such a teaching faculty, and from such teaching faculty to the research group, and this for each adviser. So, given all the links for the courses, one can get all the pages for the teaching faculty members, then, from there, the pages for the research groups they belong to, and thus the pages of all the students.

6. Starting from the first final page, *gen-1* produces this page, then moves on to producing its antecedent pages (the ones that point directly at the first final page), since *gen-1* appends the list of these pages in front of the current list of pages to produce. So *gen-1* moves backward depth first, and the function can only generate pages arranged as a superposition of rooted trees where each page can only reference the pages that are strictly on its left in any tree or along the path toward the last root (final) page generated so far.

   Starting from the first final page, *gen-2* produces this page, then moves on to producing all the other final pages before producing the antecedent pages of the first final page, since *gen-2* appends the list of antecedent pages at the end of the current list of pages to produce. So *gen-2* moves backward breadth first, and the function can only generate pages arranged as a superposition of rooted trees where each page can only reference the pages that are strictly above it in any tree or that are at the same level and on its left.